________1.) The point $(5, 8, q)$ is on the line passing through the points $(1, 2, 0)$ and $(3, 5, 4)$. What is the value of q?

(a) 6
(b) 7
(c) 8
(d) 4
(e) 5

________2.) What is the maximum value of

$$f(x) = x^2(x+1)^{1/3}$$

on the interval $(-\infty, 0]$?

(a) $\frac{9}{16(4^{1/3})}$

(b) $\frac{16}{25(5^{1/3})}$

(c) $\frac{25}{36(6^{1/3})}$

(d) $\frac{36}{49(7^{1/3})}$

(e) $\frac{4}{9(3^{1/3})}$

________3.) $\lim_{n\to\infty} (e^n + n)^{1/n} =$

(a) ∞
(b) e
(c) 0
(d) 2
(e) $e + 1$

__________4.) Let S denote the triangular region with vertices

$$(0,0), \quad (1,0), \quad \text{and} \quad (0,1).$$

Then $\iint_S x\sqrt{y}\,dx\,dy =$

(a) $\dfrac{35}{105}$

(b) $\dfrac{8+48\sqrt{2}}{105}$

(c) $\dfrac{35+48\sqrt{2}}{105}$

(d) $\dfrac{8}{105}$

(e) $\dfrac{20}{105}$

__________5.) Let $f(x) = \int_0^{x^2} \int_0^{t^2} e^{-u}\,du\,dt$. What is the slope of the graph of $f(x)$ at $x = 2$?

(a) $4 - e^{-16}$
(b) $4(1 - e^{-16})$
(c) $1 - e^{-4}$
(d) $1 - e^{-16}$
(e) $4(1 - e^{-4})$

__________6.) The product $(\log_3 2)(\log_2 9)$ is equal to:

(a) 3
(b) 9
(c) $9^{1/3}$
(d) $\sqrt{2}$
(e) 2

__________7.) If $g(x,y) = f(x-y) + f(x+y)$, then $\dfrac{\partial g}{\partial x} - \dfrac{\partial g}{\partial y} =$

(a) 0
(b) $2f'(x+y)$
(c) $f(1-y') + f(1+y')$
(d) $f(1-y) + f(1+y) - f(x-1) - f(x+1)$
(e) $2f'(x-y)$

__________ 8.) What is the interval of convergence of the power series

$$\sum_{n \to 0}^{\infty} \frac{x^n}{(n+1)2^n}?$$

(a) $(-1, 1)$
(b) $[-1, 1]$
(c) $[-2, 2)$
(d) $(-2, 2]$
(e) $[-\frac{1}{2}, \frac{1}{2}]$

__________ 9.) What is the volume of the solid generated by revolving the region bounded by the x–axis and the graph of $y = x - x^2$, where $0 < x < 1$, about the line $x = 2$?

(a) $\frac{\pi}{2}$
(b) $\frac{1}{4}$
(c) $\frac{17}{60}$
(d) $\frac{\pi}{6}$
(e) $\frac{17\pi}{60}$

__________ 10.) What is the directional derivative of

$$f(x, y, z) = x^3 y^2 z$$

at the point $(-1, 1, 2)$ in the direction of the vector $< 1, -2, 2 >^T$?

(a) 6
(b) 12
(c) 4
(d) $-\frac{4}{3}$
(e) $-\frac{1}{3}$

__________11.) What is the area of the region bounded by the graphs of

$$y^2 = 3 - x \text{ and } y = x - 1?$$

(a) $\frac{3}{2}$

(b) $\frac{5}{2}$

(c) $\frac{14}{3}$

(d) $\frac{9}{2}$

(e) $\frac{7}{6}$

__________12.) Which vector below is a unit normal to the surface $xy^2z = 1$ at the point $(1, -1, 1)$?

(a) $< 1, -1, 1 >^T$

(b) $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >^T$

(c) $< \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} >^T$

(d) $< 1, -2, 1 >^T$

(e) $< \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >^T$

__________13.) If $f(x) = \int_{-1}^{x} \sin(xy)\,dy$ for $x \neq 0$ and $f(0) = 0$, then

$$\lim_{x \to 0} f'(x) =$$

(a) $-\frac{1}{2}$

(b) -1

(c) 0

(d) $\frac{1}{2}$

(e) 1

__________14.) $\lim_{x\to 1}(\frac{1}{x-1} - \frac{1}{\ln x}) =$

(a) $+\infty$

(b) $-\frac{1}{2}$

(c) -1

(d) 0

(e) $\frac{1}{2}$

__________15.) $\int_2^\infty \frac{1}{x\ln^3(x)}\, dx =$

(a) $\frac{1}{3\ln^2(2)}$

(b) $\frac{1}{\ln^2(2)}$

(c) $\frac{2}{\ln^2(2)}$

(d) $\frac{3}{\ln^2(2)}$

(e) $\frac{1}{2\ln^2(2)}$

__________16.) Suppose f is continuous and

$$f(x) = 1 + \int_0^x ((f(t))^2 + 1)\, dt$$

for $0 \le x \le \frac{1}{2}$. Which of the following is equal to $f(x)$ on $[0, \frac{1}{2}]$?

(a) $\tan(x + \frac{\pi}{4})$

(b) $x^2 + 1$

(c) $\frac{1}{x^2+1}$

(d) $\tan^{-1}(x) + 1$

(e) $\tan(x) + 1$

__________17.) Which of the following is equal to $\int_0^1 \int_0^{3x} f(x,y)\,dy\,dx$ for every function f for which this integral exists?

(a) $\int_0^3 \int_{y/3}^1 f(x,y)\,dx\,dy$

(b) $\int_0^3 \int_0^{y/3} f(x,y)\,dx\,dy$

(c) $\int_0^1 \int_{3x}^3 f(x,y)\,dy\,dx$

(d) $\int_0^1 \int_0^{x/3} f(x,y)\,dy\,dx$

(e) $\int_0^3 \int_{3y}^1 f(x,y)\,dx\,dy$

__________18.) For $p > 0$ let A_p be the area of the region in the first quadrant bounded by $x = 0$, $y = x^3$, and the line normal to $y = x^3$ at (p, p^3). Then $\lim_{p\to 0} A_p =$

(a) $\frac{1}{6}$

(b) 0

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

(e) $+\infty$

__________19.) What is the minimum value of $f(x,y) = \dfrac{x^2}{9} + \dfrac{y^2}{4}$ subject to the restriction $|x + y| = 1$?

(a) $\frac{1}{13}$

(b) 0

(c) $\frac{1}{17}$

(d) $\frac{1}{9}$

(e) $\frac{1}{4}$

__________20.) If $f(x) = \int_0^x (2t - 3t^2)\,dt$, then what is the maximum value of f on $[0, 1]$?

(a) $\frac{2}{3}$

(b) $\frac{4}{27}$

(c) 0

(d) $\frac{1}{27}$

(e) $\frac{2}{27}$

__________ 21.) $\lim_{n\to+\infty}\sqrt{(1+\frac{1}{2n})^n} =$

(a) 1
(b) $e^{1/2}$
(c) e
(d) e^2
(e) $e^{1/4}$

__________ 22.) What is the smallest positive constant K such that

$$|\sin(x)-\sin(y)| \le K|x-y|$$

for all real x and y? Hint: Consider the derivative of an appropriate trig function.

(a) $\frac{2}{\pi}$
(b) $\frac{\pi}{2}$
(c) 2
(d) does not exist
(e) 1

__________ 23.) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1+x^3}\,dx\,dy =$

(a) $\frac{4\sqrt{2}}{9}$
(b) $4\sqrt{2}-2$
(c) $\frac{4\sqrt{2}}{3}$
(d) $\frac{4\sqrt{2}-2}{9}$
(e) $\frac{4\sqrt{2}-2}{3}$

__________ 24.) Suppose f is a function that has derivatives of all orders, and $f'(0)=0$. Which of the following must be equal to

$$\lim_{x\to 0}\frac{(f(x))^2-(f(0))^2}{x^2}?$$

(a) $f(0)f''(0)$
(b) 0
(c) $f''(0)$
(d) $2f''(0)$
(e) $2f(0)f''(0)$

__________25.) Which of the following vectors are orthogonal to the vector $< 1, -1, 0, -3 >^T$ and have a length less than 4?

$$x = < 1, 1, 0, 0 >^T, \quad y = < 3, -3, 0, 2 >^T, \quad \text{and} \quad z = < 1, 1, -1, 0 >^T$$

(a) x and z only
(b) y only
(c) z only
(d) x and y only
(e) $x, y,$ and z

__________26.) Which of the following is an equation of the tangent plane to the surface $z = x^2 + 4y^2$ at the point $(0, 1, 4)$?

(a) $8y - z - 4 = 0$
(b) $8y + z - 12 = 0$
(c) $y + 4z - 4 = 0$
(d) $2x^2 + 8y^2 - 8y - z + 4 = 0$
(e) $y - 1 = 0$

__________27.) If $f(x) = x^{1/\ln(x^2)}$ for $0 < x < \frac{1}{2}$ and f is continuous at $x = 0$, then $f(0) =$

(a) 1
(b) e
(c) $e^{1/2}$
(d) 0
(e) $e^{-1/2}$

__________28.) At what point does the line normal to the curve

$$x^2y^3 + y + 2 = 0$$

at $(1, -1)$ intersect the line $2x - 3y + 7 = 0$?

(a) $(3, \frac{13}{3})$
(b) $(-\frac{1}{2}, 2)$
(c) $(-5, -1)$
(d) $(-\frac{2}{3}, \frac{17}{9})$
(e) $(\frac{1}{3}, \frac{23}{9})$

__________29.) How many solutions are there to the differential equation $xy' = y$ for which $y(0) = 0$?

(a) 3
(b) infinitely many
(c) 0
(d) 1
(e) 2

__________30.) $\int_0^1 \int_0^{2y} (6y^2 - x)\, dx\, dy =$

(a) $\frac{5}{3}$
(b) 8
(c) $\frac{7}{3}$
(d) $-\frac{2}{3}$
(e) $-\frac{1}{6}$

__________31.) $\lim_{x\to+\infty} \dfrac{\int_2^x \dfrac{2t^2 + 3\ln(t)}{t + t^2}\, dt}{x} =$

(a) 0
(b) 1
(c) 3
(d) ∞
(e) 2

__________32.) Which of the following statements is true about the sum

$$S = \sum_{n=1}^{\infty} \frac{n}{3^n}?$$

Hint: Expand S and consider $S - \frac{1}{3}S$.

(a) $\frac{3}{4} \le S < 1$
(b) $S < \frac{3}{4}$
(c) $1 \le S < 3$
(d) $3 \le S < \infty$
(e) $S = \infty$

__________33.) $\int_0^3 \sqrt{9-x^2}\,dx =$

(a) 3

(b) $\dfrac{9\pi}{2}$

(c) $\dfrac{9\pi}{4}$

(d) 18

(e) 9

__________34.) If $f(x) = \sqrt{x} - 2$ for $x > 0$, then $\lim_{h\to 0} \dfrac{f(2+h) - f(2-h)}{2h} =$

(a) 1

(b) does not exist

(c) $\dfrac{\sqrt{2}}{4}$

(d) 0

(e) $\dfrac{\sqrt{2}}{8}$

__________35.) Which of the following is a necessary and sufficient condition on the real number p for the equation

$$4x^2 + 4px + 4 - 3p = 0$$

to have two distinct real roots?

(a) $p < -4$ or $p > 1$

(b) $p < 4$ and $p > -1$

(c) $p > 4$ or $p < -1$

(d) $p > 1$

(e) $p > 4$

__________36.) What is the area of the triangle with the following vertices?

$$(1,0,2), \quad (-1,1,1), \quad \text{and} \quad (0,3,2)$$

(a) $\dfrac{\sqrt{35}}{4}$

(b) $\dfrac{\sqrt{59}}{2}$

(c) $\sqrt{35}$

(d) $\sqrt{59}$

(e) $\dfrac{\sqrt{35}}{2}$

__________37.) If $F(x) = \int_0^x (x+1)e^t\,dt$ for $x > 0$, then $F'(1) =$

(a) $e - 1$
(b) e
(c) $2e - 2$
(d) $2e$
(e) $3e - 1$

__________38.) What is the limit of the convergent sequence defined recursively by

$$x_0 = 5 \text{ and } x_n = \frac{1}{2}(x_{n-1} + \frac{8}{x_{n-1}})$$

for $n \geq 1$?

(a) $-2\sqrt{2}$
(b) $-\sqrt{2}$
(c) 1
(d) $\sqrt{2}$
(e) $2\sqrt{2}$

__________39.) What is the length of the plane curve with parametric equations

$$x = \frac{2}{3}(t^2 + 6)^{3/2} \text{ and } y = 6t,$$

where $0 \leq t \leq 3$?

(a) 27
(b) 72
(c) $60\sqrt{15}$
(d) $60\sqrt{15} - \frac{144\sqrt{6}}{15}$
(e) 36

__________40.) The value of $\int_0^1 \frac{1}{1+x^4}\,dx$ lies in which of the following open intervals?

(a) $(\frac{26}{45}, \frac{31}{45})$
(b) $(\frac{31}{45}, \frac{36}{45})$
(c) $(\frac{41}{45}, \frac{46}{45})$
(d) $(\frac{36}{45}, \frac{41}{45})$
(e) $(\frac{21}{45}, \frac{26}{45})$

__________41.) $\int x^2 e^{x/2}\,dx =$

(a) $2x^2e^{x/2} - 8e^{x/2} + C$
(b) $2x^2e^{x/2} + 8e^{x/2} + C$
(c) $e^{x/2}(2x^2 + 8x + 16) + C$
(d) $e^{x/2}(2x^2 - 8x + 16) + C$
(e) $x^2e^{x/2} + C$

__________42.) If $y'' - 3y' - 4y = 0$, $y(0) = 2$, and $y'(0) = -3$, then $y(10) =$

(a) $(\frac{11}{5})e^{40} - (\frac{1}{5})e^{-10}$

(b) $(\frac{5}{11})e^{40} + (\frac{1}{11})e^{-10}$

(c) $(-\frac{1}{5})e^{40} + (\frac{11}{5})e^{-10}$

(d) $(\frac{1}{5})e^{40} + (\frac{11}{5})e^{-10}$

(e) $(\frac{11}{5})e^{40} + (\frac{1}{5})e^{-10}$

__________43.) What values of x produce a relative minimum and relative maximum, respectively, for

$$f(x) = 2x^3 + 3x^2 - 12x - 5?$$

(a) $1, -2$
(b) $-5, 0$
(c) $-2, 1$
(d) $1, -3$
(e) $2, -1$

__________44.) Let $I = \int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dy\,dx$. Which of the following integrals is equivalent to f?

(a) $\int_0^{2\pi}\int_0^{\infty} e^{-r^2}\,dr\,d\theta$
(b) $\int_0^{2\pi}\int_{-\infty}^{\infty} |r|e^{-r^2}\,dr\,d\theta$
(c) $2\int_0^{\pi/2}\int_{-\infty}^{\infty} e^{-r^2}\,dr\,d\theta$
(d) $2\int_0^{\pi/2}\int_{-\infty}^{\infty} re^{-r^2}\,dr\,d\theta$
(e) $4\int_0^{\pi/2}\int_0^{\infty} re^{-r^2}\,dr\,d\theta$

__________45.) If $f(x) = \ln \sqrt{e^{6x}(\sin 4x)}$ for $0 < x < \frac{\pi}{4}$, then $f'(x) =$

(a) $3 + 2\tan 4x$
(b) $3 - 2\tan 4x$
(c) $e^{6x}(\sin 4x)$
(d) $3 + 2\cot 4x$
(e) $3 - 2\cot 4x$

__________46.) If $f(x, y) = x^y$ for $x > 0$ and $y > 0$, then

$$\frac{\partial f}{\partial x}(4, \frac{1}{2}) + \frac{\partial f}{\partial y}(4, \frac{1}{2}) =$$

(a) 0
(b) $\frac{1}{2} - \frac{\ln 2}{16}$
(c) $\frac{1}{2}$
(d) 2
(e) $\frac{1}{4} + 2\ln 4$

__________47.) Suppose $f(x) = x^5 + 2x^3 + 7x - 4$ and f^{-1} denotes the inverse of f. Then $(f^{-1})'(6) =$

(a) $\frac{1}{20}$
(b) $\frac{1}{12}$
(c) does not exist
(d) $\frac{1}{18}$
(e) $\frac{1}{21}$

__________ 48.) Suppose $dz = (x+y)dx + (x-y+e^y)dy$ and $z(0,0) = 2$. Which of the following equals z?

(a) $\dfrac{(x+y)^2}{2} + e^y + 1$

(b) $\dfrac{(x-y)^2}{2} + 2e^y$

(c) $\dfrac{x^2}{2} + xy - \dfrac{y^2}{2} + e^y + 1$

(d) $\dfrac{x^2}{2} + xy - \dfrac{y^2}{2} + 2e^y$

(e) $\dfrac{(x-y)^2}{2} + e^y + 1$

__________ 49.) $\lim_{x\to\pi/2}((1-\sin x)\tan(x)) =$

(a) $-\infty$
(b) -1
(c) 1
(d) ∞
(e) 0

__________ 50.) If S is the region bounded by $y = x^2$ and $y = x+2$, then

$$\int\int_S x\,dx\,dy =$$

(a) $\dfrac{111}{4}$

(b) $\dfrac{9}{4}$

(c) $-\dfrac{111}{4}$

(d) $-\dfrac{9}{4}$

(e) $\dfrac{19}{12}$

__________ 51.) For what real value of $x > 1$ is $e^{2\ln(x-1)} = 4$?

(a) 3
(b) 9
(c) 17
(d) $1 + e^{-2}$
(e) $\dfrac{3+e^4}{2}$

__________52.) Let $a_n = n\sin(\frac{1}{n}) + (-1)^n \frac{\cos n}{n}$ for $n = 1, 2, \cdots$. Which statement is true of the sequence $\{a_n\}$?

(a) It converges to 0.
(b) It diverges to $+\infty$.
(c) It is unbounded and contains both arbitrarily large positive and arbitrarily negative terms.
(d) It converges to a positive number.
(e) It is bounded but does not converge.

__________53.) Let $f(x) = \frac{1}{x+1}$ for $x \neq -1$. What is the nth derivative $f^{(n)}(x)$?

(a) $n!(1+x)^{n+1}$
(b) $-\frac{n!}{(1+x)^{n+1}}$
(c) $\frac{n!}{(1+x)^{n+1}}$
(d) $-\frac{n!}{(1+x)^{n}}$
(e) $\frac{(-1)^n n!}{(1+x)^{n+1}}$

__________54.) For a real number α, consider the series $\sum_{n=1}^{\infty} n^{\alpha n}$. A necessary and sufficient condition for this series to be convergent is

(a) $\alpha < -1$
(b) $\alpha \leq -2$
(c) $\alpha < 0$
(d) $\alpha \leq 0$
(e) $\alpha \leq -1$

__________55.) $\int_0^{\infty} \frac{x+1}{(x^2+2x+2)^2}\, dx =$

(a) 1
(b) $+\infty$
(c) $\frac{1}{4}$
(d) 0
(e) $\frac{1}{2}$

__________56.) If $u = e^{x/y}$, $x = 2r - s$, and $y = r + 2s$ for $r + 2s \neq 0$, then in terms of x and y, $\dfrac{\partial u}{\partial r} =$

(a) $(\dfrac{2y - x}{y})e^{x/y}$

(b) $\dfrac{(2y - x)}{y^2}e^{x/y}$

(c) $(\dfrac{2 - x}{y})e^{x/y}$

(d) $3e^{x/y}$

(e) $\dfrac{2}{y}e^{x/y}$

__________57.) Which expression below is equal to $\int_0^1 (\int_0^{\sqrt{y}} 4x^3\, dx)\, dy$?

(a) $\int_0^1 (\int_{x^2}^1 4x^3\, dy)\, dx$

(b) $\int_0^1 (\int_0^{\sqrt{x}} 4x^3\, dy)\, dx$

(c) $\int_0^1 (\int_0^{\sqrt{x}} 4x^3\, dx)\, dy$

(d) $\dfrac{1}{4}\int_0^1 y^4\, dy$

(e) $\int_0^1 x^4\, dy$

__________58.) Let $u = < 1, 2, 1 >^T$ and $v = < 2, 0, -2 >^T$. For which of the following vectors w are the three vectors u, v, and w orthogonal?

(a) $< 4, 4, 4 >^T$

(b) $< 5, -4, 3 >^T$

(c) $< 1, -1, 1 >^T$

(d) $< 0, 1, -2 >^T$

(e) $< 0, -1, 0 >^T$

__________59.) Let f be differentiable at $x = 0$ and $f'(0) = 2$. The $\lim_{h \to 0} \dfrac{f(h) - f(-2h)}{2h} =$

(a) 1

(b) 2

(c) 3

(d) -1

(e) 0

__________60.) What is the largest possible subset of the real number line that can be a domain for the real–valued function

$$f(x) = \sqrt{x^3 - x}?$$

(a) $(-\infty, -1] \cup [1, \infty)$
(b) $(-\infty, -1] \cup \{0\} \cup [1, \infty)$
(c) $[-1, 0] \cup [1, \infty)$
(d) $[1, \infty)$
(e) $\{0\} \cup [1, \infty)$

__________61.) If $f(x) = \dfrac{xe^{3x}}{1+x}$ for $x \neq -1$, then $f'(1) =$

(a) $4e^3$
(b) $\dfrac{7e^3}{4}$
(c) $\dfrac{3e^3}{4}$
(d) $\dfrac{5e^3}{4}$
(e) $\dfrac{9e^3}{4}$

__________62.) If $u = \sqrt{x^2+9}$ and $v = 3x^2 - 2x$, then what is $\dfrac{du}{dv}$ as a function of x?

(a) $\dfrac{3x-1}{2\sqrt{x^2+9}}$
(b) $\dfrac{2x(3x-1)}{\sqrt{x^2+9}}$
(c) $\dfrac{x(3x-1)}{2\sqrt{x^2+9}}$
(d) $\dfrac{x}{2(3x-1)\sqrt{x^2+9}}$ for $x \neq \dfrac{1}{3}$
(e) $\dfrac{1}{4(3x-1)\sqrt{x^2+9}}$ for $x \neq \dfrac{1}{3}$

__________ 63.) What is the average value of xe^{x^2} on the interval $[2, 4]$?

(a) $2e^{16} + e^4$

(b) $\dfrac{e^{16} - e^4}{4}$

(c) $\dfrac{e^{16} - e^4}{2}$

(d) $\dfrac{2e^{16} - e^4}{2}$

(e) $2e^{16} - e^4$

__________ 64.) If $F(x)$ is a strictly decreasing continuously differentiable function on the closed interval $[a, b]$, then $\int_a^b |F'(x)|\,dx$ must equal

(a) $|F(a)| - |F(b)|$

(b) $F(-b) - F(-a)$

(c) $F(a) - F(b)$

(d) $|F(b)| - |F(a)|$

(e) $F(b) - F(a)$

__________ 65.) What is the directional derivative of $f(x, y) = 5 - 4x^2 - 3y$ at (x, y) toward $(0, 0)$?

(a) $\dfrac{-8x - 3}{\sqrt{64x^2 + 9}}$

(b) $8x^2 + 3y$

(c) $\dfrac{8x^2 + 3y}{\sqrt{x^2 + y^2}}$

(d) $-8x - 3$

(e) $\dfrac{-8x^2 - 3y}{\sqrt{x^2 + y^2}}$

__________ 66.) Let $f(x) = 2x + 1$ for $0 \leq x \leq 1$. If the interval $[0, 1]$ is partitioned into 4 subintervals of equal length, then what is the smallest Riemann sum for $f(x)$ and this partition?

(a) $\dfrac{15}{8}$

(b) 2

(c) $\dfrac{7}{2}$

(d) 7

(e) $\dfrac{7}{4}$

__________67.) The equations below define a line

$$x + y = 1, \text{ and } x + z = 1.$$

Which of the following equations defines a plane that is perpendicular to this line?

(a) $x + z = 1$
(b) $y - z = 1$
(c) $x - y - z = 0$
(d) $2x + y + z = 2$
(e) $x + y = 1$

__________68.) $\lim_{x \to 0} \dfrac{x + \ln(1 - x)}{x - \ln(1 + x)} =$

(a) -1
(b) 0
(c) 1
(d) 2
(e) ∞

__________69.) $\sum_{k=1}^{\infty} \dfrac{2^{k+2}}{3^k} =$

(a) 8
(b) 4
(c) 6
(d) 12
(e) ∞

__________70.) $\int_0^{\pi} x \sin(x)\, dx =$

(a) π^2
(b) π
(c) $-\dfrac{\pi^2}{2}$
(d) $-\pi$
(e) 0

__________71.) Let f be a function with Taylor series converging to $f(x)$ for all real numbers x. If

$$f(0) = 2, \quad f'(0) = 2, \quad \text{and} \quad f^{(n)}(0) = 3 \text{ for } n \geq 2,$$

then $f(x) =$

(a) $3e^x - x - 1$
(b) $3e^x + 2x - 1$
(c) $e^{3x} + 2x + 1$
(d) $e^{3x} - x + 1$
(e) $3e^x + 5x + 5$

__________72.) What is the area of the closed region bounded by

$$x = -1, \quad x = 0, \quad y = x^2, \quad \text{and} \quad y = x^3?$$

(a) $\frac{7}{12}$
(b) $\frac{1}{12}$
(c) $\frac{1}{6}$
(d) $\frac{1}{4}$
(e) $\frac{5}{12}$

__________73.) What is the maximum of $f(x, y) = x^2y$, given $x^2 + y^2 = 1$?

(a) $\frac{\sqrt{2}}{3}$
(b) $\frac{2}{3}$
(c) $\frac{2}{9}\sqrt{3}$
(d) $\frac{4}{27}\sqrt{3}$
(e) $\frac{\sqrt{6}}{9}$

__________74.) $\lim_{x\to 0^+}(\sqrt{\dfrac{1+x}{x^2}} - \dfrac{1}{x}) =$

(a) 0
(b) 1
(c) $+\infty$
(d) $\dfrac{1}{2}$
(e) $-\infty$

__________75.) Let S be the closed region bounded by the curves

$$y^2 = 2x, \text{ and } y^2 = 8 - 2x.$$

Then $\int\int_S (y^2+4)\,dx\,dy =$

(a) $\dfrac{128}{3}$
(b) $\dfrac{256}{3}$
(c) $\dfrac{256}{5}$
(d) $\dfrac{64}{5}$
(e) $\dfrac{128}{5}$

__________76.) Which of the following is an equation of the plane tangent to the surface

$$x^2 + y^2 - 3z = 2$$

at the point $(-2, -4, 6)$?

(a) $-2x - 4y + 6z = 0$
(b) $-2x - 4y + 6z - 2 = 0$
(c) $4x + 8y + 3z = 0$
(d) $4x + 8y + 3z + 22 = 0$
(e) $x + y + z - 2 = 0$

__________77.) What is the graph of $x^2 - 6y^2 = 0$?

(a) an ellipse
(b) a line through the origin
(c) the union of two intersecting straight lines
(d) a hyperbola
(e) a point

__________78.) The following functions are defined for all values of x except $x = 0$. Which of the functions can be defined so that the function is continuous at $x = 0$?

(a) $\dfrac{\tan(x)}{x}$

(b) $\sin(\dfrac{1}{x})$

(c) $\cos(\dfrac{1}{x})$

(d) $\dfrac{\sqrt{x^2}}{x}$

(e) $\dfrac{x}{x^2}$

__________79.) Which of the following series converge?

$I.\quad \sum_{m=1}^{\infty} \dfrac{\ln(m^{-3})}{m^{-3}}$

$II.\quad \sum_{m=1}^{\infty} \dfrac{\ln(3)}{3m}$

$III.\quad \sum_{m=1}^{\infty} \dfrac{m}{3^m}$

(a) II only
(b) none of the other answers is correct
(c) III only
(d) none
(e) I only

__________80.) Let $y = \sqrt{3x}$ for $x \geq 0$. What is the x–coordinate of the point on the graph of y that is closest to the point $(5, 0)$?

(a) $\dfrac{2}{7}$

(b) 1

(c) $\dfrac{5}{2}$

(d) $\dfrac{13}{2}$

(e) $\dfrac{7}{2}$

__________81.) Let $f(x) = \sqrt{x}$ for $x \geq 0$. With respect to the closed interval $[1, 4]$, what value of x satisfies the statement of the mean value theorem for derivatives?

(a) $\frac{3}{2}$

(b) 3

(c) 4

(d) $\frac{9}{4}$

(e) 1

__________82.) The closed region in the first quadrant bounded by the curves

$$y = x^{1/3}, \quad \text{and} \quad y = x^3$$

is rotated about the x–axis. What is the volume of the resulting solid?

(a) $\frac{1}{2}$

(b) $\frac{128\pi}{455}$

(c) $\frac{\pi}{2}$

(d) $\frac{32\pi}{35}$

(e) $\frac{16\pi}{35}$

__________83.) If $f(x) = \int_{\pi/2}^{x} (\sin(t))^{1/3}\, dt$, then at what value of x in the interval $[0, 2\pi]$ is $f(x)$ a maximum?

(a) $\frac{\pi}{2}$

(b) $\frac{3\pi}{2}$

(c) 2π

(d) π

(e) 0

__________84.) $\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx =$

(a) $e + 1$

(b) $e - 1$

(c) e

(d) $\frac{e-1}{3}$

(e) $\frac{e}{3}$

__________85.) Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1}$$

for all n. Then $\lim_{n\to\infty} a_n =$

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 4
(e) 0

__________86.) If $f(x) = \int_0^{x^2} \cos(t^2)\, dt,$ then $f'(x) =$

(a) $2x\cos(x^4)$
(b) $\cos(x^4)$
(c) $-4x^3\sin(x^4)$
(d) $-\sin(x^4)$
(e) $4x^3\cos(x^4)$

__________87.) What is the slope of the line tangent to the curve $y^3 - x^2y + 6 = 0$ at the point $(1, -2)$?

(a) $-\frac{2}{5}$
(b) $\frac{4}{11}$
(c) $\frac{11}{4}$
(d) 8
(e) $-\frac{4}{11}$

__________88.) Let $f(x) = \dfrac{\sin(x)}{x}$ for $x \neq 0,$ and $f(0) = 1.$ What is $f''(0)$?

(a) 0
(b) $\frac{1}{2}$
(c) does not exist
(d) $-\frac{1}{3}$
(e) $-\frac{2}{3}$

__________89.) What is the x–coordinate of the centroid of the closed region

$$R = \{(x, y) : \quad 0 \leq x \leq 1, \quad \sqrt{1 - x^2} \leq y \leq 1\}?$$

(a) $\dfrac{4}{3\pi}$

(b) $\dfrac{6}{4 - \pi}$

(c) $\dfrac{2}{3(4 - \pi)}$

(d) $\dfrac{1}{6}$

(e) $\dfrac{1}{4}$

__________90.) If $u = xy^z$ for $x, y,$ and $z > 0$, then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$$

(a) $u(1 + y\ln(z) + z\ln(y))$

(b) $u(1 + \dfrac{z^2}{x} + y\ln(z))$

(c) $u(1 + z + z\ln(y))$

(d) $3u$

(e) $u(1 + z + \dfrac{z^2}{x})$

__________91.) $\int_0^1 \int_0^1 (x + y)^2 \, dx \, dy =$

(a) $\dfrac{5}{4}$

(b) $\dfrac{4}{3}$

(c) $\dfrac{7}{2}$

(d) $\dfrac{15}{4}$

(e) $\dfrac{7}{6}$

__________92.) $\int_{-1}^{1} x^{-2}\,dx =$

(a) does not exist
(b) -2
(c) -1
(d) 1
(e) 2

__________93.) Which of the following is a solution to the differential equation

$$y\ln(y) + xy' = 0 \text{ for } x > 0?$$

(a) $x\ln(y) = 1$
(b) $xy\ln(y) = 1$
(c) $(\ln(y))^2 = 2$
(d) $-y(\ln(y))(\ln(x)) = 1$
(e) $\ln(y) + \frac{x^2}{2}y = 1$

__________94.) A rectangle is to be inscribed in a semicircle of radius 10, with one side lying on the diameter of the semicircle. What is the maximum possible area of the rectangle?

(a) $5\sqrt{2}$
(b) 50
(c) $60\sqrt{5}$
(d) 145
(e) 100

__________95.) $\int_{1}^{6} \frac{x^2+3x-5}{x^2}\,dx =$

(a) $\frac{61}{6} + 3\ln(6)$
(b) $\frac{17}{6} + 3\ln(6)$
(c) $\frac{17}{6} - 3\ln(6)$
(d) $\frac{5}{6} + 3\ln(6)$
(e) $\frac{5}{6} - 3\ln(6)$

__________96.) $\lim_{n\to\infty} 2^{-n}\ln(n) =$

(a) $\frac{1}{2}$

(b) $+\infty$

(c) 0

(d) $-\frac{1}{2}$

(e) $\frac{1}{4}$

__________97.) Let $f(x,y) = 3y^2\ln(x^3+4) + \frac{2y}{x}$ for $x > 0$ and $-\infty < y < \infty$. What value of y minimizes $\frac{\partial f}{\partial x}$ when $x = 2$?

(a) -1

(b) $-\frac{1}{3}$

(c) 0

(d) 1

(e) $\frac{1}{12}$

__________98.) $\int_0^1 \int_0^{\sqrt{1-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy =$

(a) $2\pi(e-1)$

(b) $\frac{\pi}{2}$

(c) $\frac{\pi(e-1)}{2}$

(d) $\frac{\pi}{4}$

(e) 2π

__________99.) $\int_1^3 x\ln(x)\,dx =$

(a) $-2 + \frac{9}{4}\ln(3)$

(b) $-4 + \frac{9}{2}\ln(3)$

(c) $-\frac{5}{2} + \frac{9}{2}\ln(3)$

(d) $-\frac{1}{4} = \frac{9}{4}\ln(3)$

(e) $-2 + \frac{9}{2}\ln(3)$

__________100.) Suppose that over the time period 1970–1990 the population of Central City at time t was proportional to b^t, for some $b > 0$. If the population of Central City was 36,000 inhabitants in 1970 and 48,000 inhabitants in 1980, what was the population in 1990?

(a) 58,000
(b) 60,000
(c) 62,000
(d) 64,000
(e) 56,000

__________101.) Let $f(x, y, z) = e^{xyz} + \ln(1 + x^2 + y^2 + z^2)$ for $-\infty < x, y, z < \infty$. What is the direction of maximum increase of f at the point $(1, 1, 0)$?

(a) $\frac{1}{\sqrt{17}} < 1, 1, 1 >^T$

(b) $\frac{1}{3\sqrt{2}} < 1, 1, 4 >^T$

(c) $\frac{1}{\sqrt{11}} < 1, 1, 1 >^T$

(d) $\frac{1}{\sqrt{2}} < 1, 1, 0 >^T$

(e) $\frac{1}{\sqrt{17}} < 2, 2, 3 >^T$

__________102.) What is the volume of the closed region bounded by the surfaces

$$z = 0 \text{ and } z = 4 - x^2 - y^2?$$

(a) $\frac{8}{3}\pi$

(b) 4π

(c) $\frac{32}{3}\pi$

(d) 8π

(e) 2π

__________103.) For all real x, let the functions $f, g,$ and h be defined as follows:

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 1$$

$$g(x) = x^3 + x^2 + x + 1$$

$$h(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x + 1.$$

Which of these functions are one–to–one over $(-\infty, \infty)$?

(a) g and h only
(b) $f, g,$ and h
(c) none of the other answers is correct
(d) f and g only
(e) f and h only

__________104.) Let $f(x) = 5 + 6x + 12x^2 - 2x^3 - x^4$, and let $g(x) = f'(x)$ for $-\infty < x < \infty$. At what value of x is $g(x)$ increasing most rapidly?

(a) -2
(b) $\frac{1}{2}$
(c) 1
(d) 2
(e) $-\frac{1}{2}$

__________105.) Which expression below is equal to $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{(n+i)^2}$?

(a) $\int_1^2 \frac{1}{x}\,dx$
(b) $\int_0^1 \frac{1}{x^2}\,dx$
(c) $\int_{-1}^0 \frac{1}{x}\,dx$
(d) $\int_2^3 \frac{1}{x^2}\,dx$
(e) $\int_1^2 \frac{1}{x^2}\,dx$

__________106.) Which of the following are subsequences of the sequence $\{a_n\}$ defined by

$$a_n = (-1)^n(1+\frac{1}{n}) \text{ for } n = 1, 2, \ldots?$$

$I.\quad \{b_n\},$ where $b_n = 1 + \dfrac{1}{n}$ for $n = 1, 2, \cdots$

$II.\quad \{c_n\},$ where $c_n = 1 + \dfrac{1}{2n}$ for $n = 1, 2, \cdots$

$III.\quad \{d_n\},$ where $d_n = 1 + \dfrac{1}{2n-1}$ for $n = 1, 2, \cdots$

(a) I only
(b) III only
(c) none of the other answers is correct
(d) II only
(e) none

__________107.) Let $z = x^2 + y^2$ where x and y are increasing at the constant rates of 2 units per second and 3 units per second, respectively. What is the set of points for which the time rate of change of z is 0 ?

(a) $\{(x, y): \quad 2y = -3x\}$
(b) $\{(x, y): \quad 2y = 3x\}$
(c) $\{(x, y): \quad 3y = -2x\}$
(d) $\{(x, y): \quad 3y = 2x\}$
(e) $\{(x, y): \quad x = y = 0\}$

__________108.) Which of the following is an equation of the normal line to the graph of $y^2 + 6y - x = 4$ at the point $(3, 1)$?

(a) $x - 8y = -5$
(b) $8x + y = -25$
(c) $8x + y = 25$
(d) $8x - y = 23$
(e) $x + 8y = 11$

__________109.) Let f be continuous on the closed interval $[0,1]$. Which of the following statements about f must be true?

$I.\quad \int_0^1 f(x^2)\,dx = \int_0^1 (f(x))^2\,dx$

$II.\quad \int_0^2 f(\frac{x}{2})\,dx = 2\int_0^1 f(x)\,dx$

$III.\quad \left(\int_0^1 f(x)\,dx\right)^2 = \int_0^1 (f(x))^2\,dx$

(a) none of the other answers is correct
(b) II only
(c) none
(d) I only
(e) III only

__________110.) Which of the following is an equation of the tangent plane to the surface $z = x^2 + y^2x - 2$ at the point $(1,1,0)$?

(a) $2x + 2y - z = 4$
(b) $2x + 2y + z = 4$
(c) $3x + 2y - z = 5$
(d) $3x + 2y + z = 5$
(e) $3x + 2y - z = 4$

__________111.) Which of the following are asymptotes for the graph of

$$xy + y = (x-2)^2?$$

(a) $x = -1$ and $5 - x$
(b) $x = -1, \quad y = 0, \quad$ and $\quad y = x - 5$
(c) $x = -1$ and $y = x - 5$
(d) $x = 0$ and $y = 0$
(e) $x = -1$ and $y = 0$

__________112.) Let C be the curve defined by the parametric equations

$$x(t) = \cos(e^t)$$

$$y(t) = \sin(e^t)$$

$$z(t) = e^t$$

for $0 \le t \le 2$. What is the length of C?

(a) $\int_0^2 \sqrt{1 + e^{2t}}\, dt$
(b) $e^4 - 1$
(c) $\dfrac{e^4 + 3}{2}$
(d) $\int_0^2 \sqrt{e^t(\cos(e^t) - \sin(e^t) + 1)}\, dt$
(e) $\sqrt{2}(e^2 - 1)$

__________113.) Which of the following conditions are necessary for a function f to be Riemann integrable on the closed interval $[a, b]$, where $a < b$?
$I.$ f is bounded on $[a, b]$
$II.$ f is continuous on $[a, b]$
$III.$ f is differentiable on $[a, b]$

(a) II only
(b) III only
(c) none of the other answers is correct
(d) I only
(e) none

__________114.) Let $\{a_n\}$ be the sequence such that

$$a_0 = 1 \text{ and } (n^2 + 2)a_{n+1} - (n^2 + 1)pa_n = 0 \text{ for } n \ge 0.$$

What are the values of p for which the series $\sum_{n=0}^{\infty} a_n$ is absolutely convergent?

(a) $|p| > 1$
(b) $p < -1$
(c) $|p| < 2$
(d) $|p| < \dfrac{1}{2}$
(e) $|p| < 1$

__________115.) Let $f(x,y) = x^3 + 6xy + y^3 + 3$. What are the points at which f has a relative maximum?

(a) $(-2,2)$ and $(2,-2)$
(b) $(0,0)$, $(-2,2)$, and $(2,-2)$
(c) $(-2,-2)$
(d) $(0,0)$
(e) $(0,0)$ and $(-2,-2)$

__________116.) $\lim_{n\to\infty} \dfrac{\sqrt{n}\cos(\pi e^n)}{2n+1} =$

(a) does not exist
(b) 0
(c) $\dfrac{1}{2}$
(d) 1
(e) $\dfrac{\pi}{2}$

__________117.) A rectangle with one side lying along the x–axis is to be inscribed in the closed region bounded by the lines

$$y = 0, \quad y = 3x, \quad \text{and} \quad y = 30 - 2x.$$

What is the largest possible area of such a rectangle?

(a) $\dfrac{135}{8}$
(b) 90
(c) 270
(d) $\dfrac{135}{2}$
(e) 45

__________118.) A particle moves along a straight line so that its acceleration at time t is $(t+1)^2$ centimeters per square second. The particle's position at time $t=0$ is at the origin, and its initial velocity is 1 centimeter per second. What is the position of the particle, in centimeters, at time t seconds?

(a) $\dfrac{(t+1)^4+2t+1}{3}$

(b) $\dfrac{(t+1)^4-1}{4}$

(c) $\dfrac{(t+1)^4}{12}+\dfrac{2}{3}t-\dfrac{1}{12}$

(d) $\dfrac{(t+1)^4}{12}+\dfrac{2}{3}t+\dfrac{1}{12}$

(e) $\dfrac{(t+1)^4+2t-1}{3}$

__________119.) Which of the following is the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(\sin(x))^n}{n}?$$

(a) $(-\infty, \infty)$

(b) $[-\frac{\pi}{2}, \frac{\pi}{2})$

(c) $(0, \pi)$

(d) $(-\frac{\pi}{2}, \frac{\pi}{2}]$

(e) $[-\frac{\pi}{2}, \frac{\pi}{2}]$

__________120.) $\lim_{x\to 0^+}(\sqrt{\frac{1+x}{x^2}}-\frac{1}{x}) =$

(a) $+\infty$

(b) $\frac{1}{2}$

(c) $-\infty$

(d) 0

(e) 1

__________121.) $\int_0^1 \int_{x^2}^{2x^2} x\cos(y)\,dy\,dx =$

(a) $\frac{1}{4}(\cos(1) - 1)$

(b) $\frac{1}{4}(\sin(2) - 2\sin(1))$

(c) $\frac{1}{4}(2\cos(1) - \cos(2) - 1)$

(d) $-\frac{1}{2}(\sin(2x^2) - \sin(x^2))$

(e) $\frac{1}{2}(\sin(2x^2) - \sin(x^2))$

__________122.) Let a and b be real numbers with $a < b$, and let f be a real–valued function that is defined on the interval (a, b). Which of the following statements implies f is continuous on (a, b)?

$I.$ The range of f is an interval.

$II.$ The graph of f has a highest and a lowest point.

$III.$ The graph of f intersects any horizontal line at most once.

(a) I only

(b) II only

(c) III only

(d) more than one of the other answers is correct.

(e) none

__________123.) A water tank in the shape of a right circular cone has a height of 10 feet. The top rim of the tank is a circle with a radius of 4 feet. If water is being pumped into the tank at the rate of 2 cubic feet per minute, what is the rate of change of the water depth, in feet per minute, when the depth is 5 feet? The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, where r is the radius and h is the height.

(a) $\frac{1}{2\pi}$

(b) $\frac{1}{\pi}$

(c) $\frac{2}{2\pi}$

(d) $\frac{2}{\pi}$

(e) $\frac{5}{2\pi}$

__________124.) What is the area of the closed region bounded by $y = x^2 - |x|$ and the x–axis, between $x = -1$ and $x = 1$?

(a) $\frac{5}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{12}$

(d) $\frac{1}{6}$

(e) $\frac{2}{3}$

__________125.) What is the Taylor series for the function $f(x) = e^{2x+1}$ about $x = -1$?

(a) $\sum_{n=0}^{\infty} \frac{2^n}{e^n} \frac{(x-1)^n}{n!}$

(b) $\sum_{n=0}^{\infty} \frac{2^n}{e} \frac{(x-1)^n}{n!}$

(c) $\sum_{n=0}^{\infty} \frac{2^n}{e} \frac{(x+1)^n}{n!}$

(d) $\sum_{n=0}^{\infty} \frac{2^n}{e^n} \frac{(x+1)^n}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

__________126.) If D is the triangular region in the xy–plane with vertices

$$(-2,1), \quad (1,1), \quad \text{and} \quad (1,4),$$

then $\int\int_D y\, dx\, dy =$

(a) 9

(b) 5

(c) $\frac{19}{3}$

(d) $\frac{21}{2}$

(e) 11

__________127.) Which of the following are sufficient conditions for the convergence of $\sum_{n=1}^{\infty} a_n$?

$I.$ $\lim_{n\to\infty} a_n = 0$.

$II.$ The sequence of partial sums is bounded.

$III.$ $\sum_{n=1}^{\infty} |a_n|$ converges.

(a) II only
(b) more than one of the other answers is correct.
(c) III only
(d) none
(e) I only

__________128.) Let $f(x) = e^{x^3+x^2+x}$ for $-\infty < x < \infty$, and let g be the inverse function for f. What is $g'(e^3)$?

(a) $6e^3$
(b) $\frac{1}{6e^3}$
(c) $\frac{1}{34e^{39}}$
(d) $\frac{1}{6}$
(e) 6

__________129.) Let the functions f, g, and h be defined as follows.

$$f(x) = \begin{cases} x\sin(\frac{1}{x}) \text{ if } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 \text{ if } x = 0, \end{cases}$$

$$g(x) = \begin{cases} x^2\sin(\frac{1}{x}) \text{ if } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 \text{ if } x = 0, \end{cases}$$

and

$h(x) = |x|^3$ if $-1 \le x \le 1$.

Which of these functions are differentiable at $x = 0$?

(a) g and h only
(b) f and g only
(c) f and h only
(d) none
(e) more than one of the other answers is correct

__________ 130.) Let f be a differentiable function on the open interval (a, b). Which of the following statements must be true?

$I.$ f is continuous on the closed interval $[a, b]$.

$II.$ f is bounded on the open interval (a, b).

$III.$ If $a < a_1 < b_1 < b$, and $f(a_1) < 0 < f(b_1)$, then there exists a number c such that $a_1 < c < b_1$ and $f(c) = 0$.

(a) I and II only

(b) I and III only

(c) II and III only

(d) I, II, and III

(e) the correct answer is not given among the other choices

__________ 131.) What is the average value of the function $f(x) = x\sin(x^2)$ over the interval $[2, 4]$?

(a) $\dfrac{\cos(4) - \cos(16)}{2}$

(b) $\dfrac{-\cos(4) + \cos(16)}{4}$

(c) $\dfrac{\cos(4) - \cos(16)}{4}$

(d) $\sin(4) + 2\sin(16)$

(e) $-\sin(4) + 2\sin(16)$

__________ 132.) What is the cosine of the angle between the vectors

$$< 0, -6, 8 >^T \text{ and } < 1, 1, 1 >^T?$$

(a) $\frac{1}{2}$

(b) 2

(c) $\frac{\sqrt{3}}{15}$

(d) $-\frac{3}{4}$

(e) $\frac{1}{150}$

__________133.) $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy =$

(a) $\dfrac{e-1}{3}$

(b) $\dfrac{e}{3}$

(c) 1

(d) $e-1$

(e) e

__________134.) If $f(x) = \dfrac{xe^x}{\sin(x)}$ for $0 < x < \pi$, then $f'(x) =$

(a) $\dfrac{e^x}{\cos(x)}$

(b) $\dfrac{e^x(x+1)}{\cos(x)}$

(c) $\dfrac{e^x(\sin(x)+\cos(x))}{\sin^2(x)}$

(d) $\dfrac{e^x\{x(\sin(x)+\cos(x))+\sin(x)\}}{\sin^2(x)}$

(e) $\dfrac{e^x\{x(\sin(x)-\cos(x))+\sin(x)\}}{\sin^2(x)}$

__________135.) What is the y–coordinate of the point on the curve $y = 2x^2 - 3x$ at which the slope of the tangent line is the same as that of the secant line between $x = 1$ and $x = 2$?

(a) -1

(b) 1

(c) 3

(d) 9

(e) 0

__________136.) $\int_{-1}^2 \dfrac{1}{x^3}\, dx =$

(a) $\dfrac{3}{8}$

(b) $-\dfrac{5}{12}$

(c) $\ln(8)$

(d) does not exist

(e) $\dfrac{1}{12}\ln(8)$

__________137.) The position vector for a curve is given by

$$< \cos(t), \cos(t), \sqrt{2}\sin(t) >^T.$$

What is the unit tangent vector at $t = \dfrac{\pi}{3}$?

(a) $< \dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{2}}{2} >^T$

(b) $< -\dfrac{\sqrt{3}}{2}, -\dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{2}}{2} >^T$

(c) $< \dfrac{1}{2}, \dfrac{1}{2}, \dfrac{\sqrt{2}}{2} >^T$

(d) $< -\dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{-\sqrt{2}}{2} >^T$

(e) $< -\dfrac{\sqrt{6}}{4}, -\dfrac{\sqrt{6}}{4}, \dfrac{1}{2} >^T$

__________138.) $\int_0^\infty \int_0^x (1 + x^2 + y^2)^{-2}\, dy\, dx =$

(a) does not exist

(b) $\dfrac{\pi}{8}$

(c) $\dfrac{\pi}{16}$

(d) $\dfrac{\pi}{4}$

(e) π

__________139.) The volume (in gallons) of water in a tank after t hours is given by

$$f(t) = 600 \quad \sin^2(\frac{\pi t}{12}) \text{ for } 0 \le t \le 6.$$

What is the rate of flow of water into the tank in gallons per hour?

(a) $1200\pi \cos(\dfrac{\pi t}{12}) \sin(\dfrac{\pi t}{12})$

(b) $50\pi \cos^2(\dfrac{\pi t}{12})$

(c) $600\pi \cos^2(\dfrac{\pi t}{12})$

(d) $100\pi \cos(\dfrac{\pi t}{12}) \sin(\dfrac{\pi t}{12})$

(e) $100\pi \sin(\dfrac{\pi t}{12})$

__________140.) $\int_1^\infty \frac{1}{e^x+1}\,dx =$

(a) $\ln(1+e)$
(b) $\tan^{-1}(e^{1/2})$
(c) does not exist
(d) $\ln(1+e^{-1})$
(e) $-\ln(1+e^{-1})$

__________141.) What is the slope of the tangent line to the graph of

$$\int_0^{x^2} u(\sin u)^{1/3}\,du$$

at $x = \sqrt{\frac{\pi}{2}}$?

(a) $\frac{\pi}{2}$
(b) π
(c) $\pi^{3/2}$
(d) $2\pi^{3/2}$
(e) $\frac{\pi^{3/2}}{2^{1/2}}$

__________142.) Let $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$ for x, y, z not all zero. Then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$$

(a) $-\frac{x+y+z}{(x^2+y^2+z^2)^{3/2}}$
(b) $\frac{9}{2}(\frac{1}{(x^2+y^2+z^2)^{5/2}})$
(c) $\frac{1}{(x^2+y^2+z^2)^2}$
(d) 0
(e) 1

__________143.) What is the slope of the tangent line to the curve $x^3 + y^3 - 3xy = 13$ at the point $(2, -1)$?

(a) -4

(b) $-\frac{1}{5}$

(c) 4

(d) 5

(e) -5

__________144.) $\int_0^3 \frac{x}{\sqrt{x+1}}\, dx =$

(a) $\frac{3}{2}$

(b) $\frac{9}{4}$

(c) $\frac{8}{3}$

(d) $\frac{3}{8}$

(e) $\frac{2}{3}$

__________145.) The amount of a chemical increases at a rate equal to the product of elapsed time (in minutes) and the amount of the chemical. If the initial amount of the chemical is 10 units, what is the number of units at 4 minutes?

(a) $10e^{16}$

(b) $10e^{8}$

(c) 14

(d) $10 + e^{8}$

(e) $10 + e^{16}$

__________146.) Let $u(x,y) = \sqrt{a^2x - by^2}$ for $a^2x - by^2 > 0$. Which of the following is equal to

$$b(\frac{\partial u}{\partial x}) + a(\frac{\partial u}{\partial y})?$$

(a) $ab(\frac{a-2y}{\sqrt{a^2x-by^2}})$

(b) $\frac{ab}{2}(\frac{2x-y^2}{\sqrt{a^2x-by^2}})$

(c) $ab(\frac{2x-y^2}{\sqrt{a^2x-by^2}})$

(d) $ab(\frac{x-y}{\sqrt{a^2x-by^2}})$

(e) $\frac{ab}{2}(\frac{a-2y}{\sqrt{a^2x-by^2}})$

__________147.) What is the area of the closed region in the xy –plane bounded by

$$y^2 = 4 - x \text{ and } y^2 = 4 - 4x?$$

(a) $\int_{-2}^{2}\int_{(1-y^2)/4}^{4-y^2} dx\, dy$

(b) $\int_{-2}^{2}\int_{(1-y^2)/4}^{y^2-4} dx\, dy$

(c) $\int_{-2}^{2}\int_{4-y^2}^{1-y^2/4} dx\, dy$

(d) $\int_{-2}^{2}\int_{1-y^2/4}^{4-y^2} dx\, dy$

(e) $\int_{-2}^{2}\int_{-1+y^2/4}^{y^2-4} dx\, dy$

Answers for Actuarial Exam Questions

1.) C 2.) D 3.) B 4.) D 5.) B 6.) E 7.) E 8.) C 9.) A 10.) C 11.) D 12.) C 13.) A 14.) B 15.) E 16.) A 17.) A 18.) A 19.) A 20.) B 21.) E 22.) E 23.) D 24.) A 25.) A 26.) A 27.) C 28.) B 29.) B 30.) C 31.) E 32.) A 33.) C 34.) C 35.) A 36.) E 37.) E 38.) E 39.) E 40.) D 41.) D 42.) C 43.) A 44.) E 45.) D 46.) E 47.) D 48.) C 49.) E 50.) B 51.) A 52.) D 53.) E 54.) C 55.) C 56.) B 57.) A 58.) C 59.) C 60.) C 61.) B 62.) D 63.) B 64.) C 65.) C 66.) E 67.) D 68.) A 69.) A 70.) B 71.) A 72.) A 73.) C 74.) D 75.) C 76.) D 77.) C 78.) A 79.) C 80.) E 81.) D 82.) E 83.) D 84.) D 85.) E 86.) A 87.) E 88.) D 89.) C 90.) C 91.) E 92.) A 93.) A 94.) E 95.) D 96.) C 97.) E 98.) B 99.) E 100.) D 101.) E 102.) D 103.) D 104.) E 105.) E 106.) D 107.) C 108.) C 109.) B 110.) C 111.) C 112.) E 113.) D 114.) E 115.) C 116.) B 117.) D 118.) C 119.) B 120.) B 121.) C 122.) E 123.) A 124.) B 125.) C 126.) A 127.) C 128.) B 129.) A 130.) E 131.) C 132.) C 133.) A 134.) E 135.) E 136.) D 137.) E 138.) B 139.) D 140.) D 141.) E 142.) D 143.) D 144.) C 145.) B 146.) E 147.) D