

This proof goes
||| backwards!

$x^3 < c \cdot x^4$ premise

Don't do it!
 $1 < c \cdot x$
true, therefore
 $x^3 \in O(x^4)$

$47 = 55$

$0 = 0$
true

Raising to a power, via repeated squaring:

What is 6^8 ?

$6, 36, 216, 1296$
 $6^1, 6^2, 6^3, 6^4, \dots$

Naive version:
 8 steps

$$6^8 = (6^2)^4 = 36^4$$

$$= (36^2)^2 = 1296^2 = \underline{\underline{\quad}}$$

By repeated squaring, raising 6^n takes $\log_2(n)$ steps, if n is a perfect power of 2 (e.g. $n=128$)
 $= [10000000]_2$

What if not?

$$6^{129} = 6 \cdot 6^{128}$$

What of

$$6^{136} = 6^8 \cdot 6^{128}$$

Note: $136 = [10001000]_2$

We compute "power"

$$6^{127} = 6^{(64+32+16+8+2+1)}$$

$$= 6^4 \cdot 6^3 \cdot 6^2 \cdot 6^1 \cdot 6^1 \cdot 6^1$$

Note: $127 = [1111111]_2$

$6^{[1]}$
 $6^{[10]}$
 $6^{[100]}$, $6^{[1000]}$

Sums - tricks

$$\sum_{i=2}^5 i^2 = 2^2 + 3^2 + 4^2 + 5^2 = \sum_{i \in \{2,3,4,5\}} i^2$$

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    sst=0;
    for (int i=2; i<=5; ++i) {
        sst += i^2;
    }
  
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$$\sum_{i=0}^3 \frac{1}{2^i} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} =$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 2 - \frac{1}{8}$$

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 2$$

$$\sum_{i=0}^n 2^i = 1 + 2 + 4 + 8 + \dots + 2^n = 2^{n+1} - 1$$

$\underbrace{\quad\quad\quad}_{n \text{ digits}}$ $\underbrace{\quad\quad\quad}_{n+1 \text{ digits}}$

$$\sum_{j=0}^8 3 \cdot 2^j = 3 + 3 \cdot 2 + 3 \cdot 4 + 3 \cdot 8 + 3 \cdot 16 + \dots + 3 \cdot 2^8$$

$$= 3 \cdot \sum_{j=0}^8 2^j = 3(1 + 2 + 4 + 8 + 16 + \dots + 2^8)$$

$$= 3(2^9 - 1)$$

$$\sum_{i=0}^2 \left(\sum_{j=0}^{36} i \cdot j \right) = \sum_{i=0}^2 (i \cdot 0 + i \cdot 1 + i \cdot 2 + \dots + i \cdot 36)$$

$$= \sum_{i=0}^2 (0 + 1 + 2 + \dots + 36)$$

$$= \sum_{i=0}^2 \left(i \cdot \left(\sum_{j=0}^{36} j \right) \right)$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=50}^{100} k = 50 + 51 + \dots + 100$$

$$= (1 + 2 + \dots + 49) + 50 + 51 + \dots + 100$$

$$= \sum_{k=1}^{100} k - \sum_{l=1}^{49} l$$

$$= \left(100 \cdot \frac{100+1}{2} \right) - 49 \cdot \left(\frac{1+49}{2} \right)$$

$$\begin{aligned}
 & \sum_{i=4}^7 (2i-6) \quad \text{Set } j=i-3 \\
 & = (8-6) + (10-6) + (12-6) + (14-6) \\
 & = (8+10+12+14) - (6+6+6+6) \\
 & = 2 \sum_{i=4}^7 i - 6 \sum_{i=4}^7 1 \\
 & \quad \quad \quad \sum_{j=1}^4 2j
 \end{aligned}$$

The diagram shows the derivation of the sum $\sum_{i=4}^7 (2i-6)$. It starts with the expression $\sum_{i=4}^7 (2i-6)$ and notes the substitution $j=i-3$. The sum is then expanded as $(8-6) + (10-6) + (12-6) + (14-6)$. This is further simplified to $(8+10+12+14) - (6+6+6+6)$. The final result is expressed as $2 \sum_{i=4}^7 i - 6 \sum_{i=4}^7 1$, with a note that $\sum_{j=1}^4 2j$ is equivalent to the second term.

Induction

A rule of inference:

If you know

- $P(1)$ is true
- $\forall k. (P(k) \rightarrow P(k+1))$

Then we conclude $\forall n \in \mathbb{N}. P(n)$