# SAT & ACT – How Equitable are They?

## I. UNIT OVERVIEW & PURPOSE:

Have you ever wondered if gender or race causes significant changes to SAT or ACT test scores? How do students perform within different states?

This unit allows students to explore both SAT and ACT data to answer the questions above. Calculations to produce box-and-whisker plots as well as normal curves are completed. Once students understand the implications of data that fits a normal curve they will be asked to compare scores on the SAT and ACT tests using z-scores. Probabilities will also be found using the normal curve.

## II. UNIT AUTHORS:

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## III. COURSE:

Mathematical Modeling: Capstone Course

## IV. CONTENT STRAND:

Probability and Statistics

## V. OBJECTIVES:

**SOL: A.9, A.10; AII.11; PS.1, PS.2, PS.3, PS.16**

- Compare two data sets statistically
- Determine the dispersion (spread) of the data visually using a box-and-whisker plot
- Determine the difference between a box-and-whisker plot and a boxplot
- Use quartiles to compare and rank data values within the set
- Encourage students to discuss controversial topics such as gender as it pertains to the data set
- Students will recognize the shape of data that fits the normal curve
- Students will use the necessary statistical descriptors (mean and standard deviation) to sketch the normal curve of a given data set
- Students will use standard deviation to discuss the consistency of data values within a given data set
- Students will use mean to discuss the overall performance of values within a given data set
- Students will connect the normal curve to the empirical rule to determine the

percentage of values contained within certain intervals
- Students will use the normal curve to understand the concept of z-score
- Students will use z-score and the normal curve to find the probability of a data value and vice versa
- Students will use z-scores to standardize and compare data sets that are otherwise uncomparable
- Students will connect the probability to the area under the normal curve as well as percentile
- Allow students to organize and compare data
- Allow students to justify the validity of comparing two data sets
- Allow students to use statistics to describe data set(s)
- Allow students to collaborate on an open-ended group project
- Allow students to present mathematical concepts within the classroom

## VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.8 – The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

MPE.22 – The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

MPE.23 – The student will analyze the normal distribution. Key concepts include
a) characteristics of normally distributed data;
b) percentiles;
c) normalizing data, using z-scores; and
d) area under the standard normal curve and probability.

## VII. CONTENT:

Mathematically this unit specifically addresses using the spread of data through box-and-whisker plots, standard deviation, z-scores and the normal curve to compare two data sets. Upon completion students will be able to discuss the consistency of a data set as well as compare data from dissimilar sets by standardizing the values.

The overall goals of this unit are to make students aware that someone who understands statistics can present the data in a manner that supports their views. However, these views might not be able to be justified. Students will also realize how to use data to show the equity or lack of equity for given scenarios.

**VIII. REFERENCE/RESOURCE MATERIALS:**

Graphing calculators will be required. Numerous documents will be needed on a daily basis – Data Tables, Algebra II Formula Sheets, Focus Activities, Student Exploration Worksheets, and Exit Slip Assessments will be needed for all lessons.

Information on testing bias can be researched or reviewed on the following websites.

- ➔ http://www.insidehighered.com/news/2010/06/21/sat
- ➔ http://diverseeducation.com/article/49830/#
- ➔ http://www.policymic.com/articles/8582/sat-racial-bias-proves-standardized-tests-are-geared-toward-white-students

**IX. PRIMARY ASSESSMENT STRATEGIES:**

Classroom Explorations (participation, completion, and discussion)
Exit Slips

**X. EVALUATION CRITERIA:**

Students will participate in daily class explorations and discussions and complete the exit slip. A rubric is attached to assist in the assessment process. The same rubric will be used on each day of the lesson.

**XI. INSTRUCTIONAL TIME:**

This unit is intended to take five 60 minute class periods.

# Lesson 1 – The Battle of the Sexes

**Strand**
Probability and Statistics

**Mathematical Objective(s)**
- Compare two data sets statistically
- Determine the dispersion (spread) of the data visually using a box-and-whisker plot
- Determine the difference between a box-and-whisker plot and a boxplot
- Use quartiles to compare and rank data values within the set
- Encourage students to discuss controversial topics such as gender as it pertains to the data set

**Mathematics Performance Expectation(s)**
- MPE.8 – The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
- MPE.22 – The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

**Related SOL**
- A.10; PS.1, PS.2, PS.3

**NCTM Standards**
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Recognize how linear transformations of univariate data affect shape, center, and spread
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Make and investigate mathematical conjectures
- Solve problems that arise in mathematics and in other contexts
- Monitor and reflect on the process of mathematical problem solving
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas

**Materials/Resources**

- Classroom Set of Graphing Calculators
- Copies of Focus Activity (Lesson 1 – Basic Statistical Descriptors)
- Copies of Focus Activity (Lesson 1 – Viewing Testing Bias)
  - http://lhs.loswego.k12.or.us/z-hoppesk/APHIR/Quarter2/activities/Measuring%20Mental%20Fitness.pdf
  - http://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&docid=6Nbqgu9qtI9YsM&tbnid=DPq4Zb3XJOcsrM:&ved=0CAUQjRw&url=http%3A%2F%2Fwww.learntoquestion.com%2Fclass%2Farchives%2F2000-2001%2Fcurrlesson%2Ftext%2F12.html&ei=rjWQUYzoGYXo9ATawoCoBg&bvm=bv.46340616,d.eWU&psig=AFQjCNGHsLrptlhVh8AdCTosWAkLvZuysA&ust=1368491793838839
- Copies of Student Exploration Packet (Lesson 1 – Battle of the Sexes)
- Copies of Assessment/Exit Slip (Lesson 1 – Battle of the Sexes)
- Copies of Data Table (Lesson 1 – Battle of the Sexes)
- Copies of Algebra II SOL Formula Sheet.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_algebra2.pdff
- Copies of Algebra II Standard Normal Probabilities Table.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_z_table.pdf

**Assumption of Prior Knowledge**

- Students should be able to draw on prior knowledge of measures of central tendency, box-and-whisker plots, normal distributions, and z-scores.
  - Students can recognize and name statistic descriptors of a data set.
  - Relationships between statistic descriptors cannot be concretely connected.
  - Students can reason informally about the statistical characteristics of the data set.
- Students will begin to compare statistical descriptors of two data sets. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly identify outliers or construct a box-and-whisker plot and will not be able to make meaningful comparisons between two data sets.
- Students should be familiar with calculating univariate statistics using the TI-84+.
- Relevant contexts: Analysis and impact of ACT and SAT scores.

## Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify and successfully calculate statistical descriptors (mean, median, mode, range, outliers, upper quartile, and lower quartile) of multiple data sets and be able to graphically represent the

dispersion of the data in a box-and-whisker plot or boxplot. The calculations and plot will be used to compare the effects that gender has on the test and then judge the equity of the test based on the comparison.

- Describe planned time outline.
    - Focus Activity and Discussion (Two Options)
    - Basic Statistical Descriptors (Stats Review) or 1917 Army IQ Test (Viewing Testing Bias)
        - Basic Statistical Descriptors (15 minutes)
          Think – Complete focus worksheet individually.
          Pair – Group students together to discuss the terminology and then extend the activity into calculations.
          Share – Use the class discussion to formally define the terms.
          Check for student understanding of necessary prior knowledge and skills. Be sure to include other terms that students remembered in the discussion (i.e. standard deviation).
        - 1917 Army IQ Test (15 minutes)
          http://lhs.loswego.k12.or.us/z-hoppesk/APHIR/Quarter2/activities/Measuring%20Mental%20Fitness.pdf
          Have students work as many of the six pieces of this test as wanted. Discuss with students the answers. Ask students if they found any evidence of bias in the test. Lead a discussion on how the results of this test affected the events in 1917. It is important for students to realize that data can legitimize racist or sexist views. Recognizing bias within forms of standardized testing can shed some light on the reasons for the differences in testing scores based on gender or race.
          (Some may want to take this part of the activity and use it for an introduction to the unit on the day prior if time allows. You may want to use the full version of the "picture completion" portion of the test.
          http://www.google.com/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&docid=6Nbqgu9qtI9YsM&tbnid=DPq4Zb3XJOcsrM:&ved=0CAUQjRw&url=http%3A%2F%2Fwww.learntoquestion.com%2Fclass%2Farchives%2F2000-2001%2Fcurrlesson%2Ftext%2F12.html&ei=rjWQUYzoGYXo9ATawoCoBg&bvm=bv.46340616,d.eWU&psig=AFQjCNGHsLrptlhVh8AdCTosWAkLvZuysA&ust=1368491793838839)

    - Student Exploration
        - Battle of the Sexes (20-25 minutes)

In small groups have students work through the exploration packet.
    - Classroom Discussion of Activity (10 minutes)
        - See questions below.
    - Exit Slip Assessment (10 minutes)
- Introduce the task.
    - By comparing SAT scores for males and females over thirty-one years students will discuss trends in the data and the effects gender seems to play on testing results. Students will then discuss the equity of the test based on gender.
- Questions or prompts to pose.
    - What are the differences between mean, median, and mode?
    - When is the mean used to "best" describe the measure of central tendency? The median? The mode?
    - What percent of data falls within the interquartile range (IQR)? Why?
    - How do you identify an outlier? (Bring up the mathematical way $(1.5 \cdot IQR)$ if it hasn't been discussed yet.)
    - How does a box-and-whisker plot differ from a boxplot? (In some texts the box plot does not include outliers in plot. Instead the create the plot without the outliers and then place the outliers as dots outside of the range of the box-and-whisker plot.)
    - Why does the dispersion of data matter?
    - How do we use quartiles to compare data values?
    - Why do you think gender effects the results of the SAT test?
    - Is it fair to compare SAT scores between males and females?

## Student Exploration: Battle of the Sexes

**Student/Teacher Actions:**

- Students should use the formula sheet and/or calculator to obtain the necessary statistical descriptors needed to answer the questions in the exploration activity.
- Teacher(s) will be guiding student groups as needed if questions/problems arise. Specifically when asked to compare two box-and-whisker graphs to obtain meaningful results.
- It might be necessary to ask students to explain the difference between mean, median, and mode of a data set.
- Students will have 10 minutes to discuss results and ask questions in the classroom setting.

**Monitoring Student Responses**

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties by clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to compare other data sets of interest to the student. The student will most likely need computer/internet access to retrieve the needed data.

## Assessment

- See the Rubric provided in Lesson 1.

## Extension and Connections (for all students)

- Students are encouraged to think deeply about the results they generate during today's activities. Did you predict the result? If so, can you explain why? If not, can you explain why?
- Students are encouraged to question why they are comparing the data sets. Is it "fair" to compare apples to oranges? If so, why and how? If not, why?

## Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 1 – Basic Statistical Descriptors

Name: _____

Think – Statistics Terminology

Below are some vocabulary words that you have studied in the past. Working individually please define or give an example for as many of the terms listed below as possible. There are a few rows left "blank". If you can remember any other term related to the topic of statistics that you have used in the past please include them in the table.

| **Vocabulary Term** | **Definition/Example** |
|---|---|
| **Mean** | |
| **Median** | |
| **Mode** | |
| **Range** | |
| **Outlier** | |
| **Box-and-whisker Plot** | |
| **Upper Quartile** | |
| **Lower Quartile** | |
| **Interquartile Range** | |
| | |
| | |

Pair!

Now with a partner discuss both of your ideas on the terms listed and any others that either of you included. You may want to edit your work above before submitting this table that will be presented to the class.

Focus Activity – Lesson 1 – Basic Statistical Descriptors

Name: _____

Pair!

Several values have been calculated for a given data set. With your partner match each value to its proper descriptor in the table provided.

| | |
|---|---|
| Data Set<br>$63, 42, 17, 75, 48, 82, 57, 48, 72$ | |
| Match each of the following to one of the descriptors below.<br>$28.5 \quad 48 \quad 56 \quad 73.5 \quad 45 \quad 57 \quad 65$ | |
| Mean | |
| Median | |
| Mode | |
| Range | |
| Upper Quartile | |
| Lower Quartile | |
| Interquartile Range | |
| Given the data set about, is there an outlier? Why or why not? | |
| Which measure of central tendency ***BEST*** describes the data? Why? | |

Share – Statistics

After completing this activity each pair will participate in a class discussion about statistics (specifically measures of central tendency) by sharing their results with the class.

# Data for Lesson 1 – Battle of the Sexes

**SAT Mean Scores of College Bound Seniors, by Sex**

| School Year | Critical Reading Score | | Mathematics Score | |
|---|---|---|---|---|
| | Male | Female | Male | Female |
| 1976-1977 | 509 | 505 | 520 | 474 |
| 1977-1978 | 511 | 503 | 517 | 474 |
| 1978-1979 | 509 | 501 | 516 | 473 |
| 1979-1980 | 506 | 498 | 515 | 473 |
| 1980-1981 | 508 | 496 | 516 | 473 |
| 1981-1982 | 509 | 499 | 516 | 473 |
| 1982-1983 | 508 | 498 | 516 | 474 |
| 1983-1984 | 511 | 498 | 518 | 478 |
| 1984-1985 | 514 | 503 | 522 | 480 |
| 1985-1986 | 515 | 504 | 523 | 479 |
| 1986-1987 | 512 | 502 | 523 | 481 |
| 1987-1988 | 512 | 499 | 521 | 483 |
| 1988-1989 | 510 | 498 | 523 | 482 |
| 1989-1990 | 505 | 496 | 521 | 483 |
| 1990-1991 | 503 | 495 | 520 | 482 |
| 1991-1992 | 504 | 496 | 521 | 484 |
| 1992-1993 | 504 | 497 | 524 | 484 |
| 1993-1994 | 501 | 497 | 523 | 487 |
| 1994-1995 | 505 | 502 | 525 | 490 |
| 1995-1996 | 507 | 503 | 527 | 492 |
| 1996-1997 | 507 | 503 | 530 | 494 |
| 1997-1998 | 509 | 502 | 531 | 496 |
| 1998-1999 | 509 | 502 | 531 | 495 |
| 1999-2000 | 507 | 504 | 533 | 498 |
| 2000-2001 | 509 | 502 | 533 | 498 |
| 2001-2002 | 507 | 502 | 534 | 500 |
| 2002-2003 | 512 | 503 | 537 | 503 |
| 2003-2004 | 512 | 504 | 537 | 501 |
| 2004-2005 | 513 | 505 | 538 | 504 |
| 2005-2006 | 505 | 502 | 536 | 502 |
| 2006-2007 | 504 | 502 | 533 | 499 |
| 2007-2008 | 504 | 500 | 533 | 500 |
| 2008-2009 | 503 | 498 | 534 | 499 |
| 2009-2010 | 503 | 498 | 534 | 500 |
| 2010-2011 | 500 | 495 | 531 | 500 |
| 2011-2012 | 498 | 493 | 532 | 499 |

*NOTE: Data for 1976-77 to 1985-86 were converted to the recentered scale by using a formula applied to the original mean and standard deviation. For 1986-87 to 1994-95, individual student scores were converted to the recentered scale and then the mean was recomputed. For 1995-96 to 1998-99, nearly all students received scores on the recentered scale; any score on the original scale was converted to the recentered scale prior to recomputing the mean. From 1999-2000 on, all scores have been reported on the recentered scale.*

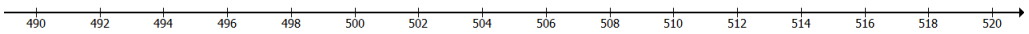
*NOTE: Data for 1976-77 through 2009-10 are for seniors who took the SAT any time during their high school years through March of their senior year. Data for 2010-11 onwards are for seniors who took the SAT any time during their high school years through June of their senior year. If a student took a test more than once, the most recent score was used. The SAT was formerly known as the Scholastic Assessment Test and the Scholastic Aptitude Test. Possible scores on each part of the SAT range from 200 to 800. The critical reading section was formerly known as the verbal section.*

*SOURCE: National Center for Education Statistics (NCES), 2012 Digest of Education Statistics, retrieved April 5, 2013 (nces.ed.gov/programs/digest/d12/tables/dt12_154.asp).*

Student Exploration – Lesson 1 – Battle of the Sexes

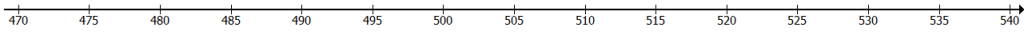
NAME: _____

1. Reading SAT Scores
   a. On the same set of axes, create a box-and-whisker plot for the male reading scores and the female reading scores.




   b. Write a paragraph comparing and contrasting the two plots.

2. Math SAT Scores
   a. On the same set of axes, create a box-and-whisker plot for the male math scores and the female math scores.

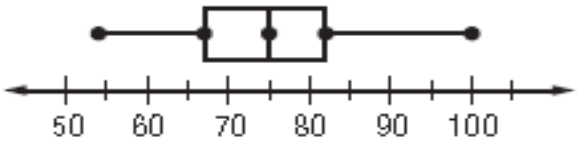



   b. Write a paragraph comparing and contrasting the two plots.

Exit Slip – Lesson 1 – Battle of the Sexes

NAME: _____

The box-and- whisker plot below represents the test scores in Mr. Gray's Algebra I class.




Algebra I Test Scores

1. What percent of the students scored between 67 and 82?

2. Approximately, what is the $25^{th}$ percentile test score in Mr. Gray's class?

3. Do you think the test was too hard? Explain your answer.

4. If there are 24 students in Mr. Gray's class, how many students scored below a 67? Explain how you arrived at your answer.

5. What score would a student have to make to be considered an outlier?

Focus Activity – Lesson 1 – Basic Statistical Descriptors

Name: ___Answer Key_____

Think – Statistics Terminology

| **Vocabulary Term** | **Definition/Example** |
|---|---|
| **Mean** | The mathematical average, found by adding all data values together and dividing by the number of data values. |
| **Median** | The number found in the middle of a data set when in numerical order. |
| **Mode** | The number occurring most often in the data set. |
| **Range** | The difference of the largest data value and the smallest data value. |
| **Outlier** | A data value that is distinctly different from the rest of the data.<br>OR<br>A data value that is more than 1.5 interquartile ranges below Q1 or above Q3. |
| **Box-and-whisker Plot** | A visual (graphical) way to show the distribution of a data set. |
| **Upper Quartile** | The median of the upper half of data values.<br>(also known as the third quartile) |
| **Lower Quartile** | The median of the lower half of data values.<br>(also known as the first quartile) |
| **Interquartile Range** | The difference of the upper quartile and the lower quartile. |
| | |
| | |

Focus Activity – Lesson 1 – Basic Statistical Descriptors

Name: _____Answer Key_____

Pair!

Several values have been calculated for a given data set. With your partner match each value to its proper descriptor in the table provided.

Data Set

$$63, 42, 17, 75, 48, 82, 57, 48, 72$$

Match each of the following to one of the descriptors below.

$$48 \qquad 56 \qquad 73.5 \qquad 45 \qquad 57 \qquad 65$$

| | |
|---|---|
| Mean | 56 |
| Median | 57 |
| Mode | 48 |
| Range | 65 |
| Upper Quartile | 73.5 |
| Lower Quartile | 45 |
| Interquartile Range | 28.5 |

Given the data set about, is there an outlier? Why or why not?

Depending on how the student defined outlier…
Most students would identify 17 as an outlier based on the first definition provided. However 17 is not 1.5 times below the lower quartile, so some students might argue that there is no outlier in the given data set.

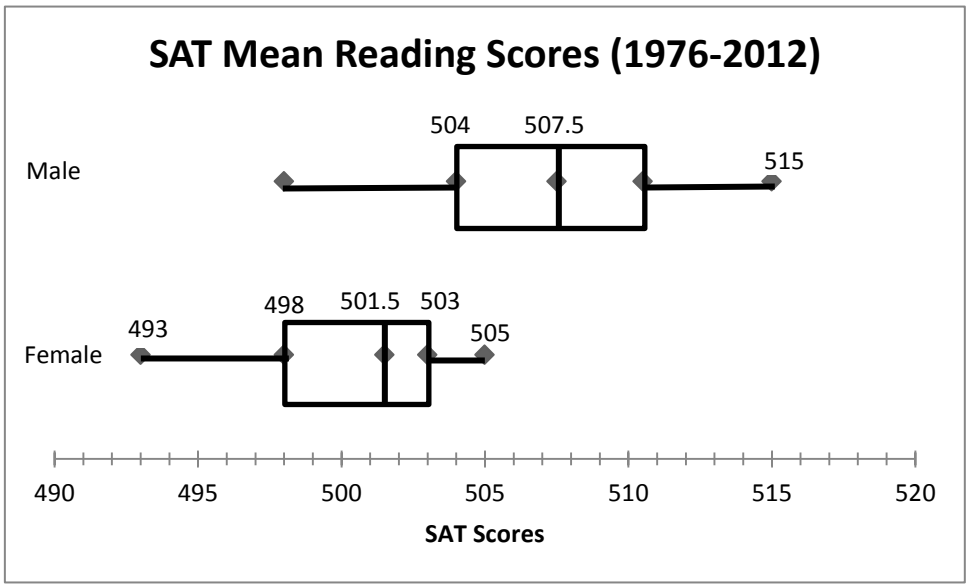
Which measure of central tendency ***BEST*** describes the data? Why?

This could also vary depending on the students' definition of outlier. The mean describes a data set without outliers the best. The median describes a data set with outliers the best. The mode is used to describe data sets involving nominal data.

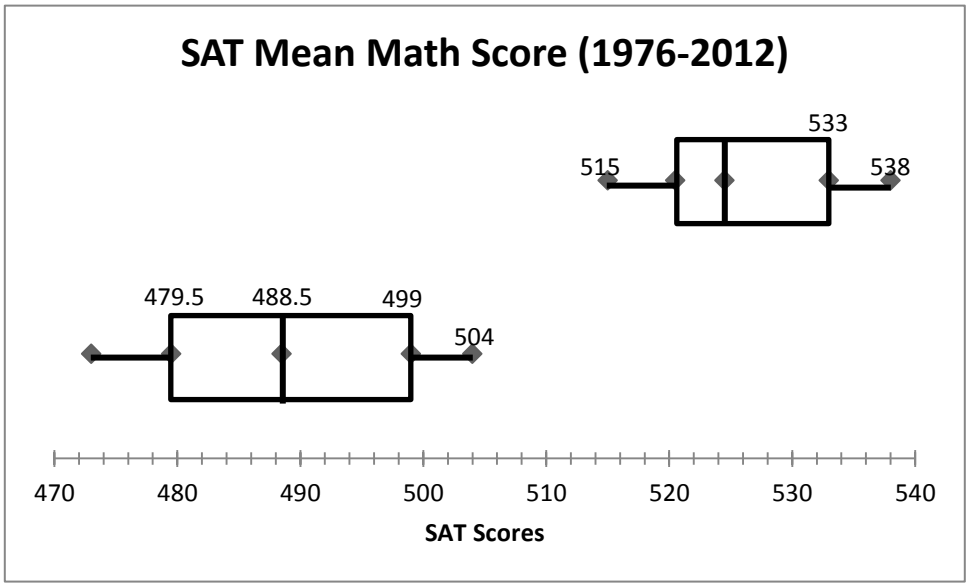
Student Exploration – Lesson 1 – Battle of the Sexes

NAME: ___Answer Key_____

1. Reading SAT Scores
   a. On the same set of axes, create a box-and-whisker plot for the male reading scores and the female reading scores.




   b. Write a paragraph comparing and contrasting the two plots. Answers will vary.
2. Math SAT Scores
   a. On the same set of axes, create a box-and-whisker plot for the male math scores and the female math scores.

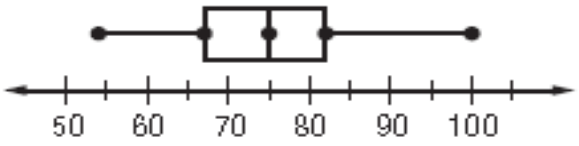



   b. Write a paragraph comparing and contrasting the two plots. Answers will vary.

Exit Slip – Day 1 – Box-and-Whisker Plots

NAME: ___Answer Key_____

The box-and- whisker plot below represents the test scores in Mr. Gray's Algebra I class.




1. What percent of the students scored between 67 and 82?

   50%

2. Approximately, what is the 25th percentile test score in Mr. Gray's class?

   67

3. Do you think the test was too hard? Explain your answer.

   Student answers will vary.

   Sample answer: No, because 75% of the class scored a 67 or higher.

4. If there are 24 students in Mr. Gray's class, how many students scored below a 67? Explain how you arrived at your answer.

   6 students

5. What score would a student have to make to be considered an outlier?

   1.5(82 - 67) = 22.5

   44.5 or below (not possible to score a 132.5 or higher)

RUBRIC: Lesson # _____ NAME: _____

(To be used for Lessons 1 – 3)

| ***Category*** | ***3*** | ***2*** | ***1*** |
|---|---|---|---|
| ***Participation in Exploration*** | Participates in **ALL** parts of the exploration | Participates in **MOST** parts of the exploration. | Little participation in the exploration activity. |
| ***Participation in Class Discussion*** | Actively participates in **ALL** parts of the classroom discussion. Makes and explains valid arguments. | Actively participates in **MOST** parts of the classroom discussion. Makes but does not justify arguments. | Little participation in classroom discussion. Does not make valid arguments. |
| ***Exit Slip*** | Answers question(s) with process, calculations, or justification. | Answers question(s) with partial process, calculations, or justification. | Answers question(s) with no process, calculations, or justification. |

# Lesson 2 – Does Gender Affect Test Scores?

**Strand**
Probability and Statistics

**Mathematical Objective(s)**
- Students will recognize the shape of data that fits the normal curve
- Students will use the necessary statistical descriptors (mean and standard deviation) to sketch the normal curve of a given data set
- Students will use standard deviation to discuss the consistency of data values within a given data set
- Students will use mean to discuss the overall performance of values within a given data set
- Students will connect the normal curve to the empirical rule to determine the percentage of values contained within certain intervals

**Mathematics Performance Expectation(s)**
- MPE.8 – The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
- MPE.22 – The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.
- MPE.23 – The student will analyze the normal distribution. Key concepts include
  a) characteristics of normally distributed data

**Related SOL**
- A.9; AII.11; PS.1, PS.2, PS.3, PS.16; AII.11

**NCTM Standards**
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Recognize how linear transformations of univariate data affect shape, center, and spread
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Make and investigate mathematical conjectures
- Solve problems that arise in mathematics and in other contexts
- Monitor and reflect on the process of mathematical problem solving

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas

**Materials/Resources**

- Classroom Set of Graphing Calculators
- Copies of Focus Activity (Does Gender Affect Test Scores?)
- Copies of Student Exploration Packet (Does Gender Affect Test Scores?)
- Copies of Assessment (Exit Slip – Does Gender Affect Test Scores?)
- Copies of Lesson 2 Data Table.
- Copies of Algebra II SOL Formula Sheet.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_algebra2.pdff
- Copies of Algebra II Standard Normal Probabilities Table.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_z_table.pdf

**Assumption of Prior Knowledge**

- Students should be able to draw on prior knowledge of measures of central tendency, box-and-whisker plots, normal distributions, and z-scores.
  - Students can recognize and name statistic descriptors of a data set.
  - Relationships between statistic descriptors cannot be concretely connected.
  - Students can reason informally about the statistical characteristics of the data set.
- Students will begin to compare statistical descriptors of two data sets. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly identify outliers or construct a box-and-whisker plot and will not be able to make meaningful comparisons between two data sets.
- Students should be familiar with calculating univariate statistics using the TI-84+.
- Relevant contexts: Analysis and impact of ACT and SAT scores.

## Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson the student will be able to identify and successfully calculate statistical descriptors (mean and standard deviation) of multiple data sets and be able to graphically represent the dispersion of the data as a normal curve. The calculations and graphical representation will be used to compare the effects that gender has on the test and then to judge the equity of the tested based on the comparison.
- Describe planned time outline.

    - Focus Activity and Discussion
        - Does Gender Affect Test Scores? (15 minutes)
          Students should complete the activity individually.
          Class discussion will follow. Be sure to discuss the characteristic shape of the histogram as the "normal curve".
    - Student Exploration
        - Does Gender Affect Test Scores? (20-25 minutes)
          In pairs have students work through the exploration packet.
    - Classroom Discussion of Activity (10 minutes)
        - See questions below.
    - Exit Slip Assessment (10 minutes)
- Introduce the task.
    - By comparing ACT scores for males and females over twelve years students will discuss trends in the data and the effects gender seems to play on testing results. Students will then discuss the equity of the test based on gender.
- Questions or prompts to pose.
    - What typical shape is the normal curve referred to?
    - Does the standard deviation show better performance or more consistency?
    - Does the mean show better performance or more consistency?
    - Is it possible for one group to show better performance but another to show more consistency?
    - Can you give an example comparing two groups when one group always performs better and is more consistent?
    - How is the empirical rule connected to the normal curve?
    - How does the "normal" curve get its name?

## Student Exploration: Does Gender Affect Test Scores?

**Student/Teacher Actions:**

- Students should use the formula sheet and/or calculator to obtain the necessary statistical descriptors needed to answer the questions in the exploration activity.
- Teacher(s) will be guiding student groups as needed if questions/problems arise. Specifically when asked to compare the two data sets in the context of a normal curve to obtain meaningful results.
- Students are likely to confuse consistency (standard deviation) and overall performance (mean).

- Students will have 10 minutes to discuss results and ask questions in the classroom setting.

**Monitoring Student Responses**

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties by clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to compare other data sets of interest to the student. The student will most likely need computer/internet access to retrieve the needed data.

## Assessment

- See the Rubric provided in Lesson 1.

## Extension and Connections (for all students)

- Students are encouraged to think deeply about the results they generate during today's activities. Did you predict the result? If so, can you explain why? If not, can you explain why?
- Students are encouraged to question why they are comparing the data sets. Is it "fair" to compare apples to oranges? If so, why and how? If not, why?

## Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

# *Data for Lesson 2 – Does Gender Affect Test Scores?*

***ACT Subject-Area Score Averages, by Gender***

| Subject-Area Scores | 2000 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |
| English | 20.5 | 20.2 | 20.3 | 20.4 | 20.4 | 20.6 | 20.7 | 20.6 | 20.6 | 20.5 | 20.6 | 20.5 |
| Male | 20.0 | 19.7 | 19.8 | 19.9 | 20.0 | 20.1 | 20.2 | 20.1 | 20.2 | 20.1 | 20.2 | 20.2 |
| Female | 20.9 | 20.6 | 20.7 | 20.8 | 20.8 | 21.0 | 21.0 | 21.0 | 20.9 | 20.8 | 20.9 | 20.9 |
| | | | | | | | | | | | | |
| Mathematics | 20.7 | 20.6 | 20.6 | 20.7 | 20.7 | 20.8 | 21.0 | 21.0 | 21.0 | 21.0 | 21.1 | 21.1 |
| Male | 21.4 | 21.2 | 21.2 | 21.3 | 21.3 | 21.5 | 21.6 | 21.6 | 21.6 | 21.6 | 21.6 | 21.7 |
| Female | 20.2 | 20.1 | 20.1 | 20.2 | 20.2 | 20.3 | 20.4 | 20.4 | 20.4 | 20.5 | 20.6 | 20.6 |
| | | | | | | | | | | | | |
| Reading | 21.4 | 21.1 | 21.2 | 21.3 | 21.3 | 21.4 | 21.5 | 21.4 | 21.4 | 21.3 | 21.3 | 21.3 |
| Male | 21.2 | 20.9 | 21.0 | 21.1 | 21.0 | 21.1 | 21.2 | 21.2 | 21.3 | 21.1 | 21.1 | 21.2 |
| Female | 21.5 | 21.3 | 21.4 | 21.5 | 21.5 | 21.6 | 21.6 | 21.5 | 21.4 | 21.4 | 21.4 | 21.4 |
| | | | | | | | | | | | | |
| Science Reasoning | 21.0 | 20.8 | 20.8 | 20.9 | 20.9 | 20.9 | 21.0 | 20.8 | 20.9 | 20.9 | 20.9 | 20.9 |
| Male | 21.6 | 21.3 | 21.3 | 21.3 | 21.4 | 21.4 | 21.4 | 21.3 | 21.4 | 21.4 | 21.4 | 21.4 |
| Female | 20.6 | 20.4 | 20.4 | 20.5 | 20.5 | 20.5 | 20.5 | 20.4 | 20.4 | 20.5 | 20.5 | 20.5 |

NOTE: Minimum score is 1 and maximum score is 36.

NOTE: Data are for high school graduates who took the ACT during their sophomore, junior, or senior year. If a student took a test more than once, the most recent score was used. Race categories exclude persons of Hispanic ethnicity. Some data have been revised from previously published figures.

NOTE: The composite score for the ACT includes scores from all four section (English, Math, Reading, and Science) of the ACT.

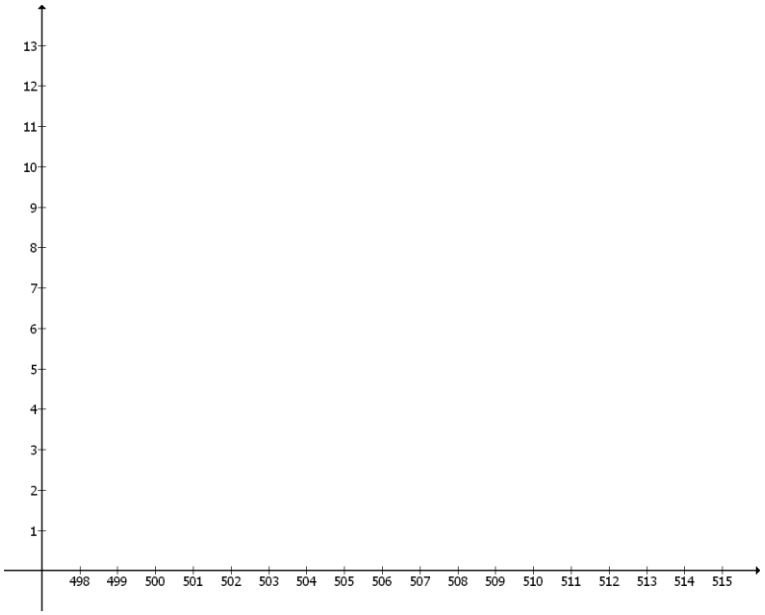
SOURCE: National Center for Education Statistics (NCES), 2012 Digest of Education Statistics, retrieved April 5, 2013 (nces.ed.gov/programs/digest/d12/tables/dt12_157.asp).

Focus Activity – Lesson 2 – Does Gender Affect Test Scores?

NAME: _____

1. Create a histogram for the male reading scores.

| Score Interval | Tally | Frequency |
|---|---|---|
| 498-500 | | |
| 501-503 | | |
| 504-506 | | |
| 507-509 | | |
| 510-512 | | |
| 513-515 | | |




Summarize your findings in a paragraph. Please include the following questions as part of your summarization: What do you notice about the shape of your distribution? What do you notice about the measures of central tendency?

2. Tom took the ACT test three times. He scored a 21, 20, and a 25. Joe also took the ACT test three times. Joe scored a 19, 21, and a 24. Whose scores do you think are more consistent? Explain your reasoning.

Student Exploration – Day 2 – Does Gender Affect Test Scores?

NAME: _____

1. Using the data table provided complete the following table.

| | Male ACT Reading Scores | Female ACT Reading Scores | Male ACT Math Scores | Female ACT Math Scores |
|---|---|---|---|---|
| Mean $(\mu)$ | | | | |
| Standard Deviation $(\sigma)$ | | | | |

Please use the calculations from the table above to answer the following questions.

2. Are males more consistent in reading or math? Justify your answer.

3. Are females more consistent in reading or math? Justify your answer.

4. Are males or females more consistent in reading? Justify your answer.

5. Are males or females more consistent in math? Justify your answer.

6. Based on the Empirical rule, estimate what percent of the male students in the U.S. scored below a 21.3 on the Math section of the ACT. Please provide a sketch to justify your answer.

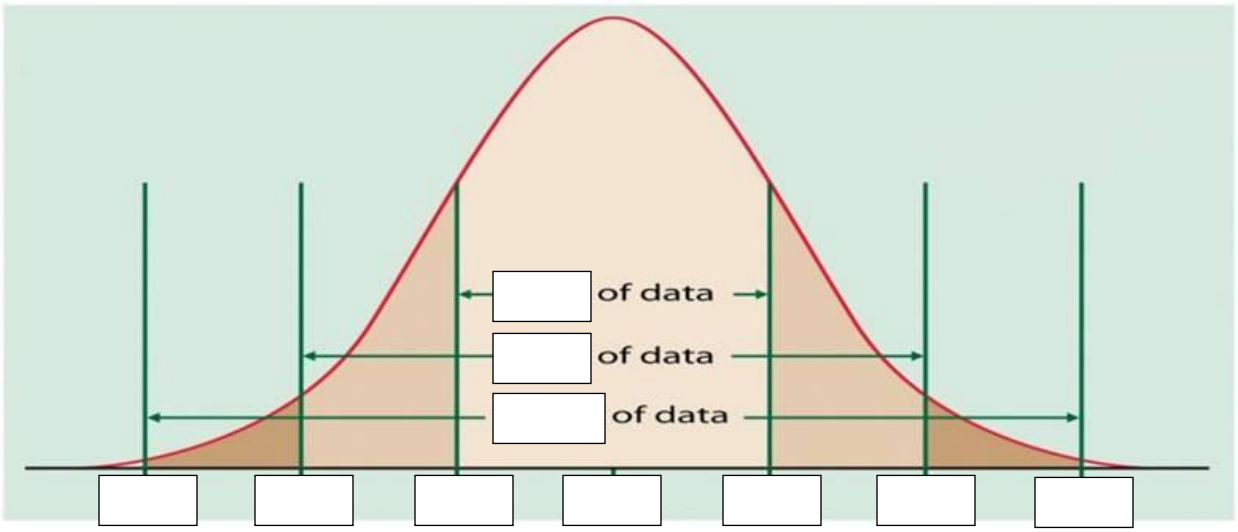
7. Based on the Empirical rule, estimate what percent of the female students in the U.S. scored between 21.3 and a 21.6 on the reading section of the ACT. Please provide a sketch to justify your answer.

Exit Slip – Lesson 2 – Does Gender Affect Test Scores?

NAME: _____

1. If one group's scores are more consistent then another does that imply that particular group did better on the ACT test? Explain.

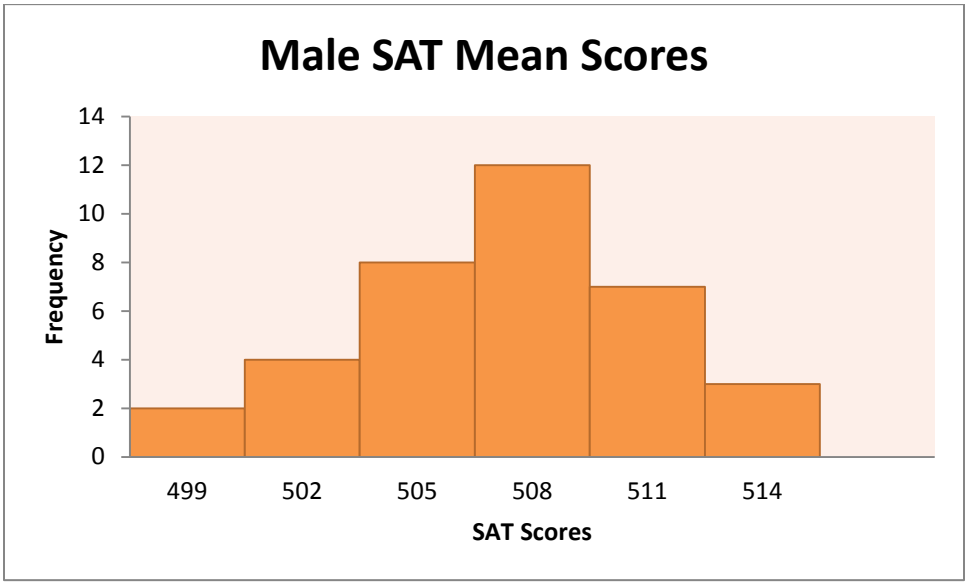
2. Label all that you know about the normal curve below.



Focus Activity – Lesson 2 – Does Gender Affect Test Scores?

NAME: ____Answer Key_____

1. Create a histogram for the male reading scores.




Summarize your findings in a paragraph. Please include the following questions as part of your summarization: What do you notice about the shape of the distribution? What do you notice about the measures of central tendency?

Student answers will vary.

- Approximately symmetric and bell shaped.
- Mean = 507.4, Median = 507.5, Mode = 509
- Approximate normal distribution.

2. Tom took the ACT test three times. He scored a 21, 20, and a 25. Joe also took the ACT test three times. Joe scored a 19, 21, and a 24. Without making any calculations, who do you think was more consistent? Explain your reasoning.

   Student answers will vary.

Range is the same for both students so this should lead into a class discussion on the need for another method to calculate the "spread" of a data set . . . variance and standard deviation.

Student Exploration – Day 2 – Does Gender Affect Test Scores?

NAME: _____

1. Using the data table provided complete the following table.

| | Male ACT Reading Scores | Female ACT Reading Scores | Male ACT Math Scores | Female ACT Math Scores |
|---|---|---|---|---|
| Mean ($\mu$) | 21.1 | 21.5 | 21.5 | 20.3 |
| Standard Deviation ($\sigma$) | .1 | .1 | .2 | .2 |

Please use the calculations from the table above to answer the following questions.

2. Are males more consistent in reading or math? Justify your answer.

   Males are more consistent in Reading because the Reading standard deviation is smaller than the math standard deviation (making the data less spread out).

3. Are females more consistent in reading or math? Justify your answer.

   Females are more consistent in Reading because the Reading standard deviation is smaller than the math standard deviation (making the data less spread out).

4. Are males or females more consistent in reading? Justify your answer.

   When rounding to one decimal place, both males and females have the same standard deviation on the Reading section of the ACT. If you round two decimal places, the males would show to be a little more consistent than the females.

5. Are males or females more consistent in math? Justify your answer.

   Equally consistent, males and females have the exact same standard deviation on the math section of the ACT.

6. Based on the Empirical rule, estimate what percent of the males students in the U.S. scored below a 21.3 on the Math section of the ACT. Please provide a sketch to justify your answer.

   Check students' sketches. $34 + 50 \approx 84\%$

7. Based on the Empirical rule, estimate what percent of the female students in the U.S. scored between 21.3 and a 21.6 on the reading section of the ACT. Please provide a sketch to justify your answer.

   Check students' sketches. $68 + 13.5 \approx 81.5\%$

Exit Slip – Lesson 2 – Does Gender Affect Test Scores?

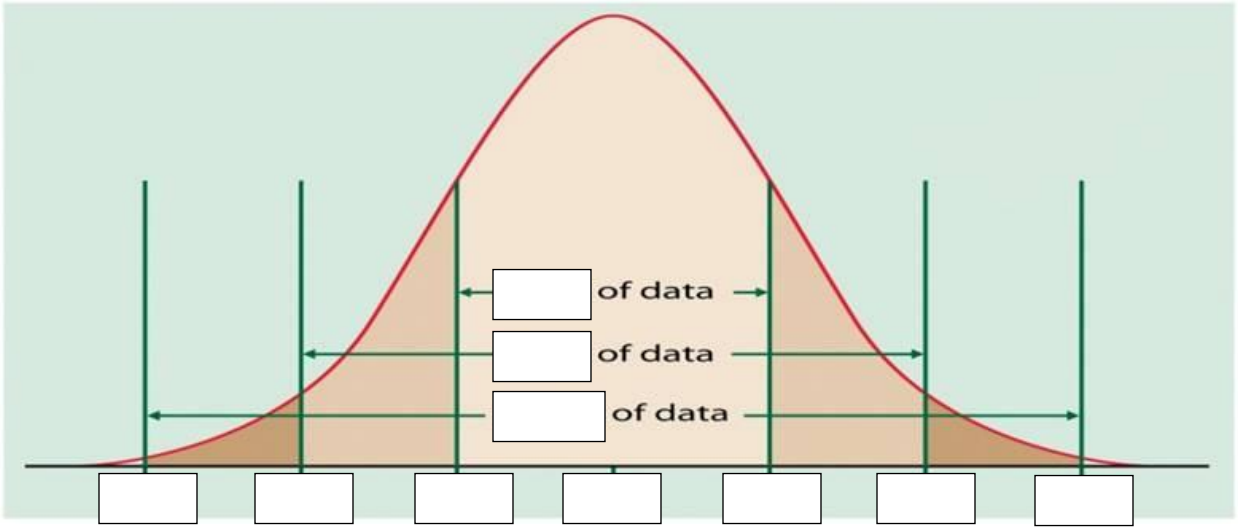
NAME: _____

1. If one group's scores are more consistent then another does that imply that particular group did better on the ACT test? Explain.

   Student answers may vary.

   Sample: No, being more consistent implies that there is less variation in the data about the mean (smaller standard deviation) but it does not tell us how well on group did.

2. Label all that you know about the normal curve below.



# Lesson 3 – Race to the Top

**Strand**
Probability and Statistics

**Mathematical Objective(s)**
- Students will use the normal curve to understand the concept of z-score
- Students will use z-score and the normal curve to find the probability of a data value and vice versa
- Students will use z-scores to standardize and compare data sets that are otherwise uncomparable
- Students will connect the probability to the area under the normal curve as well as percentile

**Mathematics Performance Expectation(s)**
- MPE.23 – The student will analyze the normal distribution. Key concepts include
  a) characteristics of normally distributed data;
  b) percentiles;
  c) normalizing data, using z-scores; and
  d) area under the standard normal curve and probability.

**Related SOL**
- A.9; AII.11; PS.16

**NCTM Standards**
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Recognize how linear transformations of univariate data affect shape, center, and spread
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Make and investigate mathematical conjectures
- Solve problems that arise in mathematics and in other contexts
- Monitor and reflect on the process of mathematical problem solving
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas

**Materials/Resources**
- Classroom Set of Graphing Calculators

- Copies of Focus Activity (Race to the Top)
- Copies of Student Exploration Packet (Race to the Top)
- Copies of Assessment (Exit Slip – Race to the Top)
- Copies of the Lesson 3 Data Tables (2 separate pages to allow for easy data comparison)
- Copies of Algebra II SOL Formula Sheet.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_algebra2.pdff
- Copies of Algebra II Standard Normal Probabilities Table.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_z_table.pdf

**Assumption of Prior Knowledge**

- Students should be able to draw on prior knowledge of measures of central tendency, box-and-whisker plots, normal distributions, and z-scores.
  - Students can recognize and name statistic descriptors of a data set.
  - Relationships between statistic descriptors cannot be concretely connected.
  - Students can reason informally about the statistical characteristics of the data set.
- Students will begin to compare statistical descriptors of two data sets. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly identify outliers or construct a box-and-whisker plot and will not be able to make meaningful comparisons between two data sets.
- Students should be familiar with calculating univariate statistics using the TI-84+.
- Relevant contexts: Analysis and impact of ACT and SAT scores.

## Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify and successfully calculate statistical descriptors (mean, standard deviation, z-score, probability, percentile, area under the normal curve) of multiple data sets. The calculations will be used to compare the effects that race has on testing results and then judge the equity of the test based on this comparison.
- Describe planned time outline.
  - Focus Activity and Discussion
    - Race to the Top? (10 minutes)
      Students should individually complete the activity.
      Class discussion should link the standard deviations of the normal curve to z-score.

    - Student Exploration
        - Race to the Top? (25-30 minutes)
          Think – Complete as much as possible on the exploration worksheet using the provided data. (~ 10 minutes)
          Pair – Group students together to compare and discuss answers to the exploration activity. It may be necessary to cue students that z-score is used to find probabilities. (~20 minutes)
    - Classroom Discussion of Activity (10 minutes)
        - See questions below.
    - Exit Slip Assessment (10 minutes)
- Introduce the task.
    - Students are asked to compare SAT to ACT scores which is impossible without standardizing the data first by finding the z-score. Students will compare race/ethnicity results on both the SAT and ACT tests. They will then discuss the equity of the tests based on these comparisons.
- Questions or prompts to pose.
    - How do you use z-score to find the area under the normal curve?
    - The table always finds the probability of scoring less that the data value. How do you adjust this process to find the probability of scoring above a given value or between two values?
    - How does the z-score allow us to standardize data?
    - Could we compare the SAT and ACT data without calculating z-score?
    - If so, would it be meaningful?
    - If not, why is the comparison not meaningful?
    - Is it possible to take a probability and work backwards to find the related score?

## Student Exploration: Race to the Top

**Student/Teacher Actions:**

- Students should use the formula sheet and/or calculator to obtain the necessary statistical descriptors needed to answer the questions in the exploration activity.
- Teacher(s) will be guiding student groups as needed if questions/problems arise. Specifically when asked to compare the two data sets in the context of a normal curve to obtain meaningful results involving z-score and probabilities.
- Students will have 10 minutes to discuss results and ask questions in the classroom setting.

**Monitoring Student Responses**

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties by clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to compare other data sets of interest to the student. The student will most likely need computer/internet access to retrieve the needed data.

## Assessment

- See the Rubric provided in Lesson 1.

## Extension and Connections (for all students)

- Students are encouraged to think deeply about the results they generate during today's activities. Did you predict the result? If so, can you explain why? If not, can you explain why?
- Students are encouraged to question why they are comparing the data sets. Is it "fair" to compare apples to oranges? If so, why and how? If not, why?

## Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

# *Data for Lesson 3 – Race to the Top*

***SAT Mean Score Averages of College Bound Seniors, by Race/Ethnicity***

| Race/Ethnicity | 2000 - 2001 | 2001 - 2002 | 2002 - 2003 | 2003 - 2004 | 2004 - 2005 | 2005 - 2006 | 2006 - 2007 | 2007 - 2008 | 2008 - 2009 | 2009 - 2010 | 2010 - 2011 | 2011 - 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **SAT—Critical reading** | | | | | | | | | | | | |
| *All students* | *506* | *504* | *507* | *508* | *508* | *503* | *502* | *502* | *501* | *501* | *497* | *496* |
| White | 529 | 527 | 529 | 528 | 532 | 527 | 527 | 528 | 528 | 528 | 528 | 527 |
| Black | 433 | 430 | 431 | 430 | 433 | 434 | 433 | 430 | 429 | 429 | 428 | 428 |
| Mexican American | 451 | 446 | 448 | 451 | 453 | 454 | 455 | 454 | 453 | 454 | 451 | 448 |
| Puerto Rican | 457 | 455 | 456 | 457 | 460 | 459 | 459 | 456 | 452 | 454 | 452 | 452 |
| Other Hispanic | 460 | 458 | 457 | 461 | 463 | 458 | 459 | 455 | 455 | 454 | 451 | 447 |
| Asian/Pacific Islander | 501 | 501 | 508 | 507 | 511 | 510 | 514 | 513 | 516 | 519 | 517 | 518 |
| American Indian/Alaska Native | 481 | 479 | 480 | 483 | 489 | 487 | 487 | 485 | 486 | 485 | 484 | 482 |
| Other Hispanic | 503 | 502 | 501 | 494 | 495 | 494 | 497 | 496 | 494 | 494 | 493 | 491 |
| **SAT—Mathematics** | | | | | | | | | | | | |
| *All students* | *514* | *516* | *519* | *518* | *520* | *518* | *515* | *515* | *515* | *516* | *514* | *514* |
| White | 531 | 533 | 534 | 531 | 536 | 536 | 534 | 537 | 536 | 536 | 535 | 536 |
| Black | 426 | 427 | 426 | 427 | 431 | 429 | 429 | 426 | 426 | 428 | 427 | 428 |
| Mexican American | 458 | 457 | 457 | 458 | 463 | 465 | 466 | 463 | 463 | 467 | 466 | 465 |
| Puerto Rican | 451 | 451 | 453 | 452 | 457 | 456 | 454 | 453 | 450 | 452 | 452 | 452 |
| Other Hispanic | 465 | 464 | 464 | 465 | 469 | 463 | 463 | 461 | 461 | 462 | 462 | 461 |
| Asian/Pacific Islander | 566 | 569 | 575 | 577 | 580 | 578 | 578 | 581 | 587 | 591 | 595 | 595 |
| American Indian/Alaska Native | 479 | 483 | 482 | 488 | 493 | 494 | 494 | 491 | 493 | 492 | 488 | 489 |
| Other Hispanic | 512 | 514 | 513 | 508 | 513 | 513 | 512 | 512 | 514 | 514 | 517 | 516 |

NOTE: Data for 2009-10 and earlier years are for seniors who took the SAT any time during their high school years through March of their senior year. Data for 2010-11 onwards are for seniors who took the SAT any time during their high school years through June of their senior year. If a student took a test more than once, the most recent score was used. Possible scores on each part of the SAT range from 200 to 800. The critical reading section was formerly known as the verbal section.

NOTE: The composite score would combine the critical reading and mathematics score together for a possible score range of 400 - 1600.

SOURCE: National Center for Education Statistics (NCES), 2012 Digest of Education Statistics, retrieved April 5, 2013 (nces.ed.gov/programs/digest/d12/tables/dt12_153.asp).

# *Data for Lesson 3 – Race to the Top*

***ACT Composite Score Averages, by Race/Ethnicity***

| Race/ethnicity | 2000 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| White | 22.7 | 21.7 | 21.7 | 21.8 | 21.9 | 22.0 | 22.1 | 22.1 | 22.2 | 22.3 | 22.4 | 22.4 |
| Black | 17.8 | 16.8 | 16.9 | 17.1 | 17.0 | 17.1 | 17.0 | 16.9 | 16.9 | 16.9 | 17.0 | 17.0 |
| Hispanic | --- | 18.4 | 18.5 | 18.5 | 18.6 | 18.6 | 18.7 | 18.7 | 18.7 | 18.6 | 18.7 | 18.9 |
| Asian/Pacific Islander | 22.4 | 21.6 | 21.8 | 21.9 | 22.1 | 22.3 | 22.6 | 22.9 | 23.2 | 23.4 | --- | --- |
| Asian | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | 23.6 | 23.6 |
| Native Hawaiian/Pacific Islander | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | 19.5 | 19.8 |
| American Indian/Alaska Native | 20.4 | 18.6 | 18.7 | 18.8 | 18.7 | 18.8 | 18.9 | 19.0 | 18.9 | 19.0 | 18.6 | 18.4 |
| Two or more races | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | 21.1 | 21.4 |

NOTE: Minimum score is 1 and maximum score is 36.

NOTE: Data are for high school graduates who took the ACT during their sophomore, junior, or senior year. If a student took a test more than once, the most recent score was used. Race categories exclude persons of Hispanic ethnicity. Some data have been revised from previously published figures.

NOTE: The composite score for the ACT includes scores from all four section (English, Math, Reading, and Science) of the ACT.

SOURCE: National Center for Education Statistics (NCES), 2012 Digest of Education Statistics, retrieved April 5, 2013 (nces.ed.gov/programs/digest/d12/tables/dt12_157.asp).

Focus – Lesson 3 – Race to the Top

NAME: _____

1. Given a set of ten normally distributed data values, how many of these numbers should fall above the mean? Why?

2. Using the data set below answer the following questions.

   $515, 480, 690, 490, 530, 600, 570, 640, 520, 585$

   a. What statistical descriptors do you need to sketch a normal curve?

   b. Sketch a normal curve of the data set provided.

c. ***z-score*** is defined as the number of standard deviations a given value is away from the mean. Using the formula provided below calculate the z-score for every data value in the given set.

$z = \frac{x - \mu}{\sigma}$ where $x$ is a data value, $\mu$ is the mean, and $\sigma$ is the standard deviation

| Data Value | z-score |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

d. What do you notice about every data value that has a negative z-score?

e. What do you notice about every data value that has a positive z-score?

Student Exploration – Lesson 3 – Race to the Top

NAME: _____

1. In 2010, did White students score better on the ACT or the SAT (be sure to use the composite scores for both tests)?

2. In 2002, did the Black students score better on the ACT or the SAT?

3. On the SAT, what is the probability of

    a. a White student scoring less than a 1059?

    b. a White Student scoring at least a 1059?

4. What is the probability of a Black student scoring between 17.3 and 17.6 on the ACT?

5. What composite score would an Asian/Pacific Islander student have to score to place in the top 10% for their race/ethnicity on the SAT?

Exit Slip – Lesson 3 – Race to the Top

NAME: _____

1. Jeff's z-score is – 1.75. What does this tell us about Jeff's SAT score?

2. In 2000, as reported by ACT Research Service, the mean ACT Math score was 20.7. If ACT Math scores are normally distributed with a standard deviation of 5.

    a. What is the probability of that a randomly selected student has an ACT Math score of at least 18?

    b. What is the probability that a randomly selected student has an ACT Math score between 24 and 27?

    c. If a student scores 29 on the ACT Math section, what is his/her percentile rank?

    d. A highly selective university will only considers applications in which the ACT Math score is in the top 10% of scores. What is the minimum score required to be considered?

Focus – Lesson 3 – Race to the Top

NAME: ____Answer Key_____

1. Given a set of ten data values, how many of these numbers should fall above the mean? Why?

   Five of the data values should fall above the mean and five should fall below the mean because the mean is a measure of "center".

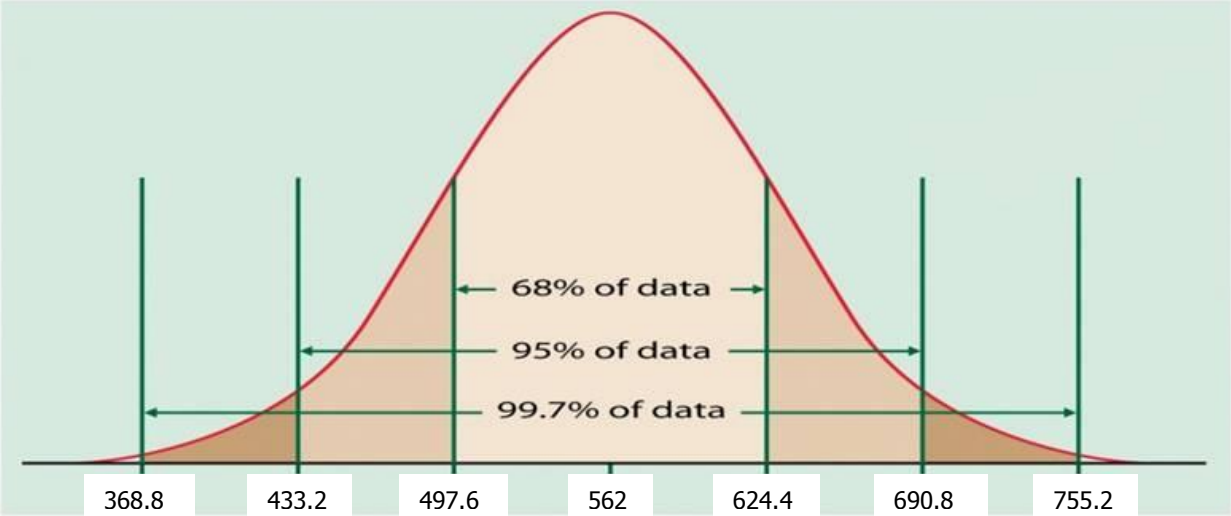
2. Using the data set below answer the following questions.

   $515, 480, 690, 490, 530, 600, 570, 640, 520, 585$

   a. What statistical descriptors do you need to sketch a normal curve?

      Mean and Standard Deviation

   b. Sketch a normal curve of the data set provided.



c. ***z-score*** is defined as the number of standard deviations a given value is away from the mean. Using the formula provided below calculate the z-score for every data value in the given set.

$z = \frac{x - \mu}{\sigma}$ where $x$ is a data value, $\mu$ is the mean, and $\sigma$ is the standard deviation

| Data Value | z-score |
|---|---|
| 515 | -0.73 |
| 480 | -1.27 |
| 690 | 1.99 |
| 490 | -1.12 |
| 530 | -0.50 |
| 600 | 0.59 |
| 570 | 0.12 |
| 640 | 1.21 |
| 520 | -0.65 |
| 585 | 0.36 |

d. What do you notice about every data value that has a negative z-score?

Student answers will vary.

Wanted response: All data values with a negative z-score fall below the mean.

e. What do you notice about every data value that has a positive z-score?

Student answers will vary.

Wanted response: All data values with a positive z-score fall above the mean.

Student Exploration – Lesson 3 – Race to the Top

NAME: ____Answer Key_____

1. In 2010, did White students score better on the ACT or the SAT (be sure to use the composite scores for both tests)?

   SAT $z = .63$ They scored higher on the SAT.
   ACT $z = .5$

2. In 2002, did the Black students score better on the ACT or the SAT?

   SAT $z = -.4$ They scored higher on the SAT.
   ACT $z = -.92$

3. On the SAT, what is the probability of

   a. a White student scoring less than a 1059?

      $P(x < 1059) = P(z < -1.58) = .0571$

   b. a White Student scoring at least a 1059?

      $P(x > 1059) = P(z > -1.58) = 1 - .0571 = .9429$

4. What is the probability of a Black student scoring between 17.3 and 17.6 on the ACT?

   $P(17.3 < x < 17.6) = P(1.08 < z < 2.28) = .9887 - .8599 = .1288$

5. What composite score would an Asian/Pacific Islander student have to score to place in the top 10% for their race/ethnicity on the SAT?

   $P(z < ?) = .90 \quad z = 1.28$

   $1.28 = \frac{x - 1092.3}{14.6} \quad x \approx 1111$

Exit Slip – Lesson 3 – Race to the Top

NAME: ____Answer Key_____

1. Jeff's z-score is – 1.75. What does this tell us about Jeff's SAT score?

   Jeff's SAT score falls 1.75 standard deviation BELOW the mean.

2. In 2000, as reported by ACT Research Service, the mean ACT Math score was 20.7. If ACT Math scores are normally distributed with a standard deviation of 5.

   a. What is the probability of that a randomly selected student has an ACT Math score of at least 18?

   $$P(x < 18) = P(z < -.74) = 1 - .2296 = .7704$$

   b. What is the probability that a randomly selected student has an ACT Math score between 24 and 27?

   $$P(24 < x < 27) = P(.66 < z < 1.26) = .8962 - .7454 = .1508$$

   c. If a student scores 29 on the ACT Math section, what is his/her percentile rank?

   $$P(x < 29) = P(z < 1.66) = .9515 \rightarrow 95th\ percentile$$

   d. A highly selective university will only considers applications in which the ACT Math score is in the top 8% of scores. What is the minimum score required to be considered?

   $$P(z < ?) = .92 \ \rightarrow \ z \approx 1.45 \ \rightarrow \ x \approx 27.95$$

# Lesson 4 – Comparing the Uncomparable?
## (Student Projects)

**Strand**
Probability and Statistics

**Mathematical Objective(s)**
- Allow students to organize and compare data
- Allow students to justify the validity of comparing two data sets
- Allow students to use statistics to describe data set(s)
- Allow students to collaborate on an open-ended group project
- Allow students to present mathematical concepts within the classroom

**Mathematics Performance Expectation(s)**
- MPE.8 – The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
- MPE.22 – The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.
- MPE.23 – The student will analyze the normal distribution. Key concepts include
  a) characteristics of normally distributed data;
  b) percentiles;
  c) normalizing data, using z-scores; and
  d) area under the standard normal curve and probability.

**Related SOL**
- A.9, A.10; AII.11; PS.1, PS.2, PS.3, PS.16
  *These may vary slightly since students are allowed to choose what data they would like to compare.*

**NCTM Standards**
*These may vary slightly since students are allowed to choose what data they would like to compare.*
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Recognize how linear transformations of univariate data affect shape, center, and spread
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Make and investigate mathematical conjectures
- Solve problems that arise in mathematics and in other contexts

- Monitor and reflect on the process of mathematical problem solving
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas

**Materials/Resources**

- Classroom Set of Graphing Calculators
- Access to laptops or computer lab (Word or PowerPoint required)
- Copies of Student Exploration Packet (Lesson 4 – Comparing the Uncomparable?)
- Copies of Lesson 4 Data Tables (4 separate sheets to allow for easy comparison)
- Copies of Algebra II SOL Formula Sheet.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_algebra2.pdff
- Copies of Algebra II Standard Normal Probabilities Table.
  http://www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_z_table.pdf

**Assumption of Prior Knowledge**

- Students should be able to draw on prior knowledge of measures of central tendency, box-and-whisker plots, normal distributions, and z-scores.
  - Students can recognize and name statistic descriptors of a data set.
  - Relationships between statistic descriptors cannot be concretely connected.
  - Students can reason informally about the statistical characteristics of the data set.
- Students will begin to compare statistical descriptors of two data sets. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly identify outliers or construct a box-and-whisker plot and will not be able to make meaningful comparisons between two data sets.
- Students should be familiar with calculating univariate statistics using the TI-84+.
- Relevant contexts: Analysis and impact of ACT and SAT scores.

## Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, students will be broken into three member groups to complete a data analysis project and presentation using SAT and ACT data as provided. Students will have to choose two data sets from those given and meet certain criteria (calculations) within this project. Students will present their interpretation of the data tomorrow in a class presentation.

- Describe planned time outline.
    - Focus Activity
        - Comparing the Uncomparable – Data Review (5 minutes)
          Individually students will be given four data tables. Students will be informed that they will be completing a statistical comparison project of their choosing. Students will have to decide if they want to compare 2 sets of SAT data, 2 sets of ACT data, or 1 set of ACT data to 1 set of SAT data. After 5 minutes students will give you their selections – these preferences will be used to form the three member groups for the day.
    - Student Exploration
        - Comparing the Uncomparable? – Data Analysis (40 minutes)
          In groups students will analyze two sets of data following the criteria given in the student exploration packet. Students should make sure they perform all calculations necessary to answer the questions listed in the packet.
        - Comparing the Uncomparable? – Prepare Presentation (15 minutes)
          Students will highlight their findings in a 10 minute PowerPoint presentation. Students should be sure that the questions in the packet are answered during their presentation. Groups will present these during class during Lesson 5.
- Questions or prompts to pose.
    - Do the two data sets chosen relate in anyway?
    - If so, how? What are you trying to find by comparing the two data sets?
    - If not, why compare the two data sets?
    - Do you prefer the visual clues that the box-and-whisker graph shows or the sketch of the normal curve? Why?
    - Were there any surprises found when analyzing the data sets chosen?
    - Are there any reasons that you can think of to explain the results of your data analysis?

## Student Exploration: Comparing the Uncomparable?

**Student/Teacher Actions:**

- Students should use the formula sheet and/or calculator to analyze their chosen data sets.
- Teacher(s) will be guiding student groups as needed if questions/problems arise. The exploration packet asks leading questions and states specific criteria that should

cue students to what they must include in the project. Students are welcome to extend to other areas in time allows.
- It might be necessary to ask students to explain the difference pictures that the box-and-whisker graph and the normal curve show about the dispersion of the data.
- Students will present their findings in Lesson 5. Students are encouraged to ask questions about other group presentations. Students should be able to answer questions about their own presentation.

**Monitoring Student Responses**

- Students are expected to collaborate on the student exploration activity (project). Teachers should monitor progress and insure that all students are contributing to the group project.
- The teacher will assist students who have difficulties by clarifying directions and/or prompting students with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to consider other data sets that could be compared or to find a data set that they feel is uncomparable and be able to justify their conclusions with characteristics of the data.

## Assessment

- Students will be assessed on their participation and contributions within the group project and presentation. This will be assessed after the group presentation in Lesson 5.

## Extension and Connections (for all students)

- Students are encouraged to think deeply about the results they generate during today's activities. Did you predict the result? If so, can you explain why? If not, can you explain why?
- Students are encouraged to question why they are comparing the data sets. Is it "fair" to compare apples to oranges? If so, why and how? If not, why?
- Students need to be ready to discuss these types of questions when presenting tomorrow to their classmates and teacher.

## Strategies for Differentiation

The graphic organizers/handouts were designed with the needs of a diverse classroom of students in mind by providing a visual representation (as clear as possible) of the data. Use of the graphing calculator to perform statistical calculations is encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.

- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Auditory learners will benefit from the classroom and group discussions. Teachers may assist in reading the data to these students to further enforce understanding.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to use technology to assist in finding necessary calculations.
- High ability students may choose a second data set to compare. These similarities or differences will help students form questions about other group presentations in Lesson 5. Another option is to allow these students to serve as peer leaders with groups that are struggling to complete the task(s).

## *2009-2010 Average SAT Scores by State*

| State | Percent of Graduates Tested | Critical Reading | Math | State | Percent of Graduates Tested | Critical Reading | Math |
|---|---|---|---|---|---|---|---|
| Alabama | 7 | 556 | 550 | Montana | 24 | 538 | 538 |
| Alaska | 48 | 518 | 515 | Nebraska | 4 | 585 | 593 |
| Arizona | 25 | 519 | 525 | Nevada | 43 | 496 | 501 |
| Arkansas | 4 | 566 | 566 | New Hampshire | 77 | 520 | 524 |
| California | 50 | 501 | 516 | New Jersey | 76 | 495 | 514 |
| Colorado | 18 | 568 | 572 | New Mexico | 11 | 553 | 549 |
| Connecticut | 84 | 509 | 514 | New York | 85 | 484 | 499 |
| Delaware | 71 | 493 | 495 | North Carolina | 63 | 497 | 511 |
| District of Columbia | 76 | 474 | 464 | North Dakota | 4 | 580 | 594 |
| Florida | 59 | 496 | 498 | Ohio | 21 | 538 | 548 |
| Georgia | 74 | 488 | 490 | Oklahoma | 5 | 569 | 568 |
| Hawaii | 58 | 483 | 505 | Oregon | 54 | 523 | 524 |
| Idaho | 19 | 543 | 541 | Pennsylvania | 71 | 492 | 501 |
| Illinois | 6 | 585 | 600 | Rhode Island | 67 | 494 | 495 |
| Indiana | 64 | 494 | 505 | South Carolina | 66 | 484 | 495 |
| Iowa | 3 | 603 | 613 | South Dakota | 3 | 592 | 603 |
| Kansas | 6 | 590 | 595 | Tennessee | 10 | 576 | 571 |
| Kentucky | 6 | 575 | 575 | Texas | 53 | 484 | 505 |
| Louisiana | 7 | 555 | 550 | Utah | 6 | 568 | 559 |
| Maine | 92 | 468 | 467 | Vermont | 66 | 519 | 521 |
| Maryland | 70 | 501 | 506 | Virginia | 67 | 512 | 512 |
| Massachusetts | 86 | 512 | 526 | Washington | 54 | 524 | 532 |
| Michigan | 5 | 585 | 605 | West Virginia | 16 | 515 | 507 |
| Minnesota | 7 | 594 | 607 | Wisconsin | 4 | 595 | 604 |
| Mississippi | 3 | 566 | 548 | Wyoming | 5 | 570 | 567 |
| Missouri | 4 | 593 | 595 | ***National*** | 47 | **501** | **516** |

NOTE: Participation rate is based on the projection of high school graduates by the Western Interstate Commission for Higher Education (WICHE), and the number of seniors who took the SAT in each state.

NOTE: Beginning with the spring SAT administration in 2006, all Maine high school juniors, including all students in their third year of high school, are required to take SAT tests in critical reading, writing, and mathematics.

NOTE: Data for 2009-10 and earlier years are for seniors who took the SAT any time during their high school years through March of their senior year. If a student took a test more than once, the most recent score was used. The SAT was formerly known as the Scholastic Assessment Test and the Scholastic Aptitude Test. Possible scores on each part of the SAT range from 200 to 800. The critical reading section was formerly known as the verbal section.

SOURCE: College Entrance Examination Board, College-Bound Seniors Tables and Related Items, selected years, 1995-96 through 2010-11, retrieved September 14, 2011, from http://professionals.collegeboard.com/profdownload/SAT_Trends_Report_9_12_2011.pdf. (This table was prepared September 2011.)

## *2010-2011 Average SAT Scores by State*

| State | Percent of Graduates Tested | Critical Reading | Math | State | Percent of Graduates Tested | Critical Reading | Math |
|---|---|---|---|---|---|---|---|
| Alabama | 8 | 546 | 541 | Montana | 26 | 539 | 537 |
| Alaska | 52 | 515 | 511 | Nebraska | 5 | 585 | 591 |
| Arizona | 28 | 517 | 523 | Nevada | 47 | 494 | 496 |
| Arkansas | 5 | 568 | 570 | New Hampshire | 77 | 523 | 525 |
| California | 53 | 499 | 515 | New Jersey | 78 | 495 | 516 |
| Colorado | 19 | 570 | 573 | New Mexico | 12 | 548 | 541 |
| Connecticut | 87 | 509 | 513 | New York | 89 | 485 | 499 |
| Delaware | 74 | 489 | 490 | North Carolina | 67 | 493 | 508 |
| District of Columbia | 79 | 469 | 457 | North Dakota | 3 | 586 | 612 |
| Florida | 64 | 487 | 489 | Ohio | 21 | 539 | 545 |
| Georgia | 80 | 485 | 487 | Oklahoma | 6 | 571 | 565 |
| Hawaii | 64 | 479 | 500 | Oregon | 56 | 520 | 521 |
| Idaho | 20 | 542 | 539 | Pennsylvania | 73 | 493 | 501 |
| Illinois | 5 | 599 | 617 | Rhode Island | 68 | 495 | 493 |
| Indiana | 68 | 493 | 501 | South Carolina | 70 | 482 | 490 |
| Iowa | 3 | 596 | 606 | South Dakota | 4 | 584 | 591 |
| Kansas | 7 | 580 | 591 | Tennessee | 10 | 575 | 568 |
| Kentucky | 6 | 576 | 572 | Texas | 58 | 479 | 502 |
| Louisiana | 8 | 555 | 550 | Utah | 6 | 563 | 559 |
| Maine | 93 | 469 | 469 | Vermont | 67 | 515 | 518 |
| Maryland | 74 | 499 | 502 | Virginia | 71 | 512 | 509 |
| Massachusetts | 89 | 513 | 527 | Washington | 57 | 523 | 529 |
| Michigan | 5 | 583 | 604 | West Virginia | 17 | 514 | 501 |
| Minnesota | 7 | 593 | 608 | Wisconsin | 5 | 590 | 602 |
| Mississippi | 4 | 564 | 543 | Wyoming | 5 | 572 | 569 |
| Missouri | 5 | 592 | 593 | ***National*** | | **497** | **514** |

NOTE: Participation rate is based on the projection of high school graduates by the Western Interstate Commission for Higher Education (WICHE), and the number of seniors who took the SAT in each state.

NOTE: Beginning with the spring SAT administration in 2006, all Maine high school juniors, including all students in their third year of high school, are required to take SAT tests in critical reading, writing, and mathematics.

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# *Data for Lesson 4 – Comparing the Uncomparable?*

## 2010 Average ACT Scores by State

| State | % of Grads Tested | Composite Score | English Score | Math Score | Reading Score | Science Score | State | % of Grads Tested | Composite Score | English Score | Math Score | Reading Score | Science Score |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alabama | 78 | 20.3 | 20.4 | 19.5 | 20.7 | 20.2 | Montana | 58 | 22.0 | 21.3 | 21.7 | 22.7 | 21.9 |
| Alaska | 28 | 21.1 | 20.3 | 21.2 | 21.7 | 20.9 | Nebraska | 73 | 22.1 | 21.8 | 21.6 | 22.4 | 22.0 |
| Arizona | 28 | 20.0 | 18.9 | 20.4 | 20.2 | 19.9 | Nevada | 30 | 21.5 | 20.8 | 21.5 | 22.0 | 21.3 |
| Arkansas | 81 | 20.3 | 20.1 | 19.9 | 20.6 | 20.2 | New Hampshire | 17 | 23.7 | 23.6 | 23.5 | 24.1 | 22.9 |
| California | 22 | 22.2 | 21.7 | 22.9 | 22.3 | 21.5 | New Jersey | 17 | 23.2 | 23.1 | 23.6 | 23.2 | 22.4 |
| Colorado | 100 | 20.6 | 19.9 | 20.4 | 21.1 | 20.7 | New Mexico | 66 | 20.1 | 19.3 | 19.7 | 20.5 | 20.2 |
| Connecticut | 24 | 23.7 | 23.8 | 23.5 | 23.9 | 22.9 | New York | 27 | 23.3 | 22.7 | 23.6 | 23.3 | 23.1 |
| Delaware | 13 | 23.0 | 22.8 | 22.7 | 23.4 | 22.5 | North Carolina | 16 | 21.9 | 21.1 | 22.3 | 22.2 | 21.6 |
| D.C. | 29 | 19.8 | 19.5 | 19.9 | 20.2 | 19.2 | North Dakota | 81 | 21.5 | 20.7 | 21.4 | 21.7 | 21.6 |
| Florida | 65 | 19.5 | 18.6 | 19.7 | 20.1 | 19.1 | Ohio | 66 | 21.8 | 21.2 | 21.5 | 22.1 | 21.8 |
| Georgia | 44 | 20.7 | 20.1 | 20.7 | 20.9 | 20.5 | Oklahoma | 73 | 20.7 | 20.4 | 19.9 | 21.2 | 20.6 |
| Hawaii | 22 | 21.6 | 20.9 | 22.2 | 21.5 | 21.2 | Oregon | 34 | 21.5 | 20.5 | 21.5 | 22.0 | 21.3 |
| Idaho | 60 | 21.8 | 21.2 | 21.4 | 22.4 | 21.6 | Pennsylvania | 17 | 21.9 | 21.3 | 22.1 | 22.1 | 21.4 |
| Illinois | 100 | 20.7 | 20.3 | 20.7 | 20.8 | 20.5 | Rhode Island | 11 | 22.8 | 22.8 | 22.4 | 23.3 | 22.0 |
| Indiana | 26 | 22.3 | 21.7 | 22.4 | 22.6 | 21.9 | South Carolina | 52 | 20.0 | 19.2 | 20.1 | 20.0 | 20.0 |
| Iowa | 60 | 22.2 | 21.8 | 21.8 | 22.6 | 22.3 | South Dakota | 79 | 21.8 | 21.1 | 21.6 | 22.0 | 21.9 |
| Kansas | 75 | 22.0 | 21.4 | 21.7 | 22.3 | 21.9 | Tennessee | 100 | 19.6 | 19.4 | 19.0 | 19.9 | 19.6 |
| Kentucky | 100 | 19.4 | 18.9 | 19.1 | 19.7 | 19.5 | Texas | 33 | 20.8 | 19.7 | 21.4 | 20.8 | 20.9 |
| Louisiana | 98 | 20.1 | 20.1 | 19.6 | 20.2 | 20.2 | Utah | 71 | 21.8 | 21.4 | 21.1 | 22.5 | 21.7 |
| Maine | 10 | 23.2 | 23.2 | 23.0 | 23.6 | 22.5 | Vermont | 26 | 23.2 | 22.8 | 22.8 | 23.7 | 22.8 |
| Maryland | 18 | 22.3 | 22.0 | 22.3 | 22.4 | 21.8 | Virginia | 22 | 22.3 | 22.0 | 22.1 | 22.5 | 21.9 |
| Massachusetts | 21 | 24.0 | 24.0 | 24.3 | 24.2 | 23.1 | Washington | 19 | 23.0 | 22.6 | 23.1 | 23.5 | 22.5 |
| Michigan | 100 | 19.7 | 18.9 | 19.7 | 19.7 | 19.9 | West Virginia | 64 | 20.7 | 20.7 | 19.6 | 21.3 | 20.6 |
| Minnesota | 70 | 22.9 | 22.3 | 22.9 | 23.2 | 22.8 | Wisconsin | 69 | 22.1 | 21.5 | 22.0 | 22.3 | 22.2 |
| Mississippi | 96 | 18.8 | 18.6 | 18.3 | 18.8 | 18.8 | Wyoming | 100 | 20.0 | 19.0 | 19.8 | 20.4 | 20.1 |
| Missouri | 69 | 21.6 | 21.5 | 21.0 | 22.0 | 21.6 | ***National*** | ***47*** | ***21.0*** | ***20.5*** | ***21.0*** | ***21.3*** | ***20.9*** |

SOURCE: ACT, 2010 ACT National and State Scores, http://www.act.org/newsroom/data/2010/states.html (retrieved April 5, 2013).

NOTE: In spring 2009, all public high school eleventh graders in the states of Colorado, Illinois, Kentucky, Michigan, Tennessee, and Wyoming were tested with the ACT as required by each state. Colorado, Illinois, Kentucky, Michigan, Tennessee, and Wyoming students who met ACT's 2010 graduating class criteria are included in the 2010 graduating class average score results.

*Data for Lesson 4 – Comparing the Uncomparable?*

## 2011 Average ACT Scores by State

| State | % of Grads Tested | Composite Score | English Score | Math Score | Reading Score | Science Score | State | % of Grads Tested | Composite Score | English Score | Math Score | Reading Score | Science Score |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alabama | 81 | 20.3 | 20.4 | 19.6 | 20.7 | 20.1 | Montana | 60 | 22.1 | 21.3 | 21.9 | 22.7 | 22.0 |
| Alaska | 40 | 21.2 | 20.3 | 21.4 | 21.7 | 21.0 | Nebraska | 76 | 22.1 | 21.9 | 21.7 | 22.3 | 22.0 |
| Arizona | 34 | 19.7 | 18.6 | 20.3 | 19.9 | 19.6 | Nevada | 31 | 21.4 | 20.7 | 21.4 | 21.8 | 21.3 |
| Arkansas | 91 | 19.9 | 19.6 | 19.7 | 20.2 | 19.8 | New Hampshire | 18 | 23.7 | 23.7 | 23.7 | 24.1 | 23.0 |
| California | 24 | 22.1 | 21.6 | 22.7 | 22.0 | 21.4 | New Jersey | 19 | 23.2 | 23.1 | 23.7 | 23.3 | 22.4 |
| Colorado | 100 | 20.7 | 20.1 | 20.4 | 20.9 | 20.7 | New Mexico | 72 | 19.8 | 19.0 | 19.5 | 20.2 | 20.0 |
| Connecticut | 26 | 23.9 | 24.0 | 23.9 | 24.1 | 23.1 | New York | 28 | 23.4 | 22.7 | 23.8 | 23.5 | 23.0 |
| Delaware | 16 | 22.4 | 22.0 | 22.4 | 22.7 | 22.0 | North Carolina | 18 | 21.9 | 21.2 | 22.4 | 22.2 | 21.4 |
| D.C. | 28 | 20.0 | 19.7 | 20.2 | 20.4 | 19.4 | North Dakota | 98 | 20.7 | 19.8 | 20.8 | 20.8 | 20.8 |
| Florida | 66 | 19.6 | 18.8 | 19.9 | 20.2 | 19.1 | Ohio | 69 | 21.8 | 21.1 | 21.5 | 22.1 | 21.8 |
| Georgia | 47 | 20.6 | 20.1 | 20.7 | 20.8 | 20.3 | Oklahoma | 76 | 20.7 | 20.5 | 19.9 | 21.3 | 20.6 |
| Hawaii | 24 | 21.3 | 20.6 | 21.8 | 21.3 | 21.1 | Oregon | 0.5 | 21.5 | 20.6 | 21.6 | 21.8 | 21.2 |
| Idaho | 64 | 21.7 | 21.1 | 21.3 | 22.2 | 21.5 | Pennsylvania | 17 | 22.3 | 21.9 | 22.6 | 22.6 | 21.8 |
| Illinois | 100 | 20.9 | 20.6 | 20.9 | 20.8 | 20.7 | Rhode Island | 12 | 23.0 | 23.1 | 22.8 | 23.5 | 22.1 |
| Indiana | 29 | 22.3 | 21.7 | 22.4 | 22.6 | 21.9 | South Carolina | 56 | 20.1 | 19.4 | 20.3 | 20.3 | 20.1 |
| Iowa | 61 | 22.3 | 21.7 | 21.9 | 22.6 | 22.4 | South Dakota | 81 | 21.8 | 21.0 | 21.8 | 22.0 | 22.1 |
| Kansas | 79 | 22.0 | 21.4 | 21.8 | 22.3 | 21.9 | Tennessee | 100 | 19.5 | 19.4 | 19.0 | 19.7 | 19.4 |
| Kentucky | 100 | 19.6 | 19.2 | 19.1 | 20.0 | 19.6 | Texas | 36 | 20.8 | 19.6 | 21.5 | 20.7 | 20.8 |
| Louisiana | 100 | 20.2 | 20.4 | 19.7 | 20.3 | 20.1 | Utah | 73 | 21.8 | 21.4 | 21.2 | 22.3 | 21.7 |
| Maine | 9 | 23.3 | 23.3 | 23.2 | 23.8 | 22.5 | Vermont | 28 | 22.7 | 22.5 | 22.6 | 23.0 | 22.2 |
| Maryland | 20 | 22.1 | 21.8 | 22.2 | 22.2 | 21.6 | Virginia | 24 | 22.3 | 22.1 | 22.2 | 22.7 | 21.8 |
| Massachusetts | 22 | 24.2 | 24.1 | 24.6 | 24.4 | 23.2 | Washington | 20 | 22.8 | 22.3 | 22.9 | 23.1 | 22.3 |
| Michigan | 100 | 20.0 | 19.3 | 19.9 | 20.1 | 20.3 | West Virginia | 65 | 20.6 | 20.6 | 19.5 | 21.2 | 20.5 |
| Minnesota | 72 | 22.9 | 22.3 | 23.0 | 22.9 | 22.8 | Wisconsin | 71 | 22.2 | 21.6 | 22.1 | 22.2 | 22.3 |
| Mississippi | 100 | 18.7 | 18.6 | 18.2 | 18.8 | 18.7 | Wyoming | 100 | 20.3 | 19.4 | 20.0 | 20.8 | 20.4 |
| Missouri | 71 | 21.6 | 21.5 | 21.0 | 21.9 | 21.6 | ***National*** | ***49*** | ***21.1*** | ***20.6*** | ***21.1*** | ***21.3*** | ***20.9*** |

SOURCE: ACT, 2011 ACT National and State Scores, http://www.act.org/newsroom/data/2011/states.html (retrieved April 5, 2013).

NOTE: In spring 2009, all public high school eleventh graders in the states of Colorado, Illinois, Kentucky, Michigan, Tennessee, and Wyoming were tested with the ACT as required by each state. Colorado, Illinois, Kentucky, Michigan, Tennessee, and Wyoming students who met ACT's 2010 graduating class criteria are included in the 2010 graduating class average score results.

Student Exploration – Lesson 4 – Comparing the Uncomparable? (Student Projects)

NAME: _____

In your group please quickly evaluate what two data sets you would like to compare.

Here are some questions to help you think about comparing two data sets…

- Do the two data sets chosen relate in anyway?

- If so, how? What are you trying to find by comparing the two data sets?

- If not, why compare the two data sets?

- Investigate the details of the data. Is the collection of the data consistent between the two data sets?

Once deciding, perform a data analysis of the two chosen sets. Be sure that your analysis is complete enough that you have the following criteria or can answer the following questions.

- Do you prefer the visual clues that the box-and-whisker graph shows or the sketch of the normal curve? Why?

- Were there any surprises found when analyzing the data sets chosen?

- Are there any reasons that you can think of to explain the results of your data analysis?

Data Analysis (Space provided to record calculations and/or answer questions.)

Group Presentations

- Group presentations will be tomorrow and will last approximately 10 minutes.

- Include details of your data analysis along with answers to the questions above. Be prepared to answer questions from others about your data analysis.

- All group members must participate during the presentation.

- Groups can use the SmartBoard to present if using Word or PowerPoint.

# Lesson 5 – Comparing the Uncomparable?
## (Student Presentations)

**Strand**
Probability and Statistics

**Mathematical Objective(s)**
- Allow students to justify the validity of comparing two data sets
- Allow students to use statistics to describe data set(s)
- Allow students to present mathematical concepts within the classroom

**Mathematics Performance Expectation(s)**
- MPE.8 – The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.
- MPE.22 – The student will analyze graphical displays of univariate data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.
- MPE.23 – The student will analyze the normal distribution. Key concepts include
  a) characteristics of normally distributed data;
  b) percentiles;
  c) normalizing data, using z-scores; and
  d) area under the standard normal curve and probability.

**Related SOL**

- A.9, A.10; AII.11; PS.1, PS.2, PS.3, PS.16
  *These may vary slightly since students are allowed to choose what data they would like to compare.*

**NCTM Standards**
*These may vary slightly since students are allowed to choose what data they would like to compare.*
- For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics
- Recognize how linear transformations of univariate data affect shape, center, and spread
- Understand histograms, parallel box plots, and scatterplots and use them to display data
- Make and investigate mathematical conjectures
- Solve problems that arise in mathematics and in other contexts
- Monitor and reflect on the process of mathematical problem solving
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas

**Materials/Resources**

- Classroom Set of Graphing Calculators
- Access to SmartBoard or laptop/projector (Word or PowerPoint required)
- Copies of Student Exploration Packet (Student Critiques – 4 per student)
- Copies of Exit Slip/Assessment

**Assumption of Prior Knowledge**

- Students should be able to draw on prior knowledge of measures of central tendency, box-and-whisker plots, normal distributions, and z-scores.
    - Students can recognize and name statistic descriptors of a data set.
    - Relationships between statistic descriptors cannot be concretely connected.
    - Students can reason informally about the statistical characteristics of the data set.
- Students will begin to compare statistical descriptors of two data sets. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly identify outliers or construct a box-and-whisker plot and will not be able to make meaningful comparisons between two data sets.
- Students should be familiar with calculating univariate statistics using the TI-84+.
- Relevant contexts: Analysis and impact of ACT and SAT scores.

## Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, students will be presenting their own data analysis projects on ACT and SAT Scores (in groups of three). Each student will be given four copies of the "Student Critique" sheet. Students will present their interpretation of the data within a 10 minute window. Each student will receive four student critiques and one teacher critique providing constructive feedback on their project/presentation.
- Describe planned time outline.
    - Focus Activity (5 minutes)
        - Discuss of how to complete the Student Critique.
        - Defining constructive criticism.
    - Student Exploration (50 minutes)
        - Group Presentations (10 minutes each)

        - A second day of presentations may be necessary if your class is sufficiently large or you lengthen the amount of time each group may have to present.
    - Students will complete a Student Critique sheet for four people in the class as they present. These will be assigned by the teacher so that each student has four student critiques.
  - Exit Slip/Assessment (5 minutes)
    - Please answer the reflection questions.

- Questions or prompts to pose.
  - Do the two data sets chosen relate in anyway?
  - If so, how? What are you trying to find by comparing the two data sets?
  - If not, why compare the two data sets?
  - Do you prefer the visual clues that the box-and-whisker graph shows or the sketch of the normal curve? Why?
  - Were there any surprises found when analyzing the data sets chosen?
  - Are there any reasons that you can think of to explain the results of your data analysis?

## Student Exploration: Comparing the Uncomparable?

**Student/Teacher Actions:**

- Students should use the formula sheet and/or calculator to analyze their chosen data sets.
- Teacher(s) will be guiding student groups as needed if questions/problems arise. The exploration packet asks leading questions and states specific criteria that should cue students to what they must include in the project. Students are welcome to extend to other areas in time allows.
- It might be necessary to ask students to explain the difference pictures that the box-and-whisker graph and the normal curve show about the dispersion of the data.
- Students will present their findings in Lesson 5. Students are encouraged to ask questions about other group presentations. Students should be able to answer questions about their own presentation.

**Monitoring Student Responses**

- Students are expected to collaborate on the student exploration activity (project). Teachers should monitor progress and insure that all students are contributing to the group project.

- The teacher will assist students who have difficulties by clarifying directions and/or prompting students with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to consider other data sets that could be compared or to find a data set that they feel is uncomparable and be able to justify their conclusions with characteristics of the data.

## Assessment

- Students will be assessed on their participation and contributions within the group project and presentation. Please see the attached rubric. Students will also receive critiques from four students and the teacher.

## Extension and Connections (for all students)

- Students are encouraged to think deeply about the results they generate during today's activities. Did you predict the result? If so, can you explain why? If not, can you explain why?
- Students are encouraged to question why they are comparing the data sets. Is it "fair" to compare apples to oranges? If so, why and how? If not, why?

## Strategies for Differentiation

The graphic organizers/handouts were designed with the needs of a diverse classroom of students in mind by providing a visual representation (as clear as possible) of the data. Use of the graphing calculator to perform statistical calculations is encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Auditory learners will benefit from the classroom and group discussions. Teachers may assist in reading the data to these students to further enforce understanding.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to use technology to assist in finding necessary calculations.
- High ability students may choose a second data set to compare. These similarities or differences will help students form questions about other group presentations in Lesson 5. Another option is to allow these students to serve as peer leaders with groups that are struggling to complete the task(s).

Student Exploration – Lesson 5 – Student Presentations/Critiques

NAME: _____

Review of group project/presentation by _____

→ List two things that you found interesting or enjoyed about the project/presentation. Feel free to leave specific reasons to support your feedback.

→ List two things that you thought needed improvement in the project/presentation. Feel free to leave specific reasons to support your feedback.

Student Exploration – Lesson 5 – Unit Reflections

NAME: _____

Please answer the following reflection questions in complete sentences.

1. Please write a paragraph summarizing your thoughts on the open-ended project and presentation. Be sure to discuss your challenges and any revelations you had while collaborating with your group.

2. Please write a paragraph discussing your opinions of this statistics unit. What have you liked and disliked about the exploration activities? What concepts have you mastered? What concepts are still unclear?

RUBRIC: Group Project/Presentation

NAME: _____

(To be used at the conclusion of Lesson 5)

| ***Category*** | ***3*** | ***2*** | ***1*** |
|---|---|---|---|
| ***Participation in Project*** | Participates in **ALL** parts of the project preparation. Provides constructive feedback on student critiques. | Participates in **MOST** parts of the project preparation. Provides constructive feedback on student critiques. | Little participation in the project preparation. Does not provide constructive feedback on student critiques. |
| ***Participation in Presentation*** | Actively participates in the presentation. Presents and explains valid arguments. | Actively participates in the presentation. Presents and explains valid arguments. | Little participation in the presentation. Does not present or explain valid arguments. |
| ***Questioning of Other Groups*** | Gives **ALL** other groups respect while presenting. Asks relevant and critical questions of the presenters. | Gives **MOST** other groups respect while presenting. Asks relevant, but simple questions of the presenters. | Does not give other groups respect while presenting. Does not ask questions of the presenters. |
| ***Exit Slip (Reflection Questions)*** | Complete paragraph. Clearly reflects on the project/presentation and statistics unit activities. | Incomplete paragraph. Clearly reflects on the project/presentation and statistics unit activities. | Incomplete paragraph and/or thoughts on the project/presentation and statistics unit activities. |