1. Find the interval of convergence for the following power series \( \sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2} \).
   [hint: Use the ratio test and be sure to check the convergence at the end points.]

2. Find the power series for \( \frac{1}{(1-x)^2} \) at \( x = 0 \). [hint: First find the power series for \( \frac{1}{1-x} \), and take the derivative of \( \frac{1}{1-x} \). Answer: \( 1+2x+3x^2+4x^3+5x^4+O(x^5) \)]

3. If \( f(x) = \frac{1}{1+x} \).
   (a) Find the power series for \( f \) and find the interval of convergence.
   (b) Find the power series for \( \ln(1+x) \) and find the interval of convergence.
   [Answer: \( x-x^2+x^3-x^4+x^5+O(x^6) \); hint: \( \ln(1+x) \) is the antiderivative of \( f(x) \).]

4. Find the power series for \( \frac{x}{1+x^2} \) and indicate its interval of convergence.
   [Power series is \( x-x^3+x^5+O(x^6) \); first find the power series for \( \frac{1}{1+x} \) and multiply it by \( x \). The interval of convergence will not change when doing integration or differentiation, so it is \((-1,1)\).]

5. If \( g(x) = \int_0^x \frac{t}{1+t^2} dt \), (a) use substitution to find \( g(x) \), (b) find the power series for \( g(x) \) at \( x = 0 \). [hint: (a) \( \frac{1}{2} \ln(x^2+1) \), (b) find the antiderivative for problem 4 above, so the power series for \( g \) is \( \frac{1}{2}x^2-\frac{1}{4}x^4+\frac{1}{6}x^6+O(x^7) \).]

6. Find the power series for \( \ln(1-x^2) \) at \( x = 0 \). (answer: \(-x^2-\frac{1}{2}x^4-\frac{1}{3}x^6+O(x^7) \); hint: \( \ln(1-x^2) = \ln[(1-x)(1+x)] = \ln(1-x) + \ln(1+x) \).
   \( \ln(1-x) = -x-\frac{1}{2}x^2-\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5+O(x^6) \); \( \ln(1+x) = x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5+O(x^6) \)]

7. Find the first four nonzero terms of the Maclaurin series for \( \cos(x) \) (\( \sqrt{1+x} \)) by multiplying the Maclaurin series for \( \cos x \) and \( \sqrt{1+x} \) respectively. [ \( \cos x = 1-\frac{1}{2}x^2+\frac{1}{24}x^4+O(x^6) \); \( \sqrt{1+x} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+O(x^5) \); \( \cos(x) \) (\( \sqrt{1+x} \)) = \( 1+\frac{1}{2}x-\frac{5}{8}x^2-\frac{3}{16}x^3+\frac{25}{581}x^4+O(x^5) \)]

8. Sketch the parametric curve \( x = 3 \cos t, y = 4 \sin t \), \( t \in [0,2\pi] \), and find \( \frac{dy}{dx} \) and find places where the curve has horizontal tangency. [ \( \frac{dy}{dx} = -\frac{3}{4} \cot t \).]

9. Consider \( [e^t \cos t, e^t \sin t], 0 \leq t \leq \pi \),
   (a) graph the parametric equation,
(b) find \( \frac{dy}{dx} \),
(c) set up the integral to find the arclength,
(d) set up the integral to find the surface area by rotating the parametric equation around the \( x-axis \).

10. Consider a cone with radius is \( r \) and height of \( h \),
(a) set up the parametric equation for such cone,
(b) set up the integral for the surface area of such cone by rotating a proper equation.

11. Consider a circle whose center is \((0, 0)\) and radius is \( r \),
(a) set up the parametric equation for such circle,
(b) set up the integral for the circumference for the circle,
(c) set up the integral for the surface area of the sphere of radius \( r \) by rotating the top half of the circle around \( x-axis \).

12. Convert the following polar equations into equations in rectangular coordinate:
(a) \( r \cos \theta = 3 \),
(b) \( \theta = \frac{\pi}{4} \).

13. For polar graph \( r = \theta \),
(a) sketch the graph for \( r = \theta \) and \( \theta \in [0, \pi] \),
(b) find \( \frac{dy}{dx} \),
(c) set up the integral to find the area of \( r = \theta \in [0, \pi] \), [hint: \( \int_0^\pi \frac{1}{2} \theta^2 d\theta = \frac{1}{6} \pi^3 \)]
(d) set up the surface area by rotating the area of \( r = \theta \) and \( \theta \in [0, \pi] \) around \( x-axis \).
Ans: \( 2\pi \int_0^\pi \theta \sin \theta \sqrt{\theta^2 + 1} d\theta \).

14. For part of the polar graph \( r = 2 + 3 \cos \theta \), \( \theta \in [0, 2\pi] \),
(a) graph the polar equation,
(b) set up the integral to find the area,
(c) find the slope of the tangent line at \( \theta = \frac{\pi}{6} \). [hint: the slope of the tangent at \( \theta = \frac{\pi}{6} \) is \( \frac{dy}{dx} = \frac{dy}{d\theta} \frac{d\theta}{dx} \) at \( \theta = \frac{\pi}{6} \). Next, we note \( y = r \sin \theta = (2 + 3 \cos \theta) \sin \theta = 2 \sin \theta + 3 \cos \theta \sin \theta \), so \( \frac{dy}{d\theta} = 2 \cos \theta + 3 (-\sin^2 \theta + \cos^2 \theta) \). Similarly, we have \( x = r \cos \theta = (2 + 3 \cos \theta) \cos \theta = \)
2 \cos \theta + 3 \cos^2 \theta \quad \text{and} \quad \frac{dy}{d\theta} = -2 \sin \theta - 6 \cos \theta (\sin \theta).

Finally, we substitute \( \theta = \frac{\pi}{6} \) into \( \frac{dy}{d\theta} \).

(d) set up the integral to find the arclength. [hint: \( \int_0^{2\pi} \sqrt{(2 + 3 \cos \theta)^2 + (-3 \sin \theta)^2} \, d\theta \)].

(e) set up the surface area by rotating the polar graph around \( y = 4 \).

[Hint: Draw a picture first and note that the radius should be \( 4 - (2 + 3 \cos \theta) \sin \theta \), therefore the surface area is

\[
2\pi \int_0^{2\pi} (4 - (2 + 3 \cos \theta) \sin \theta) \sqrt{(2 + 3 \cos \theta)^2 + (-3 \sin \theta)^2} \, d\theta.
\]

(f) set up the surface area by rotating the polar graph around \( x = -4 \).

[Hint: Draw a picture first and note that the radius should be \( (2 + 3 \cos \theta) \cos \theta - (-4) \), therefore the surface area is

\[
2\pi \int_0^{2\pi} (4 + (2 + 3 \cos \theta) \cos \theta) \sqrt{(2 + 3 \cos \theta)^2 + (-3 \sin \theta)^2} \, d\theta.
\]