$\underline{0}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}$

$$\left|\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}\right|$$

$$B \times B \times B \times B \times B = \underline{B^5}$$

$$|A \times B| = |A| \times |B|$$

$$|A \cup B| = |A| + |B| \quad \text{if } A, B \text{ disjoint, i.e. } A \cap B = \emptyset$$

$$B^6 \cap B^4 = \emptyset$$

Bit Strings of length 6 or less:

$$\left|\overset{64}{B^6} \cup \overset{32}{B^5} \cup \overset{16}{B^4} \cup \overset{8}{B^3} \cup \overset{4}{B^2} \cup \overset{2}{B^1} \cup \overset{1}{B^0}\right|$$

$$= \sum_{i=0}^{6} |B^i| = 127$$

Bit strings of length $\leq 3$: $8+4+2+1 = 15$

$$[1111]_2 = [10000-1]$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} 2^n = 2^{n+1} - 1$$

# Counting

# bit strings of length ≤ 6, start & end w/ "1":

1 _ _ _ _ 1

1 choice, 2 choices, 2 choices, 1 choice {"1"}

$1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 16$

1 _ _ _ _ _ 1 $= 32$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$f(6) = 2^4$

$f(7) = 2^5$

$f(8) = 2^6$

$f(n) = 2^{n-2}$

for n ≥ 2

n bits:

$2^a \cdot 2^b = 2^{a+b}$

$2^{-1} \cdot 2^2 = 2^1$

$\frac{1}{2} \cdot 4 = 2$

1 _ 1

"11"

"1"

$f(3) = 2 = 2^{3-2}$

$f(2) = 2^{2-2} = 2^0 = 1$

$f(1) = 1 \neq 2^{1-2} = 2^{-1} = \frac{1}{2}$

$$f(n) = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ 2^{n-2} & \text{else} \end{cases}$$

$$P(n,r) = \frac{n!}{(n-r)!} \qquad P(6,6) = \frac{6!}{0!} = 6!$$

$$\underbrace{\underline{n} \cdot \underline{(n-1)} \cdot \underline{(n-2)} \cdot \cdots (r-n+1)}_{r \text{ times}} = \frac{n!}{(n-r)!}$$

Order does matter

$C(n,r)$ : order does <u>not</u> matter

"n choose r"

$$C(10,3) = \frac{P(10,3)}{3!} = \frac{10!}{7!\,3!}$$

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

$$\sum_{i=1}^{9} i = \frac{9 \cdot 10}{2}$$

$$C(10,2) = \frac{10!}{8!\,2!} = \frac{10 \cdot 9 \cdot 8!}{8!\,2!} = \frac{10 \cdot 9}{2} = 45$$

How many bit strings of length ten have exactly 3 ones?

0 0 1 0 0 1 0 1 0 0

Hi guys

$$C(10,3) = \frac{10!}{3!\,7!} = \frac{10 \cdot \cancel{9} \cdot \cancel{8}\,4 \cdot 3}{\cancel{3} \cdot \cancel{2}} = 120$$

How many bit strings of length 10, where I have exactly 7 zeroes.

$$C(10,7) = \frac{10!}{7!\,3!} = 120$$

Thm $C(n,r) = C(n, n-r)$.

MMMAAC

MMMACA

MMMCAA

MMCMAA

⋮

\# permutations but w/ repetitions

MMMAAC

MMMAAC

MACMAM

MACMAM

6! arrangements

overcounted Ms

$$\frac{6!}{2!\,3!} = \frac{6 \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{3!}\,\cancel{2!}} = 60$$

account for overcounted A

p. 367 Pascal's △

row 0: 1

row 1: 1 1

row 2: 1 2 1

1 3 3 1

1 4 6 4 1

row 5: 1 5 10 10 5 1

1 6 15 20 15 6 1

col 0, col 1, col 2

$(a+b)(a+b)$

$= 1a^2 + 2ab + 1b^2$

$(a+b)^3 = (a+b)(a^2 + 2ab + b^2)$

$= 1 \cdot a^3 + 3a^2b + \ldots b^3$

$C(5,2) = \frac{5 \cdot 4}{2} = 10$

$(a+b)(a+b)(a+b)(a+b)(a+b)$

$a^2b^3$

Pascal's triangle: the r'th entry in row n is $C(n,r)$ which is also the coefficient of $a^r b^{n-r}$, in $(a+b)^n$.

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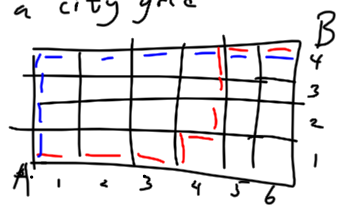
Consider a city grid

How many paths from A to B?




$C(10,4)$

EEENENNNEE

NNNNEEEEEE

---

Ordering permutations:

suppose

EENNN, 00111

ENENN, 01011

ENNEN,

# Probability:

Def'n: If there $n$ (equally likely) outcomes, and $m$ are "favorable" then the probability that the outcome is favorable is $\frac{m}{n}$.

Example: Generate a 3-character password at random. (character ← digit or a-z)

What is probability we have no digits?

$$376 = \frac{26^3}{36^3} \begin{matrix} \leftarrow m \\ \leftarrow n \end{matrix}$$

$$= \left(\frac{26}{36}\right) \gtrsim \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$\sim .3$

What is prob that a random permutation of MMMAAC spells "MCMAMA"?

(A) total permutations: $\frac{6!}{3!\,2!} = \frac{6 \cdot 5 \cdot 4}{2} = 60$

$\frac{1}{60}$

or

(B) How many perms w/ different-colored Ms, As? $6!$

How many spell "McMAMA": $3! \cdot 2!$

$$\frac{3!\,2!}{6!} = \frac{1}{60}$$

Probability that a shuffled deck is sorted A→K, clubs→spades. : $\frac{1}{52!}$

prob of royal flush:

$$\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!} = C(52,5)$$

$$\frac{\cancel{4} \cdot (\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1)}{\cancel{52} \cdot \cancel{51} \cdot \cancel{50} \cdot 49 \cdot \cancel{48}} \quad \begin{matrix} 13 \cdot 17 \cdot 5 \cdot 12 \end{matrix}$$

$$= \frac{1}{49 \cdot 5 \cdot 12 \cdot 13 \cdot 17} = \frac{1}{649740}$$

$$\frac{4 \cdot 5!}{\left(\left(\frac{52!}{\cancel{5!}}\right) \cancel{5!}\right)} = \frac{4 \cdot 5!}{52!}$$