

lect14-graph-marriage

- Questions, hw1, hw12
- FSM vs (right-regular) grammars vs. regular expressions
- The Marriage Problem
- Intro to Graphs
 - def'n
 - terms
 - representations

A FSM for recognizing floats.

Grammar \rightarrow
reg.exp. \rightarrow

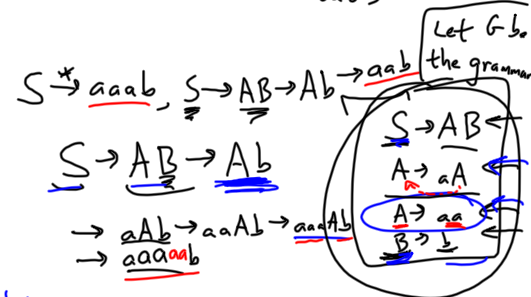
$x = 789.j$
 $x = .3j$
 $x = -.3j$

DD^* or $DD^*.DD^*$
digit 0 or more digits
or $-DD^*$ or $-DD^*.DD^*$

$([+-]DD^*)(.DD^*)$
↑ "one of" ↑ optional

$([+-]D^*.DD^*)$

A Grammar for floats



Let f' be:

$S \rightarrow aas$
 $S \rightarrow ab$
 $S \rightarrow b$

aaa^*b is a regular expression corresponding to $L(G) = \{aab, aaab, aaaab, \dots\}$

What is a grammar generating all strings match the reg. exp.

+

$$([+-])D^*N^*F$$

$S \rightarrow V$
 $S \rightarrow GV$
 $G \rightarrow +$
 $G \rightarrow -$
 $F \rightarrow \cdot DN$

$D \rightarrow 0$
 $D \rightarrow 1$
 $D \rightarrow 2$
 $D \rightarrow 3$
 \vdots
 $D \rightarrow 9$

$V \rightarrow N$
 $V \rightarrow NF$
 $N \rightarrow E$
 $N \rightarrow DN$

Th'm: Given any r.e. R, there exists a Grammar G such that $L(G) = L(R)$

Def'n: A right-regular grammar is a grammar where each rule:

- has a single non-terminal on l.h.s.
- the r.h.s. has only at most one nonterminal which occurs right-most.

Th'm: For any reg. exp R, there is a FSM M such that $L(M)$ (all sequences of input which lead to a designated "end state") = $L(R)$.

Btw - Both th'ns can go the other direction

Graphs (Rosen chpt. 9)
 def'n (chpt. 8 in Sed.)
 terms
 basic notions

Def'n: A Graph $G = \langle V, E \rangle$ where
 V is a set of vertices,
 and $E \subseteq V \times V$

Ex: $V = \{ROA, CLT, LAX, ORD\}$
 $E = \{ \langle ROA, CLT \rangle, \langle ROA, ORD \rangle, \langle LAX, ORD \rangle, \dots \}$

A graph is undirected if $\forall x, y \in V$
 $\langle x, y \rangle \in E \rightarrow \langle y, x \rangle \in E$.

Def'n: ^{"out"} degree of a vertex in a graph G :
 For any $v \in V$,
 $out\text{-deg}(v) = |\{w \mid \langle v, w \rangle \in E\}|$

Def'n: A path in $G = \langle V, E \rangle$ is
 a sequence of vertices
 $v_1, v_2, v_3, \dots, v_n$
 such that $\forall i \in \{1, \dots, n-1\}, \langle v_i, v_{i+1} \rangle \in E$
 $\langle v_1, v_2 \rangle \in E$
 $\langle v_2, v_3 \rangle \in E$
 \dots
 $\langle v_{n-1}, v_n \rangle \in E$

Next time:

- algorithms on finding a path;
- spanning tree
- topological sort

Def'n: A cycle is a path
 $v_1, v_2, \dots, v_n, v_1$

Def'n: A Tree is a graph
 with no cycles.

The marriage problem:
(n men, n women)

Q: Terminate?

Yes -

proof: Each man makes
at most n moves,
so at most n^2 moves total,
so terminates in $O(n^2)$ steps.

Def'n: a matching is optimal
for k if they are matched
with k^* , and k^* is their
highest choice in any stable
matching.

Th'm: The suitor algorithm gives
each male their optimal choice.

Proof: Consider the first
step when a guy M moves away from w ,
(his optimal choice)

This happened because he was
bumped by some other guy N .

But because w is optimal for M ,
there is some stable matching S
which includes $\langle M, w \rangle$ and $\langle N, X \rangle$.

w : ... N , ... M , ... (since M got bumped)

N : ... X , ...

- w can't occur before X in N 's list: this is the point where a guy leaves their optimal choice

- w can't occur after X in N 's list: this is the point where a guy leaves their optimal choice

Lemma: If M is matched with their optimal choice w ,
then M is w 's pessimal (least preferred) choice, of any stable matching.

Proof: Consider a matching S

(which is stable, but w is matched with N . Then w : ... N , M , ...
If w preferred M to N , then S wouldn't be stable:

~~X~~