Right Triangle Applications

I. UNIT OVERVIEW & PURPOSE:

The unit will be covering properties of right triangles, Pythagorean Theorem, Converse of Pythagorean Theorem, special right triangles, and Trigonometry of right triangles. Pythagorean Theorem is covered in Standards for Algebra 1, Algebra 2, and Geometry. Pythagorean Theorem and Trigonometry is also covered in College Algebra, Pre-calculus, and Calculus. The lesson also can encompass area of triangles as well.

II. UNIT AUTHOR:

Victor Maciel
William Byrd High School
Roanoke County

III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Problem Solving, Decision Making, and Integration

V. OBJECTIVES:

- Students should be able to recognize a right triangle using the Pythagorean Theorem
- Students should be able to calculate the lengths of sides of a special right triangle given one side
- Students should be able to find the angles or sides of a right triangle given an angle and side or given two sides

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.5 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

VII. CONTENT:

The content that will be discussed in this unit deals with using Pythagorean Theorem to solve real world problems covering distances on a map. Using Trigonometry to find angles of elevation, angles of depression, and distances to solve real world problems. Finally I will discuss a recent event (Tornado in Joplin, MO) where students will have to: plan to find the distance to Joplin, Mo from where they live, determine the path of the tornado, how many square feet the tornado affected, the shape of the tornado, and finally the height of the Tornado.

VIII. REFERENCE/RESOURCE MATERIALS:

Each student will need a TI -83 or above calculator. Ruler. Protractor. Map of the United States. Worksheets dealing with real world applications. A computer that has internet access for finding data on the Tornado that hit Joplin.

IX. PRIMARY ASSESSMENT STRATEGIES:

The assessments will be worksheets that will measure the students understanding of the material. There will also be assessments for real world problems at the end of each class.

X. EVALUATION CRITERIA:

For the first two days of the class there will be a seven question worksheets where each question is worth ten points. The final thirty points will be determined by the application question. The final day's lesson will be a project that each student will turn in. The final lesson project will be worth a total of 100 points. All three days of assessments will total 300 points.

XI. INSTRUCTIONAL TIME:

The instructional time required for this unit will be three 90-minute class periods.

Lesson 1 Pythagorean Theorem and Its Converse

Strand
Geometry

Mathematical Objective(s)
Identifying right triangles, Pythagorean theorem. In this lesson students will use the Pythagorean theorem to find distance between two points on a map. Students will also use the converse of the Pythagorean theorem to determine if points on a map form a right triangle. Students will develop a travel plan following a right triangle path. In the travel plan students will use their knowledge of right triangles to determine the distance traveled. Students will locate three points on a map that form a right triangle. Students will also have to plan a budget according to travel distance, gas prices, hotel rooms, food and drink, and mpg depending on the car they wish to drive.

Mathematics Performance Expectation(s)

1) The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
2) The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
3) The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

Related SOL

G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

NCTM Standards

- Use trigonometric relationships to determine lengths and angle measures.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
The student will need to know how to read and use a map.

Materials/Resources

- Classroom set of graphing calculators
- Classroom set of tangible road atlas map. Map can be found: http://images.nationmaster.com/images/motw/national_atlas_1970/ca000012.jpg
- Ruler
- Classroom set of computers

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- Student should know how to use the Pythagorean Theorem and its Converse.
- To succeed in this lesson students should know how to use a ruler.
- Students should know how to solve for a variable that involves using squares and square roots.
- The most common misconception in this problem would be converting different units such as inches to miles.
- The relevant real life context in this problem is budgeting for travel.

Introduction: Setting Up the Mathematical Task

Understanding Student Comprehension: Since not all students have the same level of understanding of certain concepts there will be a 30-minute review of not only the Pythagorean theorem, but also knowing how to use a map and converting inches to miles.

- Students will work in groups of two.
- Each student will get a worksheet filled with seven questions and one real world problem.
- Questions on the worksheet will consist of problems that correspond to finding distance-using points/cities on the map, proving that points/cities either make a right triangle or do not, and finally finding three points/cities on their own that make up a right triangle and proving this using the converse of the Pythagorean theorem.
- This warm-up activity should not last longer than 30-minutes

Real World Problem:

- Next I will ask all students if they were planning a road trip, to think of all the factors that would have to create a budget for a road trip.
    1. Distance
    2. Gas Prices
    3. MPG for the car they prefer to take
    4. Hotel cost using a travel website

5. Food and Beverage
6. Tourism

- Once the list has been generated students will use the last question in the warm-up as their trip.
- Students will develop a budget using the help of the Internet. Time should not take longer than 45-minutes.
- Last 15-minutes of class students will present their budget to the class.

Student/Teacher Actions:

To explore and develop this lesson the following are important points of discussions:

- Using the map given to each student one inch corresponded to 100 miles
- Since the trip will consist of interstate travel the price of gas will be the national average which was found by google.com to be 3.57
- Students may choose which car they wish to travel in and have to use the internet to find the average miles per gallon highway
- Students may choose which hotel they would wish to stay in but must use the Internet and a travel website.
- Students need to know how to spend their money according to dining. They need to know how much money they need to eat on.
- Tourism is a category in which students must plan sites to visit while on their trip. They can find points of interest by visiting the city's website.

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the questions mentioned in their worksheet in their small groups as well as during the class presentation at the end of class. Since there is more than one correct answer for the group project (Example: Not all students will be using the same cities as their three points) allow students to prove their assumption that these cities form a right angle using the converse Pythagorean theorem. Students will also discuss their budgets as well.
- Students may have a problem converting inches to miles. Make sure all students know how to convert from inches to miles.
- Make sure students know how to do the warm-up worksheet before they attempt the budget project.
- For students who tend to finish more quickly see how much their budget changes by using different cars. (Example: Instead of traveling in an SUV maybe the students use a hybrid vehicle.)

Assessment

1. The first assessment will be the warm-up worksheet that consist of seven questions
    - **Questions**
        - Find the distance from Roanoke, VA to Washington, D.C. using a right triangle.
        - Find the distance from Richmond, VA to Austin, TX using a right triangle.
        - Find the distance from New York, NY to Los Angeles, CA using a right triangle.
        - Determine if the three cities Springfield, MO, Bowling Green, KY and Huntsville, AL using the converse Pythagorean theorem.
        - Determine if the three cities Roanoke, VA, Richmond, VA and Washington, D.C. using the converse Pythagorean theorem.
        - Determine if the three cities Kansas City, MO, Sacramento, CA and Seattle, WA using the converse Pythagorean theorem.
        - On your own find three cities that are in three different states that form a right triangle.

Each question in assessment one is worth ten points each.

2. The second assessment will be the Travel Budget.
    - **Questions**
        - Using the cities that the students found formed a right triangle on their own will develop a travel budget for the following trip.
        - The budget will consist of gas price, mpg, hotel, travel distance, tourism, and food. Feel free to add or take away from the budget.

The budget will be worth 30 points and students will have to write these down.

Extensions and Connections (for all students)

- To extend material for students who are ready to move forward introduce a different concept with right triangles. Let students try to find 30-60-90 right triangles and 45-45-90 right triangles that have vertices at cities.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
- Be aware that there might be some differences in communicating the procedural knowledge of mathematics

- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Let them discuss in pairs first
- Enunciate clearly and slowly without speaking louder.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations are adopted from the following source.
http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf

Lesson 2 Right Triangle Trigonometry

Strand
Geometry

Mathematical Objective(s)
Identifying right triangles, Right Triangle Trigonometry. In this lesson students will examine real life problems using right triangle trigonometry. Students will also use angles of depression and angles of elevation to determine the length of corresponding sides of a triangle. Students will also be taken outsides and determine the angle of elevation for a kite flying in the wind.

Mathematics Performance Expectation(s)

1) The student will solve practical problems involving rational numbers (including numbers in scientific notation), percents, ratios, and proportions.
2) The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.
3) The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

Related SOL

G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

NCTM Standards

- Use trigonometric relationships to determine lengths and angle measures.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Additional Objectives for Student Learning (include if relevant; may not be math-related):
Students will need to know how to read measuring devices.

Materials/Resources

- Classroom set of graphing calculators

- Kite with string marked by feet; this way the hypotenuse is easier to measure.
- Measuring Tape in yards and feet
- Access to outside space

Assumption of Prior Knowledge

- The typical student would have already taken Algebra 1 class.
- Student should know how to use the Pythagorean Theorem and its Converse.
- To succeed in this lesson students should know how to use measuring tape.
- Students should know how to solve for a variable that involves using right triangle trigonometry.
- The most common misconception in this problem would be converting different units such as yards to feet.
- The relevant real life context in this problem is determining length, height, angle of depression, and/or angle of depression given limited facts.
- Students should know how to use the Pythagorean Theorem to find the lengths of a missing side in a right triangle.
- Students also should know the interior angle-sum for a triangle.

Introduction: Setting Up the Mathematical Task

Understanding Student Comprehension: Since not all students have the same level of understanding of certain concepts there will be a 30-minute review of solving for missing lengths on the right triangle, angle of elevation, and angles of depression. There will also be a short discussion on how many feet are in a yard.

- Students will work in groups of two. Students will partner where high-level students are with low-level students. Partnering in this way should limit the amount of review time.
- Each student will get a worksheet filled with six questions; all questions will be real life examples.
- Questions on the worksheet will consist of problems that correspond to finding lengths of a right triangle, angle of depression and angle of elevation. All questions will correspond to real life examples.
- This warm-up activity should not last longer than 30-minutes

Hands on Activity:

- Hands on Activity will begin once students are finished with warm-up/worksheet activity.
- Students should be taken outside still being with their partners. Students at no point should leave their partners.
- Once outside to a wide-open space teacher should start to fly the kite.
- Once the kite is flying well students should line up behind the teacher side by side with their partner.

- The first group should have the measuring tape and one student should walk out under the kite to determine the distance from the teacher to the kite and the other student should stay with the teacher measuring the length of the string while holding the measuring tape.
- Students will record this data and then try to find the height of the kite, angle of elevation, and the third angle in the right triangle.
- Students will share their findings at the end of class.

Student/Teacher Actions:
To explore and develop this lesson the following are important points of discussions:

- How many feet are in a yard? How many yards make up a football field? How many feet make up a football field?
- What are good conditions for flying a kite?
- Students need to know all trigonometric ratios. Students may ask how the trigonometric ratios came about.
- Students need to know that dividing by a trigonometric function is not the same as taking the inverse.
- Make sure calculator is in degree mode. Explain why there is a degree mode and radian mode for trigonometric ratios.

Monitoring Student Responses

- Students make their mathematical thinking and understanding public by responding to the questions mentioned in their worksheet in their small groups as well as during the class presentation at the end of class. Since there is more than one correct answer for the group project (Example: Not all students will have the same base and hypotenuse for their right triangle because wind will not stay constant) this allows a student to prove their assumption that the kite forms a right angle using the converse Pythagorean theorem.
- Students may have a problem converting yards to feet. Make sure all students know how to convert from yards to feet.
- Make sure students know how to do the warm-up worksheet before they attempt the kite activity.
- For students who tend to finish more quickly tell them extend the lengths of their right triangle so that it is bigger but similar. Ask the student to find the angles using trigonometric ratios. Then ask him why are the lengths the same and to determine what this means about how trigonometric ratios are found.

Assessment

1.) The first assessment will be the warm-up worksheet that consist of six questions

- **Questions**

- Victor is standing 335.5 ft from the Empire State building. The angle of elevation from Victor's feet is 77 degrees. Find the actual height of the Empire State building.
- You are about to climb the tallest mountain in the world, Mount Everest. You set up base camp 50,000 thousand feet from the peak. The angle of depression from the peak to camp is 30 degrees. Find the actual height of Mount Everest.
- You are standing at the top of the world's tallest tower the Burj Khalifa in Dubai, United Arab Emirates. You are looking at your friend at an angle of depression of 45 degrees. If the top of the tower is 2717 ft high, how far is your friend?
- You are standing at the free throw line. If the distance from the free throw line to the backboard is 15 ft and the height from the ground to the basket is 10 ft find the angle of elevation.
- You are in a hot air balloon and are 7,000 ft in the air. You can see your house using an angle of depression of 37 degrees. What is the lines of sight distance from you and your house?
- You are standing in a hotel room in Paris, France. You can see the Eiffel tower from you room. The Eiffel tower is 750 feet away. The angle of elevation from your room to the top of the Eiffel tower is 46 degrees and the angle of depression from your room to the bottom of the Eiffel tower is 20.9 degrees. Find how high you room is off the ground and find the total height of the Eiffel tower.

Each question in assessment one is worth ten points each.

2.) The second assessment will be the Kite activity.

- **Question**
  - Using the kite the students will have to use the length of the string and the distance from the teacher to directly underneath the kite (distance) to find height, angle of elevation, and the opposite angle of the base.
  - The student must confirm these by using the converse Pythagorean theorem as well.

The activity will be worth 40 points and students will have to write these down to present. Five percent of the grade for this activity should be on the clarity of the presentation.

Extensions and Connections (for all students)

- To extend material for students who are ready to move forward introduce the concept of how trigonometric identities are solved by letting them create a similar triangle to their kite

triangle and ask the students to find the angles using trigonometric ratios. Let them determine why trigonometric ratios are the values they are.
Example: A 3-4-5 right triangle could be 9-12-15 and they will determine that the angles are the same. This will help them realize that trigonometric ratios are actual ratios of the degree. No matter how long the sides how if the ratios are the same then they will have the same degree.

Strategies for Differentiation

The differentiation strategies might include but are not limited to the following list created specifically for ESL students. Feel free to adopt these to your lesson:

- Make instruction more concrete, visual, collaborative, and hands-on
- Assign roles to students in collaborative activities. Discover the strengths of EOL students and assign appropriate roles.
- Be aware that there might be some differences in communicating the procedural knowledge of mathematics
- Focus on mathematical content rather than on linguistic form (simplify word problems without changing the math meaning)
- Language and content should be presented simultaneously
- Seating (near teacher or next to a buddy, native language if possible)
- Write legibly and in print
- Step by step instructions (orally and in writing) Ask students to repeat aloud for the rest of the class.
- Give EOL student more time for questions and answers.
- Let them discuss in pairs first
- Enunciate clearly and slowly without speaking louder.
- Simplify the language used rather than the mathematical concepts taught (use known vocabulary and simple sentence constructions).
- When students speak, focus on their message rather than their grammatical skills and accuracy. Respond using the proper grammatical form rather than overtly correcting their mistakes.
- Give LEP students (especially beginners) alternate ways to participate in whole-class discussions and respond to questions (think/pair/share, flashcards to raise over head, hand and/or body movements, individual chalkboards for solving computations).
- Assess whether LEP students have mastered mathematical concepts rather than their English grammar and fluency.

The accommodations are adopted from the following source.
http://www.doe.virginia.gov/VDOE/Instruction/ESL/LEPmathResource.pdf