To start a proof: ?

- List your hypotheses

"If n odd, then $n^2$ odd"

E.g.

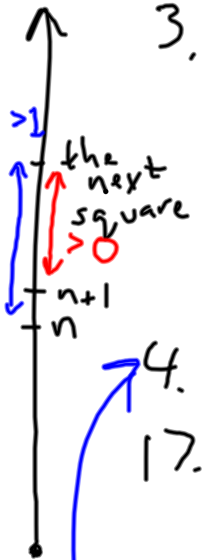
1. n is odd
2. $n = 2k+1$, for some integer k

?

3. $n^2 = (2k+1)^2 = \underline{4k^2 + 4k + 1}$
4. $n^2 = 2\underbrace{(2k^2 + 2k)}_{\text{integer}} + 1$

16. $n^2 = \underline{2j+1}$, where j is an integer
17. $n^2$ is odd

If $n$ a positive perfect square,
then $n+1$ is not a perfect square.




1. $n$ is a perfect square — hypoth
2. $n = k^2$ — def'n of perf. sq.
3. The next square bigger than $n$ is ... $(k+1)^2$

   But $(k+1)^2 - k^2 = 2k+1$

   $2k+1 > 1$ ... if $k$ is positive.
4. So: distance between $n$ and the next biggest perf. square is $> 1$.

17. $n+1$ is not a perf. square

$$(k+1)^2 - n > 1$$

$$(k+1)^2 > n+1$$

To prove "if and only if" statements:

$A \leftrightarrow B$ "iff"

Must show two things: $A \to B$

$B \to A$

A number is rational iff its decimal expansion repeats.

"A is necessary and sufficient for B"

Proof:

$A \to B$ I: ("if"): if x rational, then its decimal expansion repeats

$.142857142857142857$

$7 \overline{)\,1.00000}$

$\frac{1}{7} =$

$A \leftarrow B$

II ("only if"): Show if x's decimal repeats, then x is rational

$0.4444\overline{4} = 0.4 + 0.04 + 0.004 + 0.0004 \ldots = 4\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots\right) = 4\left(\frac{1}{1-\frac{1}{10}}\right) = 4/9$

$0.374444\overline{4} = \frac{37}{100} + \frac{4}{9} \cdot \frac{1}{100}$

$0.12345678910111213141516 \ldots$

The following are equivalent

(a) x's decimal never rpts

(b) x is irrational

(c) ....

Proof: I. (a) → (b)

⋮

II. (b) → (c)

⋮

III. (c) → (a)

⋮

# Sets (Rosen § 2.1, 2.2)

$x \in S$ "x is an element of S"

$\mathbb{R}$ – real numbers

$\mathbb{Q}$ – rationals $\{\frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$

such that

$\mathbb{Z}$ – integers $\{\ldots, -3, -2, -1, 0, 1, 3, \ldots\}$

$\mathbb{N}$ – natural numbers $= \{0, 1, 2, 3, \ldots\}$

$\mathbb{B}$ – boolean = {false, true}

$\Sigma^*$ – Strings = {"", "a", "b", ... "aa", "ab", ... "xylophone", ...}

$\Sigma$ – characters

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class List extends Object

set of values    set of values

def'n Subset: "$A \subseteq B$" cf. $\leq$

means: $\forall x. (x \in A \rightarrow x \in B)$

def'n Equal: "$A = B$" iff

$\forall x. ((x \in A) \leftrightarrow (x \in B))$

Th'm: $A = B$ iff $(A \subseteq B \wedge B \subseteq A)$

Proof:

I - (if): $(A \subseteq B \wedge B \subseteq A) \rightarrow A = B$

1. $A \subseteq B$ premise
2. $B \subseteq A$ premise
3. Take any $x$; $x \in A \rightarrow x \in B$. line 1, plus def'n of subset
4.
5. $\forall x. ((x \in A) \leftrightarrow (x \in B))$

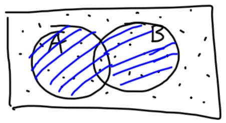
II (only if):

1.
2.
3.

$x \lhd y$

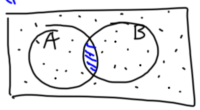
Union: $A \cup B$: cf "$\vee$"

Def'n: $x \in (A \cup B)$ iff $x \in A \vee x \in B$




Intersection: $A \cap B$

def'n $x \in (A \cap B)$ iff $((x \in A) \wedge (x \in B))$

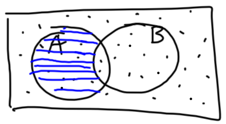



Set difference: $A - B$

def'n $x \in (A - B)$ iff $(x \in A) \wedge (x \notin B)$

$A - B$

$\equiv$ A.removeAll(B)




$\mathbb{R} - \mathbb{Q}$ : The set of irrationals

Cross Product:

$\{1, 5, 9\} \times \{\text{Ian}, \text{Penny}\}$

$= \{(1, \text{Ian}), (1, \text{Penny}),$
$(5, \text{Ian}), (5, \text{Penny}),$
$(9, \text{Ian}), (9, \text{Penny})\}$

Power Set of A: The set of all subsets of A.

$P(A)$

Example: $A = \{\text{bmw}, \text{vw}, \text{saab}\}$.

$P(A) = \{ \{\text{bmw}, \text{vw}, \text{saab}\},$
$\{\text{bmw}, \text{vw}\},$
$\{\text{bmw}, \text{saab}\},$
$\{\text{vw}, \text{saab}\},$
$\{\text{vw}\}, \{\text{bmw}\}, \{\text{saab}\},$
$\{\} \}$

size of

if $|A| = 3$, $|P(A)| = 8$.

if $|A| = n$, $|P(A)| = 2^n$

# Naive Set Theory.

The indicator function of a set: Takes in anything, returns boolean — is that thing in the set?

The constant function false ($f(x) \equiv false$) is indicator function for $\emptyset$. ← empty set.

How about: $f(x) = true$.
The universal set, $\mathcal{U}$.

$3 \in \mathcal{U}$
$Ian \in \mathcal{U}$
$\mathbb{R} \in \mathcal{U}$
$\{\} \in \mathcal{U}$

$\mathcal{U} \overset{?}{\in} \mathcal{U}$

Consider: $Bertie = \{x \mid \boxed{x \in x}\}$
Is $Bertie \in Bertie$?

Now Try: $R = \{x \mid x \notin x\}$.
Is $R \in R$?
Consider case 1, "yes $R \in R$": This means $R \notin R$ (by def'n of R).

Consider case 2, "no, $R \notin R$": This means (by def'n of R) $R \in R$

Resolution: When making a new set, $\{x \in \mathbb{Z} \mid \ldots\ldots\}$
must use ↑ some pre-existing set.