Problem Set 1.

1. If you have 10 coins, how many possible combinations of heads and tails are there for all 10 coins? Hint: how many combinations for one coin; two coins; three coins?

Here there are 2 events (heads or tails) possible for each coin. The first coin toss gives 2 possible outcomes. The second coin toss is independent of the first and also has 2 outcomes.

For 2 coins, it is 2 possible outcomes for the first coin **AND** 2 possible outcomes for the second coin

\[ 2 \times 2 = 4 \]

For three coins, there are 2 possible outcomes for the first coin **AND** 2 possible outcomes for the second coin **AND** possible outcomes for the third coin

\[ 2 \times 2 \times 2 = 2^3 = 16. \]

Here we have 10 independent occurrences each with 2 possible events:

\[ 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10} = 1024 \]

A general rule for these problems. The number of possible outcomes raised to the number of independent trials (occurrences).

2. Proteins are made up of chains of amino acids. Insulin is a relatively small protein with 53 amino acid residues. How many possible proteins of length 53 can be made with 20 possible amino acids for each position in the protein?

This is the same question as asked above. Here, we have a protein of 53 amino acid residues. Each position in that protein has 20 possible events.

\[ 20^{53} = 9.0 \times 10^{68} \]
3. Humans have 23 pairs of chromosomes. Gametes get one chromosome from each pair. How many possible gametes are possible just looking at combinations of chromosomes? Hints: Suppose an animal just has one pair of chromosomes, how many gametes are possible? In this problem, we are ignoring any sort of recombination within each pair of chromosomes.

This again is the same type of problem as above. We have 23 pairs of chromosomes (one maternal and one paternal). At meiosis a gamete will receive either the maternal or the paternal chromosome … therefore there are 2 possible events for each chromosome pair. There are 23 pair of chromosome therefore the number of possible gametes are:

\[ 2^{23} = 8,388,608 \]

As a matter of interest, Mom can make 8,388,608 possible eggs and dad can make 8,388,608 different sperm. Since a zygote is the union of egg AND sperm there are \( 8,338,608 \times 8,388,608 = 8,388,608^2 = 7,036,874,418,000 \). That is 7 quadrillion possible zygotes! And that, is even without the recombination due to crossing over! See why you and your sibling are not exactly alike (unless you are part of monozygotic twins!)

4. If you have a pair of die, how many combinations of two faces adding up to a 6 are possible? Hint: first think how many combinations of two numbers between 1 and 6 sum to six (e.g. 1 + 5, 5 + 1)?

- 1 + 5
- 5 + 1
- 2 + 4
- 4 + 2
- 3 + 3

5 different ways.

(Why not 3 + 3 twice?)

Problems 5 and 6 deal with so-called sampling without replacement and are more challenging than the other problems. We won't generally encounter this sort of problem in genetics, but they are related to conditional probability which we will encounter.
5. A standard deck of cards has 52 cards, ignoring the Joker. (If you are dealt four cards at random from a shuffled deck, how many hands are there that has all four aces? Hints: As the hand is dealt, the cards are removed from the deck. This is an example of sampling without replacement. If you are being dealt a hand, how many aces can you get on the first card? How many for the second card given that you got an ace on the first card? How about the third card given that you already have two aces? And for the fourth card?

This question is asking how many ways (combinations) you can draw 4 cards (how many hands out of the deck of 52 cards will give 4 aces).

<table>
<thead>
<tr>
<th>First draw</th>
<th>Second draw</th>
<th>Third draw</th>
<th>Fourth draw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ace of Spades</td>
<td>Ace of Clubs</td>
<td>Ace of Hearts</td>
<td>Ace of Diamonds</td>
</tr>
<tr>
<td>Or</td>
<td>Ace of Clubs</td>
<td>Ace of Diamonds</td>
<td>Ace of Hearts</td>
</tr>
<tr>
<td>Or</td>
<td>Ace of Diamonds</td>
<td>Ace of Clubs</td>
<td>Ace of Hearts</td>
</tr>
<tr>
<td>Or</td>
<td>Ace of Clubs</td>
<td>Ace of Hearts</td>
<td>Etc.</td>
</tr>
</tbody>
</table>

For the first draw, there are 4 possible choices. On the second draw, there are 3 possible choices so for the first and second draw there are 4 x 3 possible combinations. For four cards there are 4 x 3 x 2 x 1 = 4! = 24 possible combinations.

6. How many possible hands of four cards are there? Hint note that there are 52 ways of selecting the first card, 51 one ways of selecting the second card etc.

We will draw four cards (without replacement) from a universe of 52 cards. If order matters there are 52 choices for the first card AND 51 choices for the second card AND 50 choices for the third card AND 49 choices for the fourth card giving:

\[ 52 \times 51 \times 50 \times 49 = 6,497,400 \text{ permutations}. \]

However, we don't care about order of draw. Knowing there are 4! = 24 combinations that will result in the same cards we can divide.

\[ 6,497,400 / 24 = 270,725 \text{ four card hands}. \]
Probability problems

7. For problem 1, what is the probability of getting all heads for the 10 coins (i.e., \(P(HHHHHHHHHH)\))?

Probability of getting a head on each flip of the coin is the same = 0.5. In order to get ten heads in a row we must get a head on the first flip \(\text{AND}\) a head on the second flip \(\text{AND}\) a head on the third flip …\(\text{AND}\) a head on the tenth flip.

\[
\begin{align*}
H \text{ AND } H \text{ AND } H \text{ AND } H \text{ AND } H \text{ AND } H \text{ AND } H \text{ AND } H \text{ AND } H \\
0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 = 0.5^{10} = 9.76 \times 10^{-4}
\end{align*}
\]

8. For the die from problem 3 and 4, what is the probability of getting two faces adding up to a 6? Hints: First list all the events in the set of all sums equaling 6 then note that for a single die \(P(1) = 1/6\) etc and use the rule for independent probabilities. For instance what is \(P(\text{first die face }=1 \text{ and second die face }=5)\)

There are 5 combinations that will result in rolling two dice and having a total value of 6. (1 + 5, 5 + 1, 2 + 4, 4 + 2, and 3 + 3). The probability of rolling any of the 5 numbers (1, 5, 2, 4 and 3) is the same for each dice (1/6) therefore the probability of each of the above 5 events occurring is equal. The probability of rolling a 1 on the first dice is 1/6. Given that we roll a 1 on the first dice the probability of rolling a 5 on the second dice is 1/6:

\[
\text{Prob(1 first dice) AND prob(5 second dice)} = 1/6 \times 1/6 = 1/36
\]

But there is more than one way to roll a total of 6. Putting them all together, we would have:

\[
\begin{align*}
\text{Prob(1 first dice) AND prob(5 second dice)} &= 1/6 \times 1/6 = 1/36 \\
\text{Or}
\text{Prob(5 first dice) AND prob(1 second dice)} &= 1/6 \times 1/6 = 1/36 \\
\text{Or}
\text{Prob(2 first dice) AND prob(4 second dice)} &= 1/6 \times 1/6 = 1/36 \\
\text{Or}
\text{Prob(4 first dice) AND prob(2 second dice)} &= 1/6 \times 1/6 = 1/36 \\
\text{Or}
\text{Prob(3 first dice) AND prob(3 second dice)} &= 1/6 \times 1/6 = 1/36
\end{align*}
\]

\[
= 1/36 + 1/36 + 1/36 + 1/36 + 1/36 = 1/36 \times 5 = 5/36 = 0.139
\]
9. A particular hypothetical human disease occurs with a probability of 0.1 in males and with a probability of 0.4 in females.

A. Assuming that the frequency of males is 0.5 and females 0.5 in a very large population, what is the probability that an individual selected at random from this population will have the disease?

In order to pick a diseased individual we need to:

Pick a Male AND have him affected OR Pick a Female AND have her affected.

Prob (Male AND affected) OR prob(Female AND affected) =

\((0.5 \times 0.1) + (0.5 \times 0.4) = 0.05 + 0.2 = 0.25\)

B. What is the probability that an individual will be male and have the disease? Hint: One way to do this is to use Bayes' theorem but the problem is better solved with a tree diagram.

We solved this problem above. Note the approach using the tree diagram below