Corn and Popcorn

I. UNIT OVERVIEW & PURPOSE:

Students will investigate the popularity of corn and popcorn. They will use statistics of corn exports and popcorn sales to predict future sales. Through the use of popcorn, students will investigate the probability of getting all popcorn kernels to pop. Students will display the data in various rational numbers. The results of the popped kernels will be compared to a normal distribution. The confidence intervals will be determined using both types of popcorn. They will then investigate if a name brand of popcorn produces more popped kernels than a generic brand. Using this information, the students will make a decision on which product they would buy based on popped kernels. Lastly, students will determine which snacks are best for their figure and for their budget. They will compare nutritional facts about a variety of snacks and the cost of the snacks to determine the best choice for snacks.

II. UNIT AUTHOR:

Sarah Whitaker, Coeburn Middle School, Wise County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Number and Operations, Data Analysis and Probability

V. OBJECTIVES:

Rational Numbers, Conducting an Experiment, Sampling Distribution, Probability
Students will interpret the calculated results of their experiment and display the data in various rational numeric formats. The students will conduct an experiment to determine percentage. The experimental data will be used to determine the probability of the number recurring.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE 1. The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 9. The student will design and conduct an experiment/survey. Key concepts include

a) sample size;

b) sampling technique;
c) controlling sources of bias and experimental error;
d) data collection; and
e) data analysis and reporting.

MPE 12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

MPE 22. The student will analyze graphical displays of univariate data, including dot plots, stem plots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

MPE 23. The student will analyze the normal distribution. Key concepts include
a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data, using z-scores; and
d) area under the standard normal curve and probability.

MPE 31. The student will calculate probabilities. Key concepts include
a) conditional probability;
b) dependent and independent events;
c) addition and multiplication rules;
d) counting techniques (permutations and combinations); and
e) Law of Large Numbers.

VII. CONTENT:

This unit contains content from Algebra 1 and the Algebra Functions and Data Analysis courses. The experiment also calls upon students understanding of business concepts and scientific inquiry.

VIII. REFERENCE/RESOURCE MATERIALS:

Indian Corn Cobs
Corn KWL worksheet (attached)
Corn Trade Statistics worksheet (attached)
Sales of Unpopped Popcorn worksheet (attached)
Microwave popcorn bags (name brand and generic brand)
Popcorn Data Sheet for name brand and generic brand
Access to microwave
Variety of snacks with nutritional information and price sheet
Nutritional Data of Snacks Sheet
Cost of Snack Sheet
Classroom set of graphing calculators

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IX. PRIMARY ASSESSMENT STRATEGIES:

Students will be asked to use proportions to make predictions on color of kernels in Lesson 1. They will also be asked to make predictions based on the statistical data given to them. Students will be asked to show ability to calculate confidence intervals given p and z scores. They will be asked to explain why it is important to determine if the product on a production line is within a certain confidence interval after Lesson 2. In Lesson 3, the students will be asked to analyze information to determine healthy and budget friendly snacks. Using that information, they will have to create graphs that show the relation of nutritional facts and cost.

X. EVALUATION CRITERIA:

The students will be evaluated in class through observation. The teacher will observe if students are active in class discussions and in groups. The graphs required and lines of best fit will be checked based on the students' data. The exit slips will be evaluated to see if there is mathematical explanation in the reasoning. Students' mathematical knowledge will be evaluated based on their work and final answers. The explanations of any open ended questions will be graded on a rubric (included in Appendix).

XI. INSTRUCTIONAL TIME:

Four 90 minute classes.

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Lesson 1 “Pop”ularity of Corn

Strand
Number and Operations

Mathematical Objective(s)
In this lesson, students will use corn and popcorn statistics to practice using rational numbers. Students will use Indian corn to create ratios and proportions to determine percentage of kernel colors. They will track and predict sales of corn and popcorn by finding line of best fit.

Mathematics Performance Expectation(s)
MPE 1. The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Related SOL

A.11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

NCTM Standards
NUMBER AND OPERATIONS All students should:

- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.

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- judge the reasonableness of numerical computations and their results

ALGEBRA All students should:

- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

DATA ANALYSIS AND PROBABILITY All students should:

- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots.
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatter plots of the data and approximate lines of fit

PROCESS STANDARDS All students should:

- Build new mathematical knowledge through problem solving
- Use the language of mathematics to express mathematical ideas precisely

Additional Objectives for Student Learning (include if relevant; may not be math-related):
N/A

Materials/Resources
Indian Corn Cobs (one per group)
Corn KWL worksheet (attached)
Corn Trade Statistics worksheet (attached)
Sales of Unpopped Popcorn worksheet (attached)
Classroom set of graphing calculators
Graphing paper
Colored pencils
Rulers
Access to web and projector to show video on popcorn:
http://science.discovery.com/videos/how-its-made-popcorn.html

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- To succeed in this lesson, students should have understanding of ratios, percents, scientific notation, and proportions.
- Students may have misconceptions between the sales of corn and the sales of popcorn. The sale of corn is based on trade and the sale of popcorn is the retail price of popcorn.
- The relevant real life context in this problem is statistical information on corn and popcorn.

Introduction: Setting Up the Mathematical Task

In this lesson, you will investigate the prevalence of corn and popcorn in the United States. What do you know about corn? In what ways is corn used? What do you think is the main reason for corn production?

Student Exploration 1:

Student/Teacher Actions:

- After introducing the topic, have the students fill out the K and W of a KWL worksheet about corn.
- Discuss what the students already know about corn and what they would like to learn about corn. If students are having trouble coming up with ideas, bring up ways that corn is used, types of corn, etc.
- After the students have filled in the KWL worksheet, watch the following video about how popcorn is made: http://science.discovery.com/videos/how-its-made-popcorn.html
- Break the students into groups of two or three and give them an ear of Indian corn. The students should discuss what they predict is the percentage of each color on the ear.
- Use the following questions for class discussion: How can you determine the percentage of the kernel colors? Would you want to count the entire ear of corn?
- The students will count the kernel colors of 3 to 4 rows and create ratios of colors to total kernels and create proportions to predict how many kernels there are of each color. Calculations will need to be turned in with rest of work.
- Using the export statistics sheet, have the students create a graph of the export information of corn. Using their calculator, the students should find the line of best fit. Construct graph on paper. Students need to make sure that they write the linear equation on the line.
- Using the sales of unpopped popcorn statistics sheet, have the students create a graph of the sales of unpopped popcorn. Using their calculator, the students should find the line of best fit. Construct graph on paper. Students need to make sure that they write the linear equation on the line.

Monitoring Student Responses

- The students will demonstrate their thinking and understanding through the class discussion of how to predict how many colors there are on the ear of corn. Give the students time to recall past knowledge and discuss information from the project that is needed to complete the task.
- There may need to be clarification on finding proportions given ratios. Have a class discussion to see who can remember and help the class. If no one remembers, allow

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students to look it up in their notes or a computer if need be. Clarify any questions about how to use the information found.

- Simplify any information needed for students who are having difficulty. Also, allow other students to help those students who are still struggling.
- For students who are ready to move forward, have them compare their numbers with other students who have finished comparing predictions.
- Students can research their questions from the KWL online to share with the class.
- To summarize, have the students finish the L part of their KWL that they started at the beginning of class.
- Students need to submit all work for a grade.

Assessment

Questions

1. If you only wanted a specific color of kernels for a project, could you predict how many you could get off of 10 ears of corn? What about 20 ears of corn? Explain, include any mathematical computations you used.
2. If we have 200 kernels, what proportion would you expect to be red? Purple? (include any color that is on the Indian corn) What if we have 500 kernels? Explain, include any mathematical computations you used.
3. Based on the graphs you created, what is the trend of corn as a commodity? Predict the sales in 1 year, 3 years, and 5 years. What is the trend of popcorn? Predict the sales in 1 year, 3 years, and 5 years. Would you invest in a popcorn company? Explain why or why not.

Other

1. Students will be evaluated on the graphs created from the data sheets and their responses to the assessment questions.

Extensions and Connections (for all students)

- To extend the lesson, the students can go online and see if they can find more sales of both corn and popcorn to extend their graphs. Students can find data for other countries. Also, they can see if they can find the percentage of corn that is popcorn.
- This lesson connects with business growth.

Strategies for Differentiation

- Allow students to work in small groups. Observe and record students' participation in small group activities.
- Assign roles to students in collaborative activities. Discover the strengths of students and assign appropriate roles.

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- Rephrase the word problems so that the math content is not lost in the terminology.
- Evaluate students based on their understanding of the concept instead of their grammar accuracy.
- Step by step instructions both orally and in writing.

Lesson 2 “Pop”ped Corn

Strand
Data Analysis and Probability

Mathematical Objective(s)
The student will:

- Create an experiment to investigate a problem
- Convert data from experiment into multiple forms of rational numbers
- Find the probability of multiple events
- Determine if the data is normally distributed and calculate *z* scores

Mathematics Performance Expectations
MPE 1. The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 9. The student will design and conduct an experiment/survey. Key concepts include
a) sample size;
d) data collection; and
e) data analysis and reporting.

MPE 23. The student will analyze the normal distribution. Key concepts include:
a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data, using z-scores

MPE 31. The student will calculate probabilities. Key concepts include:
a) conditional probability;
b) dependent and independent events;

Related SOLs
A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

AFDA.6 The student will calculate probabilities.

AFDA.7 The student will analyze the normal distribution. Key concepts include
a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data, using z-scores

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include
a) sample size;

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d) data collection; and
e) data analysis and reporting.

NCTM Standards

NUMBER AND OPERATIONS: Students should be able to:

- compute fluently and make reasonable estimates develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios

DATA ANALYSIS AND PROBABILITY Students should:

- Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments.

PROCESS STANDARDS Students should be able to:

- Build new mathematical knowledge through problem solving
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Additional Objectives for Student Learning (include if relevant; may not be math-related):

N/A

Materials/Resources

Microwave Popcorn Bags (name brand)

Microwave

Popcorn Data Sheet (attached)

Classroom set of graphing calculators

Assumption of Prior Knowledge

- The typical student will have taken Algebra 1 and the Algebra, Functions and Data Analysis classes.
- In order to be successful in this lesson, students will need to understand how to calculate probability and confidence intervals using normal distribution data. They must be able to understand a non-biased sample when creating an experiment and be able to analyze data from the experiment.
- Students may have difficulty creating the initial experiment. They may also have difficulty understanding and using the confidence intervals.

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- This lesson will require the students to apply, analyze, synthesis, and evaluate mathematical information.
- The relevant real life context in this problem is use and analysis of production in a business model.

Introduction: Setting Up the Mathematical Task

In this lesson, you will be acting as a quality assurance statistician for a popular popcorn manufacturer. The owner of the popcorn company wants a 90% kernel pop rate for all the bags of popcorn produced. It is your job to make sure that the batches that are placed in bags for sale meet this standard.

Your goal is to create an experiment that will allow you to determine what percentage of popcorn kernels pop. (10 minutes small groups, 10 minutes whole group discussion to determine the class experiment)

As the class is determining the class experiment, discuss the following questions:

- Would you want to sample the entire population of popcorn bags? Why or why not?
- How many bags would you want to sample out of how many bags? Why?
- Would you want to use all the bags that come off the production line together? Why or why not?
- How can we replicate the experiment in class?

Give students time to work independently or in small groups to develop their own experimental plan before creating a whole group experiment.

Student Exploration 1 (Day 1):

Student/Teacher Actions:

- Have students work in groups of 3.
- Each group gets their own bag of popcorn and will share their data after everyone has done their bag.
- Allow each group to pop their popcorn bag according to the instructions.
- As the popcorn is popping, pass out the popcorn data sheet to every student.
- The students will fill in the information on the data sheet after their bag of popcorn is popped.
- Once all the groups are completed, the groups will present their data in front of the class so that everyone can include all the data from each bag on their data sheet.
- With the class data, the students calculate the ratio of non-popped kernels to total kernels as a fraction, decimal and percent. ($p =$ non-popped kernel/total kernels, $q = 1 - p$)

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- Using their graphing calculators, have the students enter the information into the lists on the calculators to create a scatter plot. Plot the data through stat plot. Does the data resemble a normal distribution curve?
- Students need to determine if the popcorn fits within the 90% confidence interval for the non-popped kernels using the z table values. (In this case, $z = 1.645$)
- The formula for the interval around the proportion is as follows:

$$\left(p - z\sqrt{\frac{pq}{N}}, p + z\sqrt{\frac{pq}{N}}\right)$$

- Students determine if this batch of popcorn fits within the standard required by the company or if the batch should be thrown out.

Monitoring Student Responses

- Students will demonstrate their understanding of conducting experiments through class discussion. Give the students time to recall past knowledge and discuss information from the project that is needed to complete it.
- There may need to be clarification on creating a confidence interval. Have a class discussion to see who can remember confidence intervals and help the class create it. If no one remembers, allow students to look it up in their notes or a computer if need be. Clarify any questions about how to use the information found.
- Simplify any information needed for students having difficulty. Also, allow other students to help those students who are still struggling.
- For students who are ready to move forward, have them recalculate the confidence interval for 95% using a z score of $z = 1.960$
- To summarize, ask the students if they think that generic popcorn would fit within the same confidence interval. Have them make a prediction if name brand or generic popcorn will fit within a smaller confidence interval as an exit slip with an explanation.

Assessment

Questions

- Using the p value of 0.016, find the confidence intervals for 80%, 90%, 95%, 98%. The z scores are as follows: 1.282, 1.645, 1.960, and 2.326.
- If you were the owner of the company, which confidence interval would you chose and why? As a consumer, which confidence interval do you want the company's to use and why?

Journal/writing prompts

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- Why does the owner of the popcorn company want to have a quality control statistician on staff? How can the quality of the product help or hurt the company?
- Do you think that our experiment had a large enough sample to make a decision for the company? Explain. Include charts, graphs, and/or calculations to support your answer.

Other

- Evaluate the exit slip, check for completion, mathematical understanding, and more than a one word sentence.

Extensions and Connections (for all students)

- An extension for the students who complete the assignment early, they can investigate the company's popcorn history. They can find the history of the popcorn company and see if they can locate the standards of popped versus non-popped kernels. If they are unable to locate the standards, encourage them to try and contact a company and see if the company will share their standards. They could also ask any other questions about the manufacturing of popcorn that they can think of or quality control in general.

Strategies for Differentiation

- Allow students to work in small groups. Observe and record students' participation in small group activities.
- Assign roles to students in collaborative activities. Discover the strengths of students and assign appropriate roles.
- Make adjustments so that there will be less differences communicating the procedural knowledge of mathematics.
- Rephrase the word problems so that the math content is not lost in the terminology. Evaluate students based on their understanding of the concept instead of their grammar accuracy.
- Step by step instructions both orally and in writing.

Student Exploration 2 (Day 2):

Materials/Resources

Microwave Popcorn Bags (generic brand)

Microwave

Popcorn Data Sheet (attached)

Classroom set of graphing calculators

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Introduction: Setting Up the Mathematical Task

Yesterday we determined the percentage of kernels popped from a name brand microwave popcorn. Today we will act as a quality assurance statistician for a generic popcorn manufacturer. The generic company wants to be competitive with name brand popcorn. They also need a 90% kernel pop rate for all the bags of popcorn produced. It is your job to make sure that the batches that are placed in bags for sale meet this standard.

Your goal is to create an experiment that will allow you to determine the percentage of popcorn kernels popped. (10 minutes small groups, 10 minutes whole group discussion to determine the class experiment)

As the class is determining the class experiment, discuss the following questions:

- Would you want to sample the entire population of popcorn bags? Why or why not?
- How many would you want to sample out of how many? Why?
- Would you want to use all the bags that come off the production line together? Why or why not?
- How can we replicate the experiment in class?

Student/Teacher Actions:

- Have students work in groups of 3.
- Each group gets their own bag of popcorn and will share their data after everyone has completed their data sheet.
- Allow each group to pop their popcorn bag according to the company's instructions.
- As the popcorn is popping, pass out the popcorn data sheet to every student.
- The students will fill in the information on the data sheet after their bag of popcorn is popped.
- Once all the groups have completed their data sheet, the groups will present their data in front of the class so that everyone can include all the data from each bag on their data sheet.
- With the class data, the students calculate the ratio of non-popped kernels to total kernels as a fraction, decimal and percent. ($p =$ non-popped kernel/total kernels, $q = 1 - p$)
- Using their graphing calculators, students enter the information into the lists on the calculators and then plot the data through stat plot. Does the data look like a normal distribution curve?
- Students need to determine if the popcorn fits within the 90% confidence interval for the non-popped kernels using the z table values. (In this case, $z = 1.645$)
- The formula for the interval around the proportion is as follows:

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$$\left(p - z\sqrt{\frac{pq}{N}}, p + z\sqrt{\frac{pq}{N}}\right)$$

- Students determine if this batch of popcorn fits within the standard required by the company or if the batch should be thrown out.
- Now that we have determined the "popability" of each brand, we want to determine which brand is a better buy for the popped corn. Give the students the cost of the name brand popcorn per box and the generic popcorn per box. Students determine the cost per bag and cost per ounce of both brands of popcorn.
- Students need to answer the following questions: Which one is a better buy? Does the amount of popcorn that pops influence your buy? Which is more important to you, the amount of popcorn popped, cost or taste?

Monitoring Student Responses

- Students will demonstrate their understanding of conducting experiments through class discussion. Give students time to recall past knowledge and discuss information from the project that is needed to complete it.
- There may need to be clarification on creating a confidence interval. Have a class discussion to see who can remember and help the class create it. If no one remembers, allow students to look it up in their notes or a computer if need be. Clarify any questions about how to use the information found.
- Simplify any information needed for students who are having difficulty. Also, allow other students to help those students who are still struggling.
- For students who are ready to move forward, have them recalculate the confidence interval for 95% using a z score of $z = 1.960$
- Students need to answer the following questions: In paragraph form, compare your data results from the past two days. Assuming that the popcorn tastes the same, is it better to buy the name brand or generic popcorn in terms of getting the most amount of popcorn?

Assessment

Questions

- Using the p value of 0.016, find the confidence intervals for 80%, 90%, 95%, 98%. The z scores are as follows: 1.282, 1.645, 1.960, and 2.326.
- If you were the owner of the company, which confidence interval would you chose and why? As a consumer, which confidence interval do you want the company's to use and why?
- Compare the confidence intervals for the name brand and generic brand popcorn.

Journal/writing prompts

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- Which brand of popcorn would you buy? Why? In your explanation, include graphs, charts, and/or mathematical computations as needed. What is your choice based on (taste, popped corn, cost, etc)?

Extensions and Connections (for all students)

- An extension for the students who complete the assignment early, they can investigate popcorn history. They can find the history of the popcorn and see if they can find what the standards of popped versus non-popped kernels. If they are unable to locate the standards, encourage them to try and contact a company and see if the company will share their standards. They could also ask any other questions about the manufacturing of popcorn that they can think of or about quality control in general.

Strategies for Differentiation

- Allow students to work in small groups. Observe and record students' participation in small group activities.
- Assign roles to students in collaborative activities. Discover the strengths of students and assign appropriate roles.
- Rephrase the word problems so that the math content is not lost in the terminology. Evaluate students based on their understanding of the concept instead of their grammar accuracy.
- Step by step instructions both orally and in writing.

Lesson 3 Which Snack is Best?

Strand
Algebra, Data Analysis and Probability

Mathematical Objective(s)
Scatter plots, Rational Functions. In this lesson, students will compare the nutritional information of a variety of snacks as well as the cost of the snacks and create two scatter plots with the information. They will use their knowledge of algebraic rational functions to see the correlation between fat and calories of snacks as well as cost and calories of snacks. They will determine which snacks are best for the nutrition and best for cost.

Mathematics Performance Expectation(s)
MPE 1. The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.

MPE 2. The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 12. The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

MPE 22. The student will analyze graphical displays of univariate data, including dot plots, stem plots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

Related SOL

A8. The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

A.11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students

will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

NCTM Standards

NUMBER AND OPERATIONS All students should:

- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- judge the reasonableness of numerical computations and their results

ALGEBRA All students should:

- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules

DATA ANALYSIS AND PROBABILITY All students should:

- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots.
- understand histograms, parallel box plots, and scatter plots and use them to display data.
- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots.
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatter plots of the data and approximate lines of fit

PROCESS STANDARDS All students should:

- Build new mathematical knowledge through problem solving
- Use the language of mathematics to express mathematical ideas precisely

Additional Objectives for Student Learning (include if relevant; may not be math-related):

N/A

Materials/Resources

Variety of snacks with nutritional information and price sheet (Students could be asked to bring in their favorite snack the day before so their favorites are included)
Nutritional Data of Snacks Sheet (attached)
Cost of Snack Sheet (attached)
Classroom set of graphing calculators

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Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- To succeed in this lesson, students should have understanding of ratios, percents, scatter plots and line of best fit.
- They should be able recognize a pattern and find a line of best fit given a set of data.
- Students may have misconceptions between serving size and information based on ounces. Usually these are the same but they will have to check.
- The relevant real life context in this problem is nutritional information.

Introduction: Setting Up the Mathematical Task

In this lesson, students investigate the relationship between fat content and calorie content in snacks as well as cost per ounce of a variety of snacks.

Teacher introduction for the class: When you go to college, there will be many late nights where you will need something to snack on but you want to be wary of the "Freshman 15". "Freshman 15" is when you gain 15 pounds in your freshman year. In order to try and prevent the "Freshman 15", you want to find the best snacks to take to college. However, you also need to be aware that you will be on a college budget so you want to be aware of the cost of the snacks.

- What types of snacks do you normally have at home? Do you know the cost of these snacks? Which snacks do you think are good for you? What makes a snack better for you than another? Which snacks are least expensive? How can you determine the cost of different snacks? (5 minutes)
- What type of representation will show us the best representation for calories versus fat content? Calories versus price? (5 minutes)

Student Exploration 1:

Student/Teacher Actions:

- Have a variety of snacks at the front of the room with the cost of each item on it. Have the students independently choose their top 6 favorite snacks (one of them must be popcorn).
- On the nutritional data sheet, have them record the information from their favorite snacks.
- Using the information of fat and calories in each item, the students should graph the data as a scatter plot on their calculator. Is there any correlation of the data? If there is correlation, is it a negative or positive correlation? Do you think this is a strong or weak correlation?
- Find the line of best fit for the data using the calculator. Students need to record the equation for the line of best fit. What do the constants mean in context of this problem?

Monitoring Student Responses

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- Students will demonstrate their understanding on conducting experiments through class discussion. Give the students time to recall past knowledge and discuss information from the project that is needed to complete it.
- There may need to be clarification on finding the line of best fit. Have a class discussion to see who can remember and help the class. If no one remembers, allow students to look it up in their notes or a computer if need be. Clarify any questions about how to use the information found.
- Simplify any information needed for students having difficulty. Also, allow other students to help those students who are still struggling.
- For students who are ready to move forward, have them find the mean, median and mode of the fat, carbohydrates, and proteins. Have them discuss what they found.
- In paragraph form, students need to summarize the meaning of the correlations between the amounts of fat and the amount of calories in the snack. Which snack will they choose that will help keep away the "Freshman 15"? Which is the worst snack out of the ones they investigated?

Student Exploration 2:

Student/Teacher Actions:

- Using the same snacks from the first exploration, students fill out the cost of each item on the cost of snacks data sheet. Students independently choose their top 6 favorite snacks (one of them must be popcorn).
- Using the information from the data sheet, the students should create a scatter plot based on cost per ounce and calories.
- Students need to answer the following questions: Is there any correlation of the data? If there is correlation, is it a negative, positive correlation? In your own words, explain the correlation.
- Find the line of best fit for the data using the calculator.

Monitoring Student Responses

- The students will demonstrate their understanding of conducting experiments through class discussion. Give students time to recall past knowledge and discuss information from the project that is needed to complete it.
- There may need to be clarification on finding the line of best fit. Have a class discussion to see who can remember and help the class. If no one remembers, allow students to look it up in their notes or a computer if need be. Clarify any questions about how to use the information found.
- Simplify any information needed for students who are having difficulty. Also, allow other students to help those students who are still struggling.

- For students who are ready to move forward, make a poster displaying the data they found about fat versus calories and cost versus calories. They can also make box and whisker plots for the nutritional data found (carbohydrates, protein, fat, calories).
- To summarize, ask the students what the correlations mean between the costs of snacks versus the amount of calories in the snack.

Assessment

Questions

1. Given the information from the snack sheet, what snack do you think is the best to keep off the "Freshman 15"? Which is the best for your new college budget?
2. Given the information presented by other people, would you choose different snacks if you were trying to lose weight? What if you were unconcerned about your weight? What if your budget had been decreased? What if your budget had been increased?

Other

1. Students will be evaluated on their computations on the data sheets. The graphs and lines of best fit will be graded.

Extensions and Connections (for all students)

- To extend the lesson, the students can make stem and leaf plots showing the amount of fat, carbohydrates, and protein in the snacks. They will find the mean, median, and mode of the data to see any if the numbers are clustered in any area for snacks. Have a discussion on which type of nutrition is most evident in snack foods.
- Students can create scatter plots comparing calories versus carbohydrates or protein and find the line of best fit to find the correlation between these factors as well. While fat is important when choosing a snack, so Is the amount of sugars/carbohydrates and protein because some snacks are more filling than others.
- This lesson connects with health class based on the nutritional information analysis.

Strategies for Differentiation

- Allow students to work in small groups. Observe and record students' participation in small group activities.
- Assign roles to students in collaborative activities. Discover the strengths of students and assign appropriate roles.
- Rephrase the word problems so that the math content is not lost in the terminology. Evaluate students based on their understanding of the concept instead of their grammar accuracy.
- Step by step instructions both orally and in writing.

Corn KWL

What I KNOW	What I WANT to know	What I LEARNED

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Corn Trade Statistics

	Export
April 2011	$\$1.340 \times 10^9$
March 2011	$\$1.266 \times 10^9$
February 2011	$\$1.091 \times 10^9$
January 2011	$\$9.31 \times 10^8$
December 2010	$\$1.108 \times 10^9$
November 2010	$\$9.76 \times 10^9$
October 2010	$\$8.32 \times 10^8$
September 2010	$\$9.66 \times 10^8$
August 2010	$\$9.72 \times 10^8$
July 2010	$\$9.36 \times 10^8$
June 2010	$\$8.69 \times 10^8$
May 2010	$\$9.6 \times 10^8$
April 2010	$\$8.38 \times 10^8$
March 2010	$\$9.58 \times 10^8$
February 2010	$\$7.62 \times 10^8$
January 2010	$\$7.73 \times 10^8$

Information from US Department of Commerce, Bureau of Economic Analysis
http://www.bea.gov/agency/uguide1.htm#_1_19

What is the line of best fit for this data?

_____

What do you think the sales will be in six months (October, 2011)? Explain why?

_____

_____

_____

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Sales of Unpopped Popcorn

Year	Retail Sales in Dollars
2009	9.90×10^8
2008	9.66×10^8
2007	9.77×10^8
2006	9.76×10^8
2005	1.007×10^9
2004	1.009×10^9
2003	9.83×10^8
2002	1.04×10^9
2001	1.038×10^9
2000	9.79×10^8
1999	9.38×10^8

Information from Popcorn Board
http://www.popcorn.org/EncyclopediaPopcornica/WelcometoPopcornica/IndustryFacts/tabid/108/Default.aspx

What is the line of best fit for this data?

_____

What do you think the sales will be this year (2011)? Explain why?

_____

_____

_____

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Popcorn Data Sheet:

Name: _____

Brand: _____ _____	Bag 1	Bag 2	Bag 3	Bag 4	Bag 5	Total
Non-Popped Kernels						
Popped Kernels						
Total Kernels						
Non to Popped						
Popped to Non						
Non to Total						
Popped to Total						
Percentage of Non-Popped						
Percentage of Popped						

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School
Popped Corn

Name: _____ Teacher: _____

Date Submitted: _____ Title of Work: _____

	Criteria				Points
	4	**3**	**2**	**1**	
Explanation	A complete response with a detailed explanation.	Good solid response with clear explanation.	Explanation is unclear.	Misses key points.	____
Use Of Visuals	Clear diagram or sketch with some detail.	Clear diagram or sketch.	Inappropriate or unclear diagram.	No diagram or sketch.	____
Mechanics	No math errors.	No major math errors or serious flaws in reasoning.	May be some serious math errors or flaws in reasoning.	Major math errors or serious flaws in reasoning.	____
Demonstrated Knowledge	Shows complete understanding of the questions, mathematical ideas, and processes.	Shows substantial understanding of the problem, ideas, and processes.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	____
Requirements	Goes beyond the requirements of the problem.	Meets the requirements of the problem.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	____

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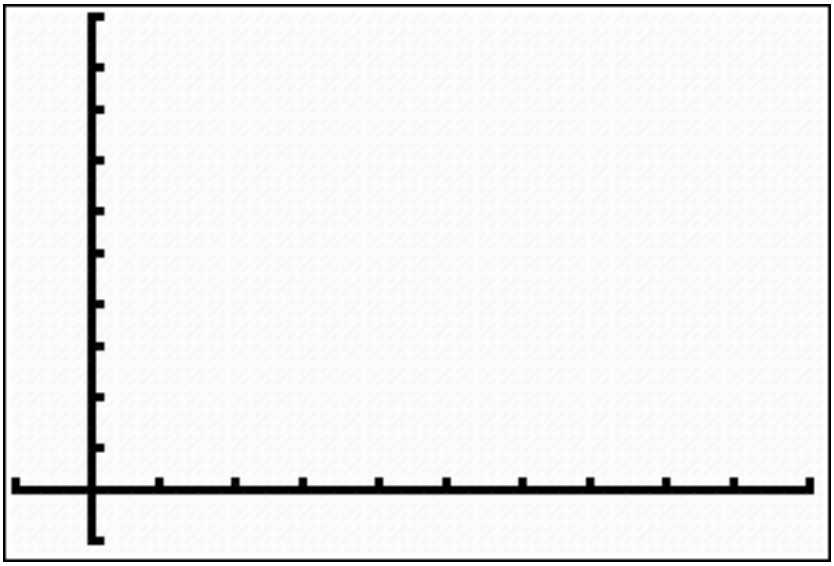
Nutritional Data of Snacks

Name: _____

Snack	Carbohydrates (grams/serving)	Protein (grams/serving)	Fat (grams/serving)	Calories (ounce/serving)
Popcorn				

Draw the scatter plot below comparing fat and calories.

Calories/serving



Fat grams/serving

Draw the line of best fit found from your calculator.

What is the equation for your line in form $y = ax + b$?

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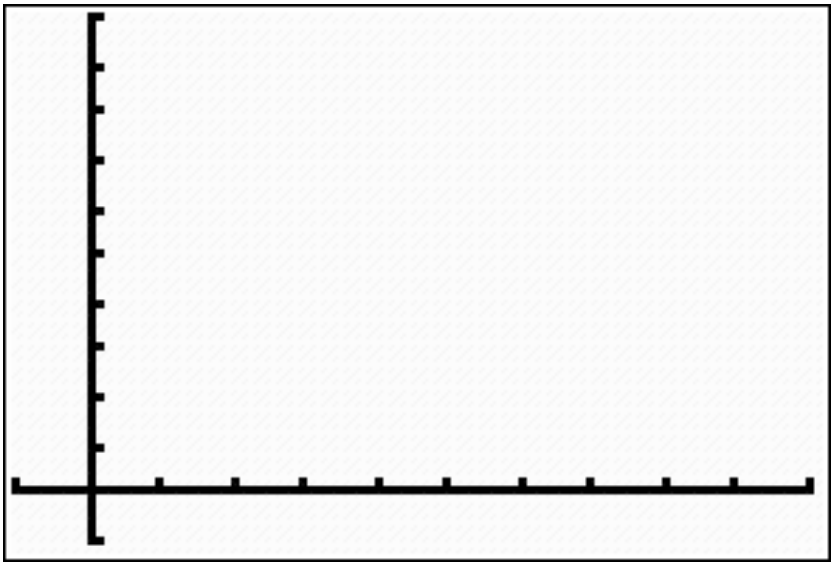
Cost of Snacks

Name: _____

Snack	Cost	Number of ounces	Cost per ounce	Cost per serving	Calories per serving
Popcorn					

Draw the scatter plot below comparing cost and calories.

Calories / serving



Cost / serving

Draw the line of best fit found from your calculator.

What is the equation for your line in form $y = ax + b$?

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Nutrition Facts

Food	Popcorn	Lays Potato Chips	Doritos Chips	Rold Gold Pretzels	Oreo Cookies	Goldfish Crackers	Planters Trail Mix	Pop Tarts (frosted strawberry)	Gushers
Price									
Serving Size		1 oz (20 chips)	1 oz (12 chips)	1 oz (18 tiny twists)	1.2 oz (3 cookies)	1.1 oz (55 pieces)	1.1 oz	1.8 oz (1 pop tart)	0.9 oz (1 packet)
Calories		150	140	110	160	140	150	200	90
Total Fat		10 g	8 g	0 g	7 g	5 g	9 g	5 g	1 g
Cholesterol		0 mg	0 mg	0 mg	0 mg	< 5 mg	0 mg	0 mg	0 mg
Sodium		180 mg	240 mg	450 mg	160 mg	250 mg	20 mg	170 mg	45 mg
Total Carbohydrates		15 g	17 g	23 g	25 g	19 g	14 g	38 g	20 g
Protein		2 g	2 g	3 g	1 g	4 g	4 g	2 g	0 g