$$\boxed{x^3 < c \cdot x^4}$$ premise

$\equiv$

$\equiv$

$$1 < \underline{c} \cdot \underline{x}$$

true, therefor $x^3 \in O(x^4)$

This proof goes backwards!

Don't do it!

$$47 = 55$$

$$0 = 0$$

true

Raising to a power, via repeated squaring:

What is 6^8?

$6, 36, 216, 1296$
$6^1, 6^2, 6^3, 6^4 \ldots$

Naive version: 8 steps

$6^8 = (6^2)^4 = \underline{36}^4$
$= (36^2)^2 = \underline{1296}^2 = \underline{\quad}$

By repeated squaring, raising 6^n takes $\log_2(n)$ steps, if n is a perfect power of 2 (e.g. $n = 128$) $= [10000000]_2$

What if not?

$6^{129} = 6 \cdot 6^{128}$

What of $6^{136} = 6^8 \cdot 6^{128}$

Note: $136 = [10001000]_2$
128 8

We compute "power"
6^1,
$6^{[10]}$,
$6^{[100]}$, $6^{[1000]}$,

$6^{127} = 6^{(64+32+16+8+2+1)}$
$= 6^{64} \cdot 6^{32} \cdot 6^{16} \cdot 6^{8} \cdot 6^{4} \cdot 6^{2} \cdot 6$

Note: $127 = [1111111]_2$

Sums – tricks

$$\sum_{i=2}^{5} i^2 = 2^2+3^2+4^2+5^2 = \sum_{i\in\{2,3,4,5\}} i^2$$

```
ssf=0;
for (int i=2; i≤5; ++i) {
  ssf += i²
  }
```

$$\sum_{i=0}^{3} \frac{1}{2^i} = \frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3} =$$

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = 2-\frac{1}{8}$$

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 2$$

$$\sum_{i=0}^{n} 2^i = 1+2+4+8+\cdots+2^n = 2^{n+1}-1$$

$$\underbrace{11111111}_{n \text{ digits}} = \underbrace{10000000}_{n+1 \text{ digits}}-1$$

$$\sum_{j=0}^{8} 3\cdot 2^j = 3+3\cdot2+3\cdot4+3\cdot8+3\cdot16+\cdots + 3\cdot 2^8$$

$$= 3\cdot\sum_{j=0}^{8} 2^j = 3(1+2+4+8+16+\cdots+2^8)$$
$$= 3(2^9-1)$$

$$\sum_{i=0}^{2}\left(\sum_{j=0}^{30} i\cdot j\right) = \sum_{i=0}^{2}\left(i\cdot0+i\cdot1+i\cdot2+i\cdot3\right)$$
$$= \sum_{i=0}^{2} i(0+1+2+3)$$
$$= \sum_{i=0}^{2}\left(i\cdot\boxed{\sum_{j=0}^{30} j}\right)$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$

$$\sum_{k=50}^{100} k = 50+51+\cdots+100$$

$$= 1+2+\cdots+49+50+51+\cdots+100$$
$$-(1+2+\cdots+49)$$

$$= \sum_{k=1}^{100} k - \sum_{l=1}^{49} l$$

$$= \left(100\left(\frac{100+1}{2}\right)\right) - 49\cdot\left(\frac{1+49}{2}\right)$$

Set $j = i-3$

$$\sum_{i=4}^{7} (2i-6) = 2+4+6+8$$

$$= (8-6)+(10-6)+(12-6)+(14-6)$$

$$= \sum_{i=4}^{7} 2i - \sum_{i=4}^{7} 6 \qquad = (8+10+12+14)-(6+6+6+6)$$

$$= 2\sum_{i=4}^{7} i - 6\sum_{i=4}^{7} 1$$

$$\sum_{j=1}^{4} 2j$$

Induction

A rule of inference:

If you know

- $P(1)$ is true
- $\forall k . (P(k) \rightarrow P(k+1))$

Then we conclude $\forall n \geq 1 \in \mathbb{N} . P(n)$