Parallel Lines Cut by a Transversal

I. **UNIT OVERVIEW & PURPOSE:**
The goal of this unit is for students to understand the angle theorems related to parallel lines. This is important not only for the mathematics course, but also in connection to the real world as parallel lines are used in designing buildings, airport runways, roads, railroad tracks, bridges, and so much more. Students will work cooperatively in groups to apply the angle theorems to prove lines parallel, to practice geometric proof and discover the connections to other topics including relationships with triangles and geometric constructions.

II. **UNIT AUTHOR:**
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III. **COURSE:**
Mathematical Modeling: Capstone Course

IV. **CONTENT STRAND:**
Geometry

V. **OBJECTIVES:**
1. Using prior knowledge of the properties of parallel lines, students will identify and use angles formed by two parallel lines and a transversal. These will include alternate interior angles, alternate exterior angles, vertical angles, corresponding angles, same side interior angles, same side exterior angles, and linear pairs.
2. Using the properties of these angles, students will determine whether two lines are parallel.
3. Students will verify parallelism using both algebraic and coordinate methods.
4. Students will practice geometric proof.
5. Students will use constructions to model knowledge of parallel lines cut by a transversal. These will include the following constructions: parallel lines, perpendicular bisector, and equilateral triangle.
6. Students will work cooperatively in groups of 2 or 3.

VI. **MATHEMATICS PERFORMANCE EXPECTATION(s):**
MPE.32 The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

VII. **CONTENT:**
Students will use prior knowledge of parallel lines cut by a transversal and geometric constructions to create a map to given specifications. The map will be a representation of a small section of a city or town and will serve to introduce students to the usefulness

of mathematics in a different context. This project will encourage students to work cooperatively and to see how these concepts are used in the real world.

VIII. REFERENCE/RESOURCE MATERIALS:

SMART Board, document camera, or LCD projector for modeling, markers, calculators, rulers, compasses, poster boards, and the following handouts:

- Pre-Assessment (Attached)
- Construction worksheets (Attached)
- Lesson 2 homework (Attached)
- Directions for map construction (Attached)
- Solution for map construction (Attached)

IX. PRIMARY ASSESSMENT STRATEGIES:

Attached for each lesson. Students will be given a pre-assessment in the form of a handout to assess prior knowledge at the beginning of the unit. The teacher will observe students as they work on tasks to see if and where any additional help may be needed. Teacher will also monitor, observe, and communicate with students as they work in groups. Homework assignments will be given after each lesson. The homework will be discussed with the class and graded by the instructor. The final assessment will be the completed project.

X. EVALUATION CRITERIA:

Attached for each lesson. Students will be observed as they work during class. Whole class discussions will also help the instructor to determine student knowledge. Students will check each others' work throughout the unit. Presentations by the students and feedback from their peers will also serve as an evaluation tool. The final assessment will consist of the construction of a map and problems regarding parallel lines from the map.

XI. INSTRUCTIONAL TIME:

The unit should take a total of three days of block scheduling, six days if standard.

LESSON 1: PARALLEL LINES & CONSTRUCTIONS

STRAND: Geometry

MATHEMATICAL OBJECTIVES:

- To review and practice geometric constructions.
- To review and practice concepts regarding parallel lines cut by a transversal.
- To determine whether two lines are parallel.
- To encourage working cooperatively in small groups.
- Recognize properties of parallel lines outside of the classroom.

MATHEMATICS PERFORMANCE EXPECTATIONS:

MPE 32.a The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.

VIRGINIA SOL: G.2a, G.4 c, f, g

- ➢ The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel.
- ➢ The student will construct and justify the constructions of:
    - A perpendicular to a given line from a point not on the line.
    - An angle congruent to a given angle.
    - A line parallel to a given line through a point not on the given line.

NCTM STANDARDS:

- Use geometric models to gain insights into, and answer questions in, other areas of mathematics.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Explore relationships among classes of two-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.

MATERIALS/RESOURCES: Classroom set of calculators, SMART Board or document camera to model constructions for students, rulers, compasses, and the following handouts: pre-assessment, and construction guides.

ASSUMPTION OF PRIOR KNOWLEDGE:

- The student should have already completed the Geometry course offered at his/her school.
- Students should understand the concepts and properties of parallel lines cut by a transversal.

- Students should be able to accurately construct a pair of parallel lines, a perpendicular bisector and an equilateral triangle.
- Students may have difficulty recalling vocabulary.

LESSON OUTLINE:

I. Introduction: Review parallel lines cut by a transversal. (20 –30 minutes)
   A. Explain to the class they will be working on a project involving parallel lines cut by a transversal and their related angles.
   B. Ask students to sketch a pair of parallel lines cut with a transversal on a piece of paper at their desks. The instructor should sketch the same figure on the dry erase board at the front of the class or using a document camera. Have students mark the angles 1 – 8.
   C. Review the following vocabulary terms encouraging students to offer their own definitions: (Note: do not give these definitions until after you have discussed the students' definitions).
      1. Parallel lines – coplanar lines that do not intersect
      2. Transversal – A line that intersects two or more coplanar lines
      3. Alternate Interior Angles – Two nonadjacent interior angles on opposite sides of a transversal
      4. Alternate Exterior Angles – Two nonadjacent exterior angles on opposite sides of a transversal
      5. Corresponding Angles – Two nonadjacent angles on the same side of a transversal such that one is an exterior angle and the other is an interior angle.
      6. Vertical Angles – Two non adjacent angles formed by a pair of intersecting lines that share a vertex
      7. Same-side Interior Angles – Two nonadjacent angles in the same side of a transversal.
      8. Linear Pairs – Two adjacent angles whose measurements sum to 180 degrees.
   D. As each vocabulary word is discussed, discuss known properties of each pair of angles. (Congruent, supplementary). Remind students to make notes on their paper if necessary. Where are these angles found in everyday life?

II. Pre-assessment (20 – 30 minutes)
   A. In order to better understand students' prior knowledge and skill, give each student a pre-assessment handout to be filled out individually
      1. Walk around the room and observe the students' solutions as they work.
      2. Go over the solutions to the handout making sure to spend extra time where needed. For questions 4 - 6, incorporate a whole class discussion and have the students justify their answers.

III. Review constructions (30 – 40 minutes)
   A. Put students into groups of 2-3 to practice constructions
   B. Give each student a set of the following construction handouts and a compass.
      1. Copy an angle

2. Perpendicular bisector
3. Parallel lines

C. Use a SMART Board or document camera to model steps for constructions. (You may want to have a student come to the camera and model for the class).
D. Let students help each other to complete the practice constructions.
E. Ask students to construct an equilateral triangle.
F. Observe and monitor as needed.
G. Homework: Tell the students to give examples of real life models for each angle relationship. Encourage them to list several examples for each angle relationship. An example for vertical angles would be when two jet streams cross each other in the sky.

IV. Extensions and connections for all students

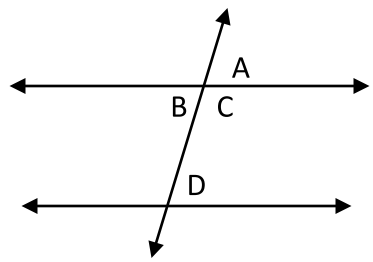
A. For the homework assignment, ask students to name some jobs that might require a working knowledge of parallel lines cut by a transversal or constructions.

V. Strategies for differentiation

A. ELL and ESL students may need extra help with vocabulary. The teacher may want to explain certain terms by relating to something else, such as, interior being inside the parallel lines. You could also offer a foldable or a fill-in-the-blank handout. Seating arrangements should be considered – sit the student near the teacher, or if possible, with a fellow student who speaks the same language. Dictionaries and online translators are good resources also.
B. Some students may have difficulty using a protractor. For these students you may wish to offer transparency paper for the constructions

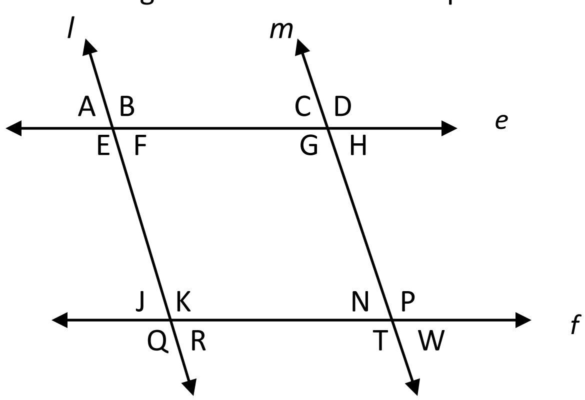
Pre-Assessment

Part 1 – Use the diagram of the two parallel lines below to answer questions 1 - 3.



1. ∠A and ∠D are called ___________________ angles.
2. ∠B and ∠D are called ___________________ angles.
3. ∠C and ∠D are called ___________________ angles.

Part 2 – Use the diagram below to answer questions 4 – 6.



4. Name all the angles that would be congruent to ∠C *if and only if* lines *e* and *f* are parallel.

5. Assume lines *l* and *m* are parallel, and lines *e* and *f* are parallel. List all the angles that would be congruent to ∠K.

6. Assume we do not know if any of the lines in the figure above are parallel. Determine which of the following angle relationships would prove lines *e* and *f* parallel. For each of the problems, explain why or why not.
   a) ∠A ≅ ∠J

b) $\angle A \cong \angle C$

c) $\angle D \cong \angle T$

d) $\angle G \cong \angle P$

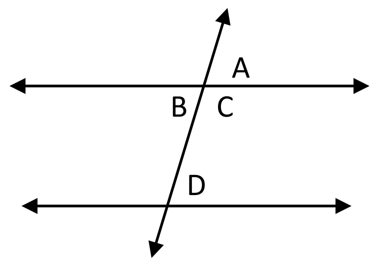
e) $\angle N \cong \angle W$

f) $\angle A + \angle Q = 180°$

g) $\angle K + \angle N = 180°$

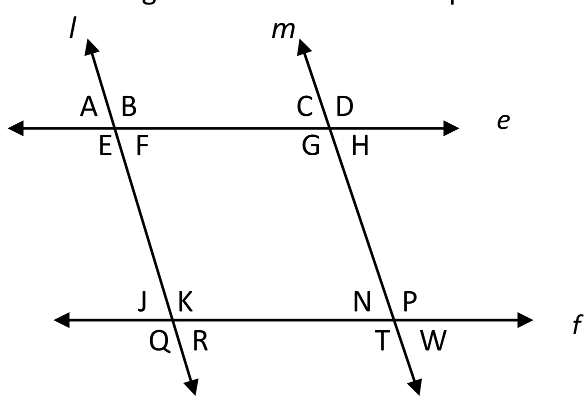
Pre-Assessment Answer Key

Part 1 – Use the diagram of the two parallel lines below to answer questions 1 - 3.



1. ∠A and ∠D are called __corresponding__angles.
2. ∠B and ∠D are called _alternate interior___ angles.
3. ∠C and ∠D are called _same-side interior or consecutive interior__ angles.

Part 2 – Use the diagram below to answer questions 4 – 6.



4. Name all the angles that would be congruent to ∠C *if and only if* lines *e* and *f* are parallel. ∠H, ∠N, ∠W

5. Assume lines *l* and *m* are parallel, and lines *e* and *f* are parallel. List all the angles that would be congruent to ∠K. ∠Q, ∠B, ∠E, ∠P, ∠T, ∠G, ∠D

6. Assume we do not know if any of the lines in the figure above are parallel. Determine which of the following angle relationships would prove lines *e* and *f* parallel. For each of the problems, explain why or why not.

   a) ∠A ≅ ∠J Yes, corresponding angles

b) $\angle A \cong \angle C$ No, both angles on line E

c) $\angle D \cong \angle T$ Yes, alternate exterior angles

d) $\angle G \cong \angle P$ Yes, alternate interior angles

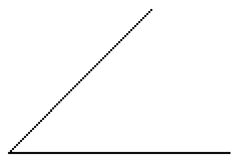
e) $\angle N \cong \angle W$ No, both angles on line F

f) $\angle A + \angle Q = 180°$ Yes, same side exterior

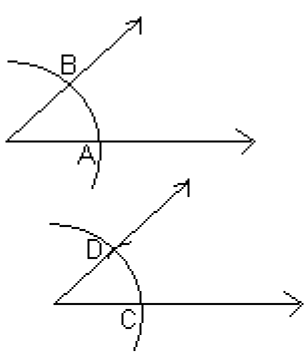
g) $\angle K + \angle N = 180°$ No, both angles on line F

Constructions!

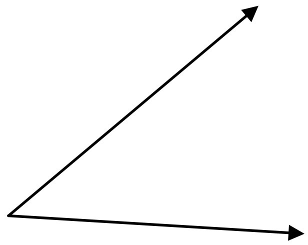
Make a copy of the angle:

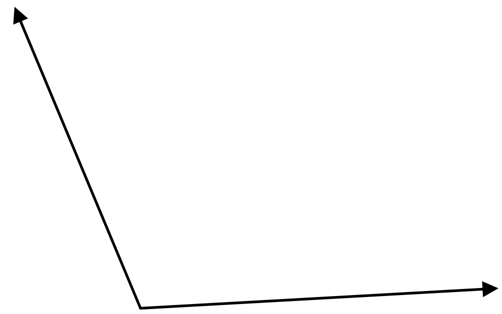


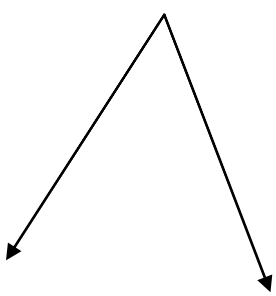
First, make a ray that will become one of the two rays of the angle.
Beginning with the vertex of the original angle, take your compass and create an arc that intersects the angle twice. The exact measurement you begin with doesn't matter. Using this same measurement, do the same thing beginning at the endpoint of the ray.
Now, using your compass, set it to be the distance from one of the intersection points on the original angle to the other intersection point on the original angle.
Keeping your compass on that same setting, make an arc coming from the intersection point on the ray, going up and intersecting the last arc you made on the ray.
This will intersect at a point. Draw the line going from this point to the endpoint of the ray to complete the copy of the angle.

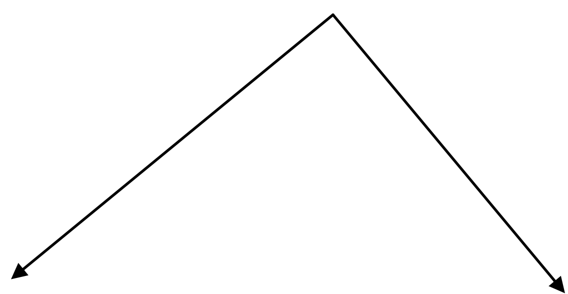


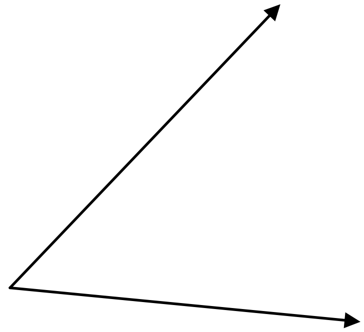
PRACTICE WITH THESE!





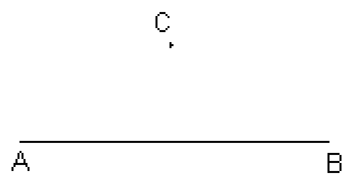




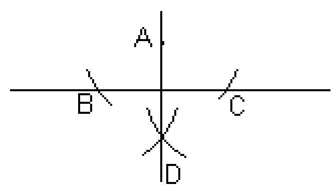


Constructions!

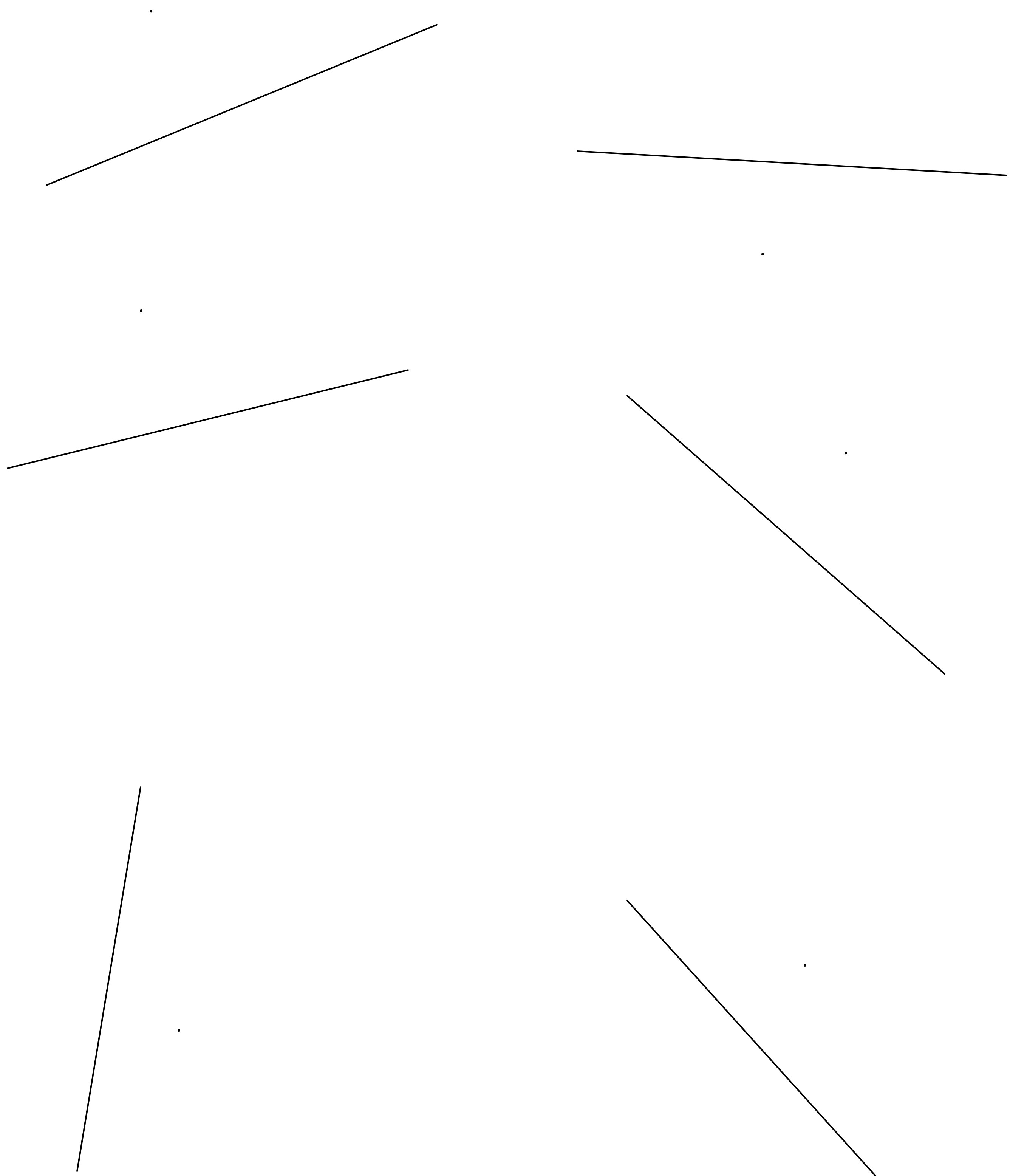
Construct a perpendicular through a point not on the line:



First, take your compass and make a big arc that hits the line segment twice. This will create two endpoints.
From each endpoint, make an arc below the line (we already found a point above, so we don't need that one!).
This will intersect at a point. Draw the line going from this point to the one we were given at first to get the perpendicular going through a point not on the line.

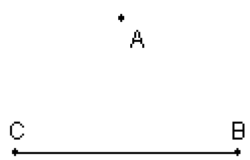


PRACTICE WITH THESE!

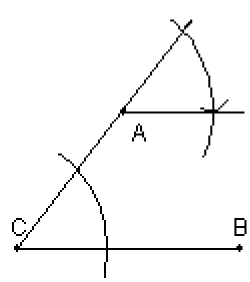


Constructions!

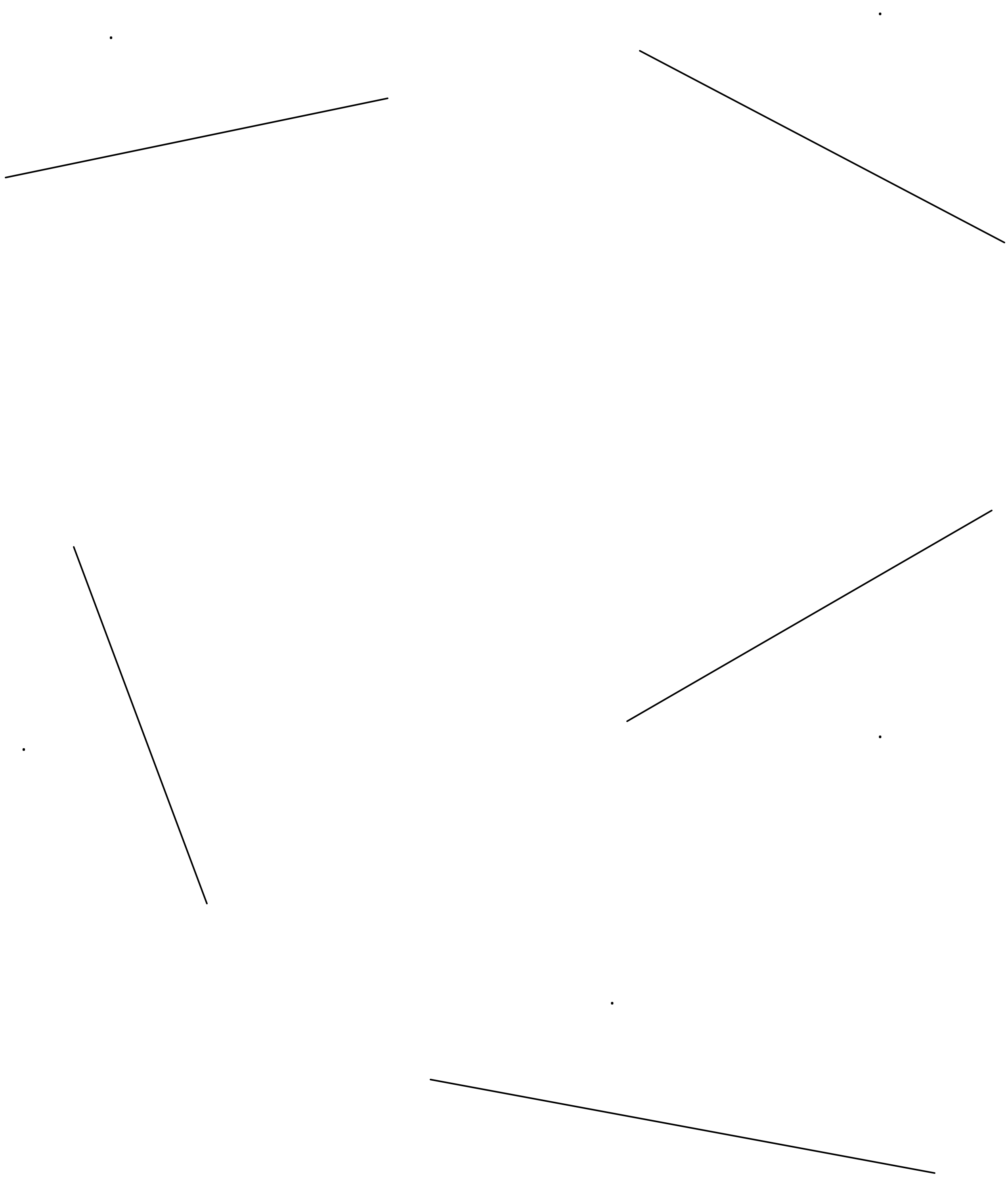
Construct a parallel through a point not the line:



First, create a ray from any point on the line to the point. This will form an angle.
Make an arc on the angle, then use the same setting to make the same arc on the original point.
Now, using your compass, set it to be the distance from one of the intersection points on the original angle to the other intersection point on the original angle.
Keeping your compass on that same setting, make an arc coming from the intersection point on the ray, going down and intersecting the last arc you made on the ray.
This will intersect at a point. Draw the line going from this point to the endpoint of the ray to complete the parallel line.



PRACTICE WITH THESE!



LESSON 2: PARALLEL LINES & PROOF

STRAND: Geometry

MATHEMATICAL OBJECTIVES:

- To review and practice concepts regarding parallel lines cut by a transversal.
- To use algebraic methods as well as deductive proof to verify if two lines are parallel.
- Use two column proofs to prove parallel line theorems.
- To encourage working cooperatively in small groups.
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MATHEMATICS PERFORMANCE EXPECTATIONS:
MPE 32.a, b The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel; verify the parallelism, using algebraic and coordinate methods as well as deductive proofs.

VIRGINIA SOL: G.2a-c
The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel, verify the parallelism, using algebraic and coordinate methods as well as deductive proofs, and solve real-world problems involving angles formed when parallel lines are cut by a transversal.

NCTM STANDARDS:

- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics.
- Explore relationships among classes of two-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

MATERIALS/RESOURCES: Rulers, document camera or LCD projector for modeling problems and ease of viewing. If these are not available, you can always sketch the diagram on a dry-erase board at the front of the classroom.

LESSON OUTLINE:

I. Introduction: Focus activity (15 – 20 minutes)
   A. Discuss answers to lesson 1 homework.
   B. Tell students to write down all the properties they can think of regarding two parallel lines cut by a transversal. Explain that you are looking for a summary of properties. Ask the students to complete the following statement as many times as necessary: “Two parallel lines that are cut by a transversal form ….”
   C. Have students share what they have written. Put these on the dry-erase board as a running list. Be sure to have students explain their reasoning whether they

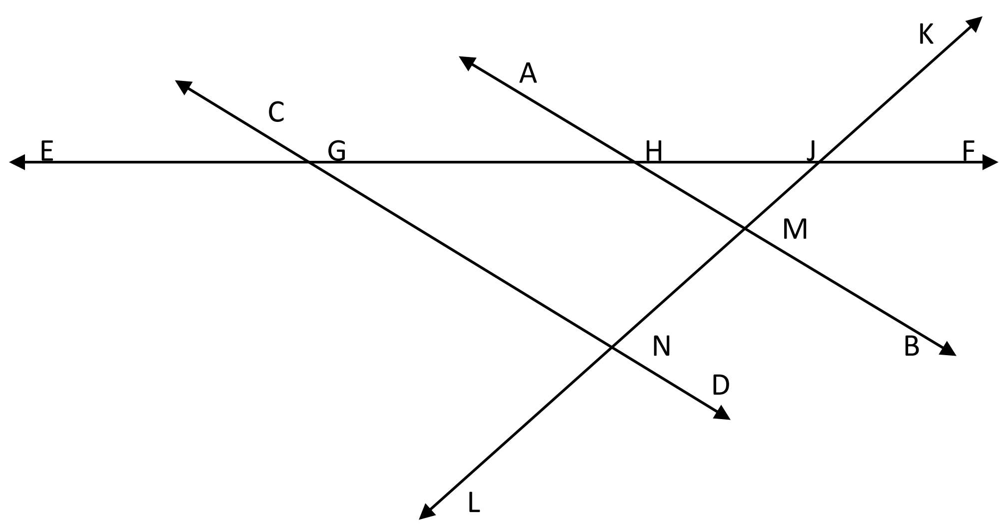
give a correct statement or an incorrect statement. As a class, determine which of the statements are valid and should stay part of the list.

1. The properties should include the following:
    a. Two parallel lines that are cut by a transversal form congruent alternate interior angles.
    b. Two parallel lines that are cut by a transversal form congruent alternate exterior angles.
    c. Two parallel lines that are cut by a transversal form congruent corresponding angles.
    d. Two parallel lines that are cut by a transversal form supplementary angles on the same side of the transversal.
    e. If two parallel lines are cut by a transversal, and the transversal is perpendicular to one line, then the transversal is also perpendicular to the other line.

II. Proving lines parallel – part 1 (20 minutes)

A. Introduction to proof.

1. Place students in groups of three or four so they can brainstorm together. Write the following problem on the dry erase board at the front of the class. Instruct students to copy the problem onto their own paper. Each student should then find the measure of each angle asked for and state the reason why they know the measurement. Ask the students to find as many valid reasons as possible. Give an example of what you are looking for: For question 1, $m\angle HMN$ = 78° because $\angle HMN$ and $\angle HMJ$ are linear pairs.



Given $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $m\angle HMJ = 102°$, $m\angle NGE = 135°$, find the measure of each angle. Be sure to state the reason you know this. Answers are given in red; students may offer different, yet valid, reasons for the measures.

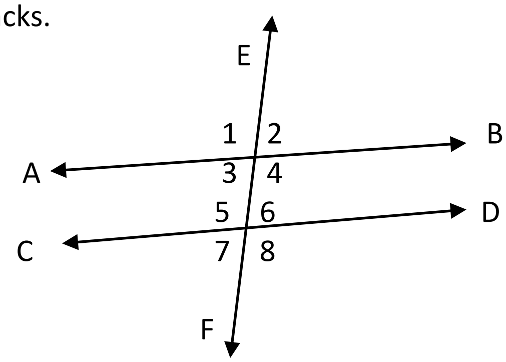
1) $m\angle HMN$ 78° $\angle HMJ$ and $\angle HMN$ are a linear pair
2) $m\angle GNM$ 102 $\angle HMJ$ and $m\angle GNM$ are corresponding angles
3) $m\angle GNL$ 78° $\angle HMN$ and $\angle GNL$ are corresponding angles

4) $m\angle NMB$ 102° $\angle HMJ$ and $\angle NMB$ are vertical angles
5) $m\angle HGN$ 45° $\angle NGE$ and $\angle HGN$ are a linear pair
6) $m\angle CGH$ 135° $\angle NGE$ and $\angle CGH$ are vertical angles
7) $m\angle AHJ$ 135° $\angle CGH$ and $\angle AHJ$ are corresponding angles
8) $m\angle AHG$ 45° $\angle AHJ$ and $\angle AHG$ are linear pairs
9) $m\angle HJM$ 33° triangle sum conjecture
10) $m\angle KJF$ 33° $\angle HJM$ and $\angle KJF$ are vertical angles

a. When going over the solutions and explanations to these problems, students may offer solutions before it is valid to do so. Remind students that even though they may "know" the measure of an angle, in order to give that measure, they must have a valid explanation.
b. Discuss solutions one at a time with the class. Let the students determine by discussion which solutions are correct and valid.
c. When going over problem 9 ask students if anyone solved the problem using the triangle sum conjecture. Lead this into a discussion of what is known about triangles GJN and HJM. (They are similar). How do we know this?
d. When going over problem 10, if no student offers this as a manner of solving the problem, discuss the exterior angle of a triangle theorem.

B. Proving lines parallel. (45 – 60 minutes)
1. Discuss with students that they already have all the knowledge necessary, (the properties discussed in the focus activity and from the exercise above), to prove two lines cut by a transversal are parallel. Tell students they will spend the next part of class proving this concept.
a. Give the students the following postulates. Explain that they will use these postulates and several of the properties of parallel lines to prove several theorems.
i. Corresponding Angles Postulate: If two coplanar lines are cut by a transversal so that two corresponding angles have the same measure, then those lines are parallel.
ii. Parallel Lines Postulate: If two coplanar lines are parallel and are cut by a transversal, corresponding angles have the same measure.

2. Write the following problem on the dry erase board at the front of the class and ask each student to copy. Go through the proof one step at a time asking students what they know that can help us.
   a. A road crosses a set of railroad tracks. If the measure of angle 5 is 106°, find the measure of angle 4.



What do we know; that is, what are we given? (Lines AB and CD are parallel and angle 5 measures 106°).
What are we trying to find? (The measure of angle 4). What do we know to get us there? (Since angles 1 and 5 are corresponding, we know angles 1 and 5 are congruent. Also, angles 1 and 4 are congruent because they are vertical angles. Therefore, angles 4 and 5 are congruent by the transitive property of equality. Thus, the measure of angle 4 is 106°).
Which theorem does this prove? (Alternate Interior Angles).

b) Use this exercise to prove the Alternate Interior Angles Theorem with the class: (Only give the left side of the table. Fill in the right side with students' responses).

Alternate Interior Angle Theorem – If two coplanar parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

Statement	Reason
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Given
$\angle 1 \cong \angle 5$	Corresponding angles
$\angle 1 \cong \angle 4$	Vertical angles
$\angle 4 \cong \angle 5$	Transitive property

3. Copy the following problems on the dry erase board at the front of the class, (or use a document camera), and tell the students to work together to construct each proof. Encourage students to talk with each other to discover different ways of approaching the problems. Teacher should observe and monitor throughout assignment. Use the diagram from Section II, B, 2. I have included solutions in red – keep in mind these are not the only valid solutions.

a. *Alternate Exterior Angle Theorem* – If two coplanar parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

Statement	Reason
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Given
$\angle 1 \cong \angle 5$	Corresponding angles
$\angle 5 \cong \angle 8$	Vertical angles
$\angle 1 \cong \angle 8$	Transitive property

b. *Same Side Interior Angle Theorem* – If two coplanar parallel lines are cut by a transversal, then each pair of same side, or consecutive, angles is supplementary.

Statement	Reason
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Given
$\angle 1 + \angle 3 = 180°$	Linear Pairs
$\angle 1 \cong \angle 5$	Corresponding angles
$\angle 3 + \angle 5 = 180°$	Substitution

c. *Perpendicular Transversal Theorem* – In a plane, if two lines are each perpendicular to the same line, then they are parallel.

Statement	Reason
$\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$, $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$	Given
$\angle 1 \cong \angle 5$	Corresponding angles
$\angle 1 \cong \angle 4$	Vertical angles
$\angle 4 \cong \angle 5$	Transitive property
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Alternate Interior angles theorem

C. Summary/Homework –

1. Discuss the solutions with the class after they have completed the proofs. Note that not all students will construct the same proof. Be sure to take the time to discuss different proofs and their validity so students will understand there is not just one correct answer.
2. Homework: Handout, Lesson 2

III. Extensions and connections for all students

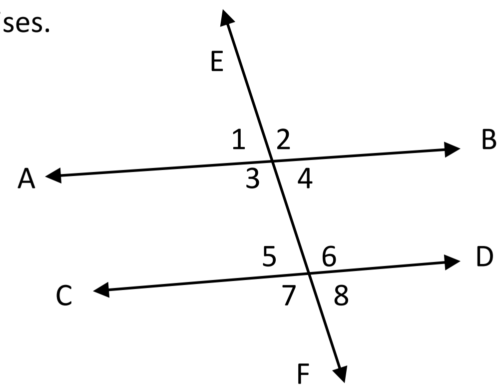
A. Ask students to write a journal entry explaining why learning how to write a proof would be helpful in ways beyond mathematics.

IV. Strategies for differentiation

A. Students may have trouble setting up the proofs. For those having a particularly difficult time, tell them to break the problem down into what they know and what they need to know. It may be easier for some students to create a more informal proof – a paragraph proof where they simply explain what they know and how to get to where they know.

Lesson 2 Homework

Refer to the diagram to the right for the following exercises.
Write your answers in complete sentences.



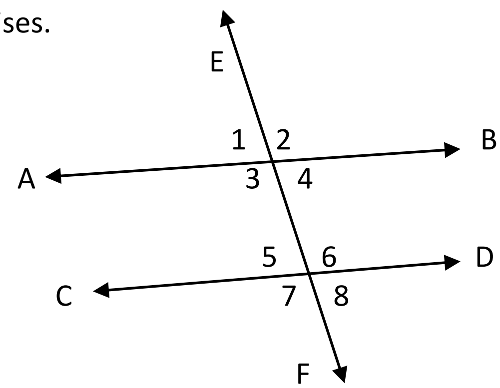
1. If the $m\angle 3 = 4x + 18$ and $m\angle 5 = 6x + 2$,
   Find the measure of x that proves $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.
   Explain why your solution is valid.

2. If the $m\angle 1 = 3x + 5$ and $m\angle 8 = 4x - 22$,
   Find the measure of x that proves $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.
   Explain why your solution is valid.

3. $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. $\angle 4$ and $\angle 6$ are supplements.
   Prove $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

Lesson 2 Homework – Answer Key

Refer to the diagram to the right for the following exercises.
Write your answers in complete sentences.



1. If the $m\angle 3 = 4x + 18$ and $m\angle 5 = 6x + 2$,
   Find the measure of x that proves $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.
   Explain why your solution is valid.

   Angles 3 and 5 are same side interior angles. Same side interior angles in parallel lines are supplementary. This means if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, then $4x + 18 + 6x + 2 = 180$. Then, $x = 16°$.

2. If the $m\angle 1 = 3x + 5$ and $m\angle 8 = 4x - 22$,
   Find the measure of x that proves $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.
   Explain why your solution is valid.

   Angles 1 and 8 are alternate exterior angles. Alternate exterior angles are congruent. This means if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, then $3x + 5 = 4x - 22$. Then, $x = 27°$.

3. $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. $\angle 4$ and $\angle 6$ are supplements.
   Prove $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

   If $\angle 4$ and $\angle 6$ are supplements, then their sum is 180°. By the same side interior angles theorem, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

   You will probably want students to write a lot more than this since proving the parallel line theorems is still fairly new to them. Encourage your students to write as much as they feel is necessary to prove the statements.

LESSON 3: CREATE A MAP

STRAND: Geometry

MATHEMATICAL OBJECTIVES:
- To construct parallel and perpendicular lines, construct an equilateral triangle.
- To solve problems regarding parallel lines cut by a transversal.
- To determine whether two lines are parallel.
- To use algebraic methods as well as deductive proof to verify if two lines are parallel.
- To encourage working cooperatively in small groups.

MATHEMATICS PERFORMANCE EXPECTATIONS:
MPE 32.a, b. The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel; verify the parallelism, using algebraic and coordinate methods as well as deductive proofs.

VIRGINIA SOL: G.2a-c, G.4 c, f, g
- ➢ The student will use the relationships between angles formed by two lines cut by a transversal to determine whether two lines are parallel, verify the parallelism, using algebraic and coordinate methods as well as deductive proofs, and solve real-world problems involving angles formed when parallel lines are cut by a transversal.
- ➢ The student will construct and justify the constructions of:
  - A perpendicular to a given line from a point not on the line.
  - An angle congruent to a given angle.
  - A line parallel to a given line through a point not on the given line.

NCTM STANDARDS:
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Explore relationships among classes of two-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

MATERIALS/RESOURCES: Classroom set of calculators, pencils, markers, rulers, compasses, and poster board.

LESSON OUTLINE:

I. Introduction: Go over previous lesson's homework. (20 –30 minutes)
   A. Put students back into groups of three or four and ask them to critique each other's group. Which proofs are valid? Which are not? Why or why not? What could you change to "fix" the proof?
   B. Ask each group to share a problem with the class. A student should come to the front of the class and model/discuss with the rest of the class. Stress that a problem does not have to be done correctly to be worthy of discussion. It would be beneficial if each group shows a proof done differently from those already discussed.

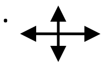
II. Construction of map (20 – 30 minutes)
   A. Keeping students in their groups, tell students they will create a map given the following criteria. (Directions for map construction attached). Each student should create their own map on a piece of poster board though discussion throughout the group is encouraged. When first considering the directions for the map, students should sketch the design somewhere other than the poster board to be certain they know what their goal is before they begin constructions. Tell students to do all constructions in pencil. Once they are happy with all the constructions and their labeling, they should then use a marker to highlight only the streets and labels to try and take the focus away from the constructions. Do not erase the constructions so these can be verified by the instructor.
      1. The teacher should observe and monitor students as they work. Encourage students to rely on each other for help and confirmation.
      2. As students complete their maps, have the students' in the same groups check each others' work.
      3. A copy of completed map is included as an attachment,
   B. Assessment of concepts (30 – 40 minutes)
   C. Students should work on the assessment individually.
   D. Parallel lines assessment attached.
   E. The teacher should discuss the various solutions to the assessment during the next class period.

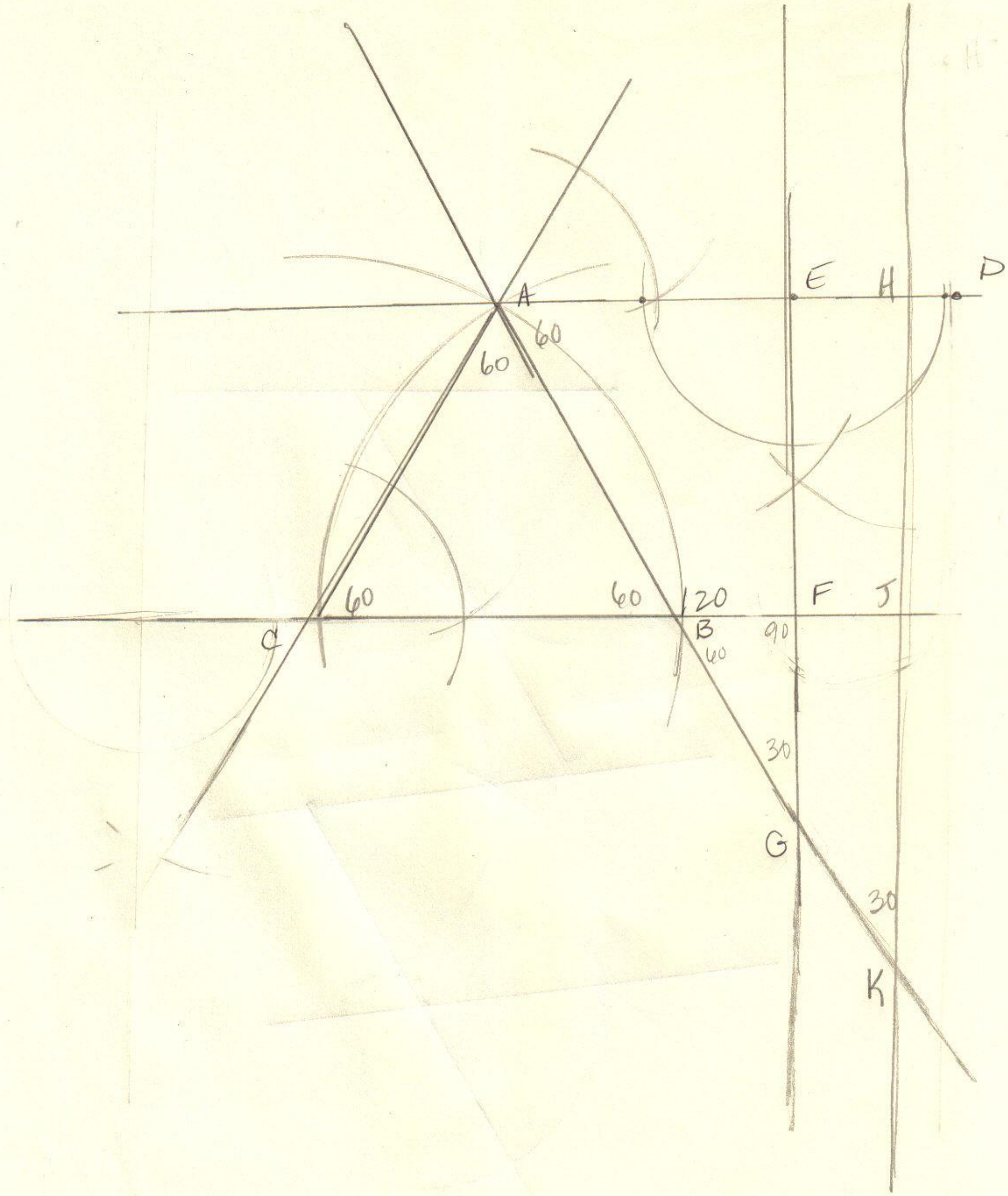
III. Extensions and connections for all students
   A. Ask students to write two questions concerning the map or an addition to the map. These should be thoughtful questions and should include solutions.

IV. Strategies for differentiation
   A. Students may have difficulty using the protractors. You should have a couple of different types of protractors available if possible – traditional protractors and safety protractors.

Directions for Map Construction:

Note* All streets, (lines), constructed should be extended to "run off" the poster board.

1. Begin by sketching a compass to indicate the directions north, south, east and west. Draw this in the upper left hand corner of the poster board. 
2. Construct an equilateral triangle in the center of your poster board so that it is sitting in a standard position:△ . The sides of the triangle should each measure 4 inches. Use a straightedge to extend the lengths of each of the sides. Label the vertices beginning at the top of the triangle in a clockwise direction A, B, and C, respectively. This will give you three streets: AB, BC and CA.
3. Construct a street parallel to street BC. Name this street AD.
4. Construct street perpendicular to street AD so that it lies to the east of the triangle but does not pass through triangle ABC. Label the intersection of the perpendicular and street AD as point E. Label the intersection of the perpendicular street and street CB as point F.
5. Label the intersection of streets AB and EF point G.
6. Construct a street perpendicular to street CB so that it lies to the east of street EF. Label the intersection of this perpendicular and street AD as point H. Label the intersection of the perpendicular and street CB as point J.
7. Label the intersection of streets AB and HJ point K.



A
60
60
E
H
D
60
60
120
F
J
C
B
90
40
30
G
30
K

Parallel Lines Assessment

Directions: Answer each question with a complete sentence. When asked to explain, prove or disprove, answer in as many ways as you find valid using the properties of parallel lines cut by a transversal.

1.
   a) What is the measure of ∠BAC? Explain how you know.
   b) What is the measure of ∠DAB? Explain how you know.
   c) What is the measure of ∠ABF? Explain how you know.
   d) What is the measure of ∠BKJ? Explain how you know.
   e) What is the measure of ∠BGF? Explain how you know.
2.
   a) Are streets AD and CB parallel?
   b) What does it mean for them to be parallel?
   c) Which pairs of angles could you use to prove them parallel? Give an explanation for each pair of angles you claim would prove the two streets parallel.
3.
   a) Are there any other pairs of streets that are parallel?
   b) Why or why not?
   c) Use properties of angles and parallel lines to prove or disprove.

4. Write a proof to explain why $\angle EAC + \angle ACB = 180°$.

5. Imagine the section of the city depicted by your map has a subway that runs entirely underground and directly beneath street EF.
   a) Is the subway parallel to street EF? Explain.
   b) Is the subway parallel to another street? Explain.
   c) Is the subway parallel to more than one street? Explain.
   d) Since the subway runs underneath the streets on the map, it will never intersect with any of the streets shown on the map. Does this mean the subway is parallel to some or all of these streets? Why or why not? Explain.