- Counting questions
- solving recurrence
- grammars + finite state machine

$$\boxed{10^{10}\cdot 10^{10} = 10^{20} = 62^{?}}$$

$$? = \left\lceil \frac{20}{\log_{10}(62)} \right\rceil = 11$$

$$\log(10^{20}) = \log(62^{?})$$

$$20\cdot\log(10) = 20 = ?\cdot\log(62)$$

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$$C(10,3) = \frac{10!}{7!\,3!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}} = 120$$

H H H — — — H H — —

So prob of flipping 10 coins and getting exactly 3 heads:

$$\frac{\#\text{desired outcomes}}{\#\text{total outcomes}} = \frac{120}{1024} \approx 12\%$$

Prob of exactly 5 heads from 10 flips

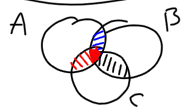
$$= \frac{\binom{10}{5}}{2^{10}} = \frac{\frac{\cancel{10}^{2}\cdot 9\cdot\cancel{8}^{2}\cdot 7\cdot\cancel{6}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}}{1024} = \frac{252}{1024} \approx 25\%$$

"Inclusion - Exclusion":

There are 1000 Redskins fans.
There are 500 with brown eyes.
→ 200 are both.
How many are either Redskins fans or brown-eyed?

$$|A\cup B| = |A| + |B| - |A\cap B|$$

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$



4 sets; how many ways to select 3 of them?

$$\binom{4}{3} = \frac{4!}{3!\,1!} = 4 = \binom{4}{1}$$

Briefly: solving recurrence relations.

Q: What is running of mergesort?

$$\big(\Theta(n \cdot \log n)\big)$$

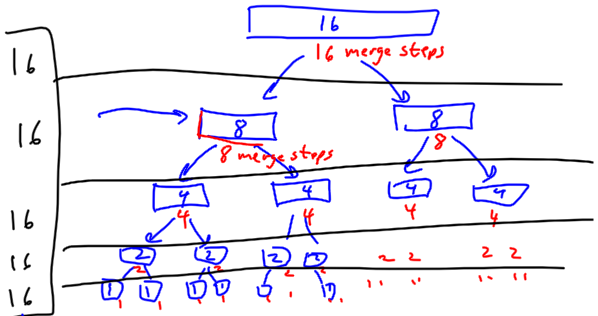
Let "$f(n)$" be the # of steps required to mergesort a list of length n.

$$f(n) = f\left(\frac{n}{2}\right) + f\left(\frac{n}{2}\right) + n$$

$$= 2f\left(\frac{n}{2}\right) + n$$

$$f(1) = 1$$

How to solve $f(n)$?



$16 \cdot \log_2(16)$ layers

In general, $n \log(n)$.

Other divide-and-conquer:

$$T(n) = aT\left(\frac{n}{k}\right) + b \cdot n^d$$

Fibonacci Numbers:

$3, -2, 1, -1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$

$2, 5, 7, 12, 19, 31, \ldots$ $g(n) = g(n-1) + g(n-2)$, $g(0) = 2$, $g(1) = 5$

How to solve $f(n) = ?$

$$f(n) = f(n-1) + f(n-2) \quad (*)$$

$$\boxed{\begin{array}{l} f(0) = 0 \\ f(1) = 1 \end{array}}$$

Cf: Amoebas: $g(n) = 2^n$

$$f(n) = 1\left(\frac{1+\sqrt{5}}{2}\right)^n + 1\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Why??

$\boxed{\text{Guess a solution of the form } f(n) = a^n.}$

If so, Then: $a^n = a^{n-1} + a^{n-2}$

$\Rightarrow \quad a^2 = a + 1$ (divide by a^{n-2})

$\Rightarrow \quad a^2 - a - 1 = 0$

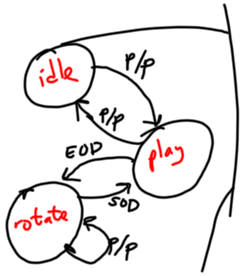
$$\Rightarrow \quad a = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

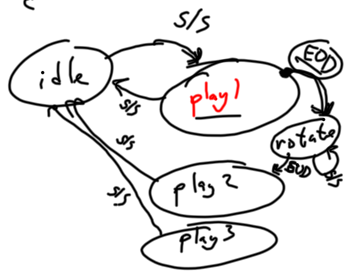
$$= \frac{1 \pm \sqrt{5}}{2}$$

Note: If a value of a makes $f(n) = a^n$ true, then $f(n) = k \cdot a^n$ also satisfies $f(n) = f(n-1) + f(n-2)$.

Grammars

Finite State Machine:





$f(idle, S/S) = play1$
$f(play1, S/S) = idle$

A FSM is:
- S, a set of states
- I, a set of inputs
- $f: S \times I \to S$
- $s_0 \in S$ the initial state

$M = \langle S, I, f, s_0 \rangle$

Write a FSM for a point of volleyball: