Function Family Fun

I. UNIT OVERVIEW & PURPOSE:

This unit emphasizes the real-world applications of parent functions and their transformations. It is crucial for high school students to have the ability to analyze and interpret data in a practical real-world setting. This unit is comprised of lessons in which students will be given various information and data to use to investigate different parent functions.

Students will use GeoGebra to explore and recall properties about the various parent functions (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic). Students will use this software to consider how each type of transformation affects each parent function to draw conclusions about these similarities (i.e. shifting left or right, or up and down).

After reviewing properties of parent functions, students will be given real-world data in order to learn which type of function best models the data set. Then, students will learn how to make predictions and answer questions using their discovered function. These lessons lead to the final project in which students will gather information, determine the best type of function to model their data, and then present their findings to their class.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course (the course title might change)

IV. CONTENT STRAND:

Algebra and Data Analysis and Probability

V. OBJECTIVES:

- Students will review properties of parent functions (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) through exploration, collaboration, and class discussion using GeoGebra.
- After reviewing, students will use properties of parent functions to determine which type of function represents specific data most accurately.

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VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.
MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

VII. CONTENT:

This unit also addresses real-world scenarios in which one would want to make predictions, such as profit & loss, marketing benefits, bacteria explosion or decay, etc. Any topic that can have data collected, have an equation regression to provide the relationship between the data, and can be used for prediction, can be utilized within this unit.

VIII. REFERENCE/RESOURCE MATERIALS:

Graphing Calculator, GeoGebra, Microsoft Excel, worksheets, rubrics, computers(for research/data collection for assessment)

IX. PRIMARY ASSESSMENT STRATEGIES:

Think/pair/share, class discussion, reflections/journals, completed activities, project and presentation.

X. EVALUATION CRITERIA:

Scoring rubrics are attached at the end of this unit for each lesson that requires them, along with all supplemental materials.
All materials (worksheets and rubrics) broken down by lesson are available at https://sites.google.com/site/functionfamilyfun/ .

XI. INSTRUCTIONAL TIME:

Approximately 7 - 90 minute blocks or 14 - 45 minute periods

Function Family Reunion

Strand
Algebra - Functions

Mathematical Objective(s)
Students will review the parent functions of linear, quadratic, absolute value, square root, rational, exponential and logarithmic equations. Students will review the graphs, equations, properties, including domain, range, and asymptotes, for each function type.

GeoGebra software and graphing calculators will be utilized to aid students in visualizing each function and allowing exploration within each family type to show that properties remain intact throughout the "child" graphs.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL AII.6, AII.7

NCTM Standards List all applicable NCTM standards related to each lesson. Example:
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior

Additional Objectives for Student Learning (include if relevant; may not be math-related):
Students will be able to compare and contrast different function families and compare functions within the same family.

Materials/Resources
- Classroom set of graphing calculators

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- Student Computers with Internet for GeoGebra (download free software at www.geogebra.org)
- Smart Board/Computer & Projector for Teacher to show GeoGebra (download free software at www.geogebra.org)
- Student Note Sheet (Appendix pages 31 – 34)
- Teacher GeoGebra Notes for function family review (Appendix pages 35 – 42)
- GeoGebra parent function demo for all functions link: http://web.psjaisd.us/auston.cron/ABCronPortal/GeoGebraMenu/GeogebraFiles/funcProps/BasicFamilyTransformations01.html
- Worksheet activity (Appendix page 43 – 45)
- Other helpful materials can be found at the following links:
  - Parent function graphs:
  http://faculty.gg.uwyo.edu/dueker/GeophysicsClass/Math%20Review/IG%20parent%20functons.pdf
  - Function Review:
  http://webhome.broward.edu/~gmaxwell/mac2233/FundamentalFxns.pdf

Assumption of Prior Knowledge

- Exposure to functions, function families and their properties
- Understanding of how equations and graphs are connected for functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students may or may not have exposure to using software such as GeoGebra (which is similar to Geometer's Sketchpad)
- Teachers may have to spend time showing students how to use the software

Introduction: Setting Up the Mathematical Task

- In this lesson, you will review different function families by examining the parent function and properties of linear, quadratic, absolute value, square root, rational, exponential and logarithmic equations and their respective graphs as well as transformations of the graphs.
- The review should last approximately 70 minutes with 15 minutes for group completion of worksheet and 5 minutes for closure reflection-journal entry.
- Let's have a math Family Reunion!! ***A Function Family Reunion***
- Do any of you take after your parents? Looks, personality, interests?
- Do functions have parents? What is a parent graph? A child graph? What is the same about them? What can be different?
- We will be discussing different types of function families by looking at the parent graphs. We will use graphing calculators and GeoGebra software to discuss the properties of each as a class as this should be a review.

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- Students will have a notes worksheet template that will help them to take notes and review the properties of each function family.
- Before beginning each function type, students will be asked to show the typical graph by using their body to recreate it. (ex. hands straight for linear, in a "U" for quadratic, etc.)
- The teacher will use the GeoGebra software to show and explain the different properties of each function parent.
- Students will be asked to participate in the review discussion by looking at graphs and answering questions about the properties, equations, domain & range.
- Students will also have a worksheet where they will have to match a parent and a child and name the function type.

Student Exploration 1:

Whole Class Sharing/Discussion

Student/Teacher Actions:

- During lesson 1 the student should be engaged in review discussion being led by the teacher during the first part of the class and filling out their own notes worksheet. Once complete, each student should be working on their family matching worksheet.
- The teacher should lead the class in review discussion and keep students engaged by asking questions and asking them their previous knowledge before each function family is introduced. Having students try to show the shape of a parent graphs with their hands or body is always an interesting way to keep them moving and motivated.

Student Exploration 2:

Whole Class Sharing/Discussion

Student/Teacher Actions:

- After the review in student exploration 1 has been completed, students will work with a partner or in a small group (this is left to the teacher to decide based on each individual class) on the "Family Function" matching worksheet (Appendix pages 43 – 45).
- While students are working on this worksheet each group can utilize their graphing calculator or the GeoGebra software as needed.
- The teacher should be walking around monitoring and helping students when they get to a problem that proves to be difficult. The teacher should ask questions that helps guide the students to the answer, rather than just providing the answer. (For example, "What do you notice about the shape of the graph?" "What type of parent function has that shape?" etc.)

Monitoring Student Responses

- Teacher and Student expectations:

- students will participate in review by answering questions and filling out notes sheet
- students will communicate with each other when working on worksheet at end of class
- teachers will answer questions and utilize software to help students visualize different function graphs and explain their properties
- teachers can allow students that are ready to move forward to explore other members (more challenging members) of the function family and see if the properties hold

- Lesson Summary
  - Emphasize that families have certain traits that are inherited and so do function families. Students should complete the parent/child matching worksheet in pairs or groups of 3.
  - In the last 5 minutes of class, have some of the groups go over their answers to the group worksheet and give explanations of why they chose each function type.
  - For closure activity, each student could respond to the following question in a journal or reflection: Describe your favorite non-linear function family and its properties.

Assessment

- Describe and attach the assessments for each lesson objective.
  - **Worksheet (attached)**
    - Students will work as a group to match parent and child graphs and decide which function family they represent
    - Questions can be asked of groups to explain their thinking methods
  - **Journal/writing prompts**
    - Writing Prompt: Describe your favorite non-linear function family and its properties.
  - **Final Assessment**
    - Final project (Lesson 6) will utilize information from this lesson to choose a proper trendline based on the graph properties.

Extensions and Connections (for all students)

- Lesson extensions- This lesson will be extended in the next 5 lessons by using the properties and identifying functions to write equations by transformation and find trendlines of data.
- Other connections will be in lessons 4 and 5 when we apply this knowledge to trendlines and predicting outcomes of scientific, business, and other data.

Strategies for Differentiation

- Ideas for addressing needs of a diverse population of students:
    - For kinesthetic learners, we will have them adding the information to a notes worksheet that will allow them to physically graph each function family. Both the auditory and visual learner styles should be met through the lecture and software application use.
    - Students with motor issues can utilize the GeoGebra software to graph their functions and print out copies in place of handwriting the notes sheet.
    - Teachers should work with the ELL teacher to help bridge the language gap for English language learners (ELLs).
    - High-ability students can look at more challenging child graphs in order to see the changes that come from multiple changes in the parent function.
- All students will have notes that will have an organized section by function family for the name, equation, graph, and properties. If a student works better without multiple function types on a page, each function family could be contained on its own page with larger graph and font.

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Transformation Techniques

Strand
Algebra-Functions

Mathematical Objective(s)
In this lesson, students will use the information they learned from the previous lesson to identify function families and then compare the graphs based on a transformational approach. Students will review what happens to the parent graph of a function when the leading coefficient is negative, if the constant changes, or if there is a sum or difference with the x-coordinate (Vertex Form: ex. $y = a(x-h)^2 + k$). The students will use GeoGebra to see how the parent graph transforms as a, h, and k change. The teacher will discuss these changes and show students how to write the equation of functions using vertex form. Students will then be able to write the equation of the function based on the transformational change. Different function families will be used to show that these transformation techniques work for all functions.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

Related SOL AII.6, AII.7

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- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Draw reasonable conclusions about a situation being modeled

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- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Materials/Resources

- Classroom set of graphing calculators
- Student Computers with Internet for GeoGebra (download free software at www.geogebra.org)
- Smart Board/Computer & Projector for Teacher to show GeoGebra (download free software at www.geogebra.org)
- GeoGebra activity-See attached file

Assumption of Prior Knowledge

- Exposure to functions, function families and their properties
- Understanding of how equations and graphs are connected for functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students should have a good handle on using GeoGebra from the last lesson

Introduction: Setting Up the Mathematical Task

- In this lesson, students will review function writing by examining transformations as it pertains to the vertex form of function (ex. $y=a(x-h)^2+k$).
- 1 - 90 minute block or 2 - 45 minute periods
    - 5 - 7 minute review about parent functions and transformations using GeoGebra lead by the teacher
    - 30 minutes - (Student exploration 1) to work with a partner choosing select amount of parent functions (this should be chosen by the instructor - there are 9 (absolute value, square root, cube root, rational, linear, quadratic, cubic, exponential, and logarithmic) parent functions that should have been reviewed, so the instructor should set a minimum amount that students must explore.)
        - For example, 4 parent functions with 2 unique transformations per function.
    - 40 - 45 minutes - (Student exploration 2) Students will work with a partner to complete the “Transformation Techniques” Worksheet, then use Microsoft Excel to make sure their equations match the given parent function.
    - 8 - 15 minutes writing journal/ reflection

- Let’s investigate the function families we learned about yesterday.
- Questions to ponder: How will the child graphs be different from the parent graph? What will be the same? How will we know if one graph belongs to a particular function family?
- A fun thing to do is to have kids do the Electric Slide or Cupid shuffle to introduce transformations.

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- The GeoGebra activity will allow students to investigate different functions and see what happens as the form of the equation changes.
- Students will have to use their previous experience about transformational graphing to analyze and review the material.
- By allowing students to manipulate multiple graphs and different types of function graphs, students are able to make connections about the standards of writing equations using a transformational approach.
- Students will present their findings in a class discussion at the end of the lesson. The teacher will ensure that all groups interact and interject their thoughts about the activities and their mathematical thinking. Students will also record their thoughts in a journal/reflection so that the teacher can see what each student thinks individually since a shyer student will hopefully be more vocal in their journal, as opposed to speaking up in front of their entire class.

Student Exploration 1:

Collaboration and Exploration

Student/Teacher Actions:

- Students will work with a partner using GeoGebra to play around with different variations of the parent graphs to explore or verify which properties remain throughout the family and what changes.
- In order to accomplish this task, students must use the function tool (“input:”) located at the bottom of the screen. (Diagram below of what the students should use)

- Students will choose a specific number (designated by the teacher) of parent functions to use for this exercise. (For example, the teacher can tell each pair of students that they must create 4 different types of parent functions. Then, they must make 2 unique transformations for each function.)
- The students will be free to create their own equations to transform the various parent functions they have chosen (students will choose from absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic).

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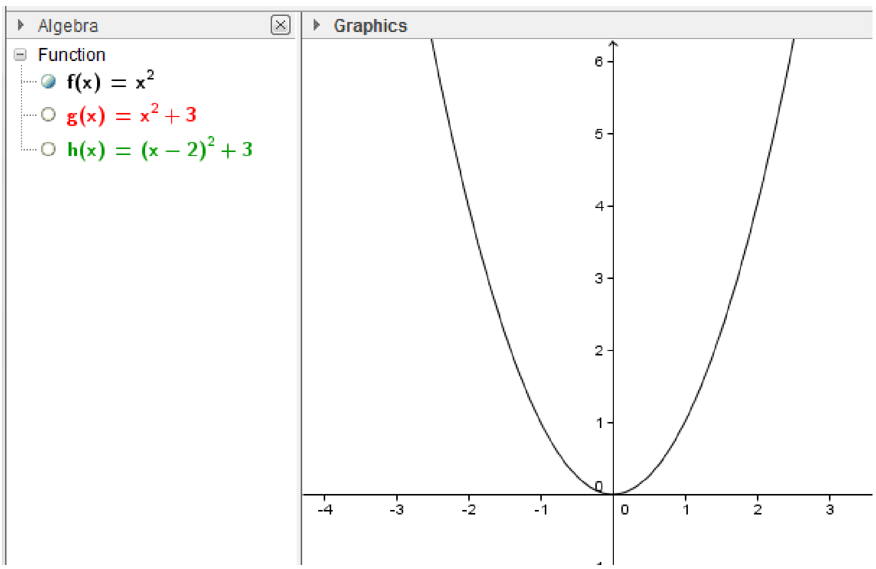
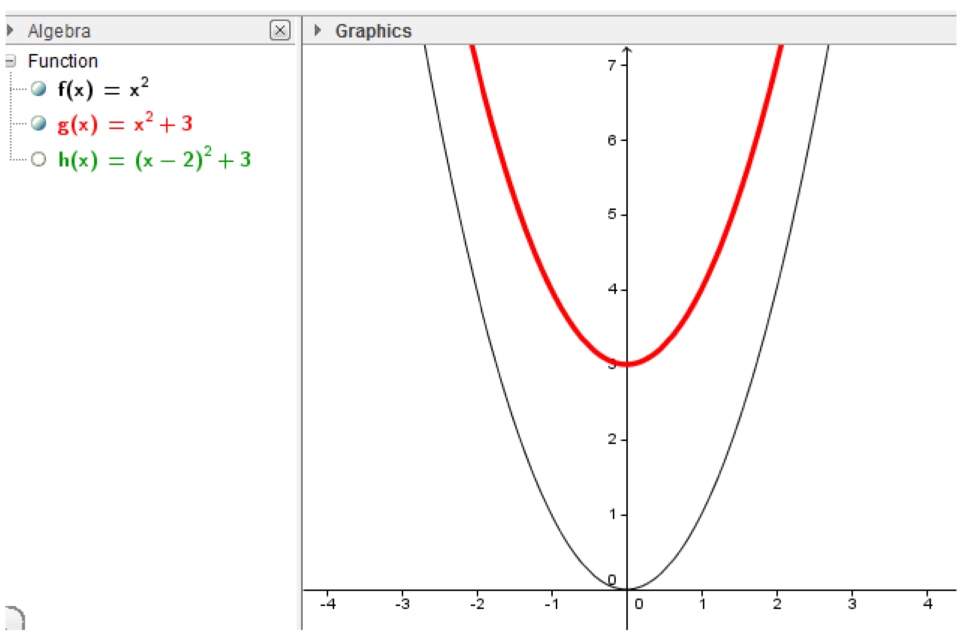
- Each pair of students should record their parent functions, then change their functions and record how they changed the function and describe what happens to the graph. Students should turn this in at the end of class.
- The only requirement for this assignment is for students to explore the various functions and create different transformations to form connections about what happens to the equation and how it affects the graph. (The teacher can determine how many transformations each function must have).

Examples of 1 set of work for a selected Parent Function in GeoGebra:



Now after a transformation:



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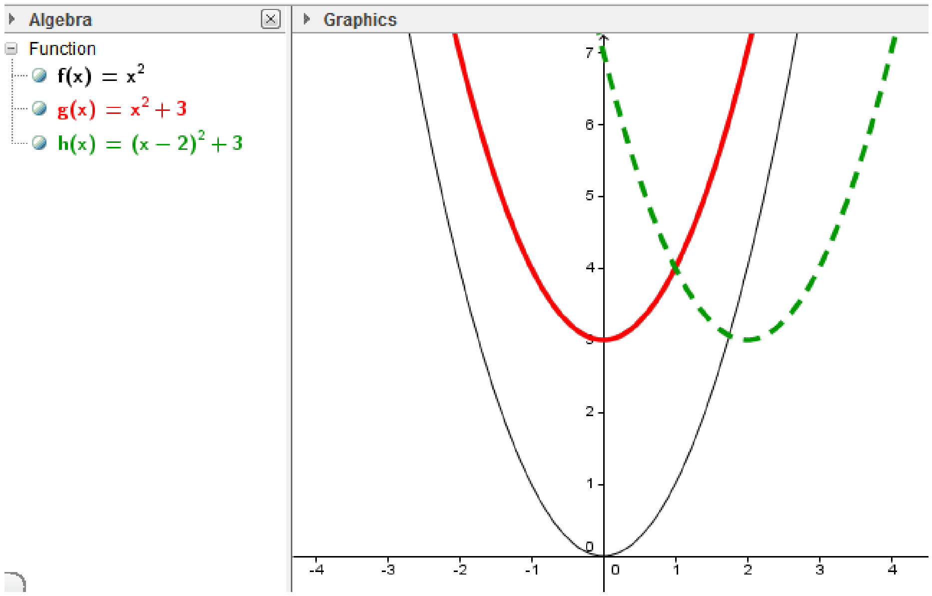
Now after a shift horizontally:



- Students should turn in their GeoGebra file(s) upon completion for a grade
  - Rubric (Appendix page 46)

Student Exploration 2:

Collaboration and Exploration

Student/Teacher Actions:

- Students will work with a partner to complete the "Transformation Techniques" Worksheet (Appendix pages 47 – 51) to write equations for various graphs.
- Students will be given graphs that have points marked (for students with disabilities, these points can also be labeled), so that they can determine the equation of that function in vertex form.
- Students should work on this exercise and they can use their graphing calculator to assist them in coming up with the equations.
- After allowing students to work on these worksheets with a partner, allow each group of students to check their equations using Microsoft Excel or GeoGebra.
  - **Microsoft Excel**: Students will have to identify the points on the graph, insert the points into Excel, and then create a scatter plot (highlight the data with headings, then click on "insert" then click on the scatter plot icon). Once students have created a scatter plot, each group can create a trendline (right click the points on the scatter plot, then click "add trendline" - then students

must choose which type of equation or function represents the data most accurately) and check the R^2 value to see if their equation is the most accurate.

 - (Directions on how to do this are located in the following youtube video titled "Plotting Data" http://www.youtube.com/watch?v=-g_Kt1cs8fo&list=PLAT_s7jzgi40_6umve55unlHosR3Fm7Yz)

- **GeoGebra**: Students can create a GeoGebra file to graph their equations from their worksheet to see if their graph in GeoGebra matches the graph on the worksheet. Students should type their equation into the input bar in vertex form. Students can type #1 as a(x), #2 as b(x), and so on to check all of their equations to ensure that each matches its corresponding graph. Students can right click each equation in the Algebra window to change the color and/or design of each line. Also, students can click on the shaded dot to the left of equation to turn the graphs on or off so that the graphs will not be on top of each other (An example of what this file should look like is located at the following: http://www.geogebratube.org/material/show/id/42229)

Individual Work (if relevant)
Small Group Work (if relevant)
Whole Class Sharing/Discussion (if relevant)
Student/Teacher Actions:

- Students should be interacting with the parent functions and transformations of their choice, using the GeoGebra software on the computer. Students should be working with a partner to discuss the graphs and the effects of the transformations by changing their equations.
- After exploring the parent functions and transformations, students should then work with their partner to complete the "Transformation Techniques" Worksheet. Students can check their work for this activity by using GeoGebra and/or Microsoft Excel.

While students are engaged in both activities, the teacher should be walking around ensuring that students are working and discussing the mathematical concepts. Also, while the teacher is circulating the room it will give students the opportunity to ask questions if they get stuck or need some assistance.

Questions to pose:

- What is the domain of the parent function you have created? What is the range?
- When you transform your function, does it affect your domain? Does it affect your range? Why or why not?

Misconceptions could arise during exploration 2 since the graphs are provided, there will only be one correct answer. This is more challenging since students in exploration 1 are free to create graphs however they choose as long as they meet the criteria, therefore, there is not just one correct answer. In order to combat this misconception students can keep their notes out about parent functions, there general shape and equation, to help assist in discovering the correct equation.

Monitoring Student Responses

Students will demonstrate their knowledge about transformations of parent functions during these activities by talking with their partner, during the review at the end of the lesson in the class discussion, and after in their reflection paper. Each student's level of understanding will determine which types of transformations will be complete; for example: do the student pairs perform more than one type of transformation within one problem (i.e. a horizontal shift and a vertical shift, or a vertical shift and a dilation-expansion or contraction).

Also, students' answers to the above questions will demonstrate their knowledge about transforming parent functions. Students will communicate their thinking from these activities in their one page reflection that should be turned in upon completion of the lesson.

Throughout this exploration the teacher will be walking around to answer any questions that may arise. Students that find this task difficult are free to discuss their problem(s) with the teacher. The student should talk out their misconception(s) and work through their problem(s) with their partner, then their teacher if necessary. In other words, the teacher will help the student come up with a solution, as opposed to just answering the question.

Assessment

Students will turn in all of their work this includes, but is not limited to, GeoGebra file(s) for exploration 1, Transformation Techniques Worksheet with all work, Microsoft Excel file(s) to check work, and journal/reflection.

Questions

Located on the "Transformation Techniques" Worksheet (Appendix pages 47 – 51)

(Answer key – Appendix page 52)

Journal/writing prompts

- What ways have you learned to transform parent functions?
- How would you describe each type of transformation? (Student should realize that a horizontal shift to the right is caused by subtracting, which means that it moves the opposite direction from what students would think. However, a vertical shift moves exactly as one would think because the graph moves up when you add and down when you subtract.)
- What were some of the concepts that you struggled with? How did you overcome these obstacles?
- List any areas of concern or confusion you still may have.

Extensions and Connections (for all students)

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Lesson extensions- This lesson will be extended in the next 4 lessons by using the properties and identifying functions to write equations by transformation and find trendlines of data. Other connections will be in lessons 4 and 5 when we apply this knowledge to trendlines and predicting outcomes of scientific, business, and other data.

Strategies for Differentiation

- This lesson is naturally differentiated for a diverse range of ability students in that gifted students could transform their parent functions multiple ways each time (i.e. a horizontal shift and a vertical shift, or a vertical shift and a dilation-expansion or contraction as opposed to just one type of transformation. Furthermore, this lesson meets the needs of students that are gifted more technologically than mathematically since they are provided with the opportunity to demonstrate their skills using GeoGebra and/or Microsoft Excel. The fact that students are creating their own parent functions and then transforming them in whatever manner they choose, allows them to express their creativity and challenge themselves at a level that is best suited to their abilities.
- Students with disabilities will be provided points on their graphs with labels and assistance using GeoGebra and Microsoft Excel.
- Ideas for addressing needs of a diverse population of students:
- For kinesthetic learners, we will have them adding the information to a notes worksheet that will allow them to physically graph each function family. Both the auditory and visual learner styles should be met through the lecture and software application use.
- Students with motor issues can utilize the GeoGebra software to graph their functions and print out copies in place of handwriting the notes sheet.
- Teachers should work with the ELL teacher to help bridge the language gap for English language learners (ELLs).
- High-ability students can look at more challenging child graphs in order to see the changes that come from multiple changes in the parent function.

Function Junction

Strand
Algebra-Functions

Mathematical Objective(s)
This lesson is intended to assess students' knowledge of function families, their graphs, their equations, and their ability to write equations of functions. Students will be given an Excel activity with hidden equations. Each of the functions will be a model of a real-world situation. The students can enter in input data and receive output data. Students will be able to collect that data and see the scatter plot graph in order to find the equation of the function. Once they have the function, they will use it to predict a problem associated with the real-world model. After the activity is complete, students will do a think/pair/share about the difficulties they had with the activity. If time remains, the group can create their own real-world functions by researching the internet for a specific type of function family. The original activity and journal reflection will be used as an assessment.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

Related SOL AII.6, AII.7

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- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Draw reasonable conclusions about a situation being modeled

- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Additional Objectives for Student Learning (include if relevant; may not be math-related):

- Students will have to deduce answers from given information.
- Students will have to predict real-world situations from an equation.

Materials/Resources

- Student Computers with Microsoft Excel (From Microsoft Office package version 2007 and higher)
- Smart Board/Computer & Projector for Teacher to demonstrate Excel application(From Microsoft Office package version 2007 and higher)
- Excel activity-Appendix

Assumption of Prior Knowledge

- Exposure to functions, function families and their properties
- Understanding of how equations and graphs are connected for functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students should know how to use Microsoft Office Products including Excel
- The teacher may have to spend time showing individual students how to use the software and application activity

Introduction: Setting Up the Mathematical Task

In this lesson, students will test their knowledge of function families, writing equations using a transformation approach and deducing equations based on coordinates and a graph.

The teacher will have a model to introduce the class to the lesson, give instruction, and answer questions (20 min). Then the students can work as individuals or in pairs on the Excel Activity for finding Mystery Functions-***Function Junction*** (Appendix pages 53 – 56)-(60 min). The last 10 minutes can be used for discussion and to complete a journal reflection.

The teacher can ask "What is a junction?" Students should have a response similar to the following: "A place where two things meet." This leads us to what we will be doing today. Today we will focus on applying the knowledge we have been working on the last two days and adding a MYSTERY factor!

The teacher should introduce the task by having a sample application to do with the class. Showing the students how to manipulate Excel, collecting data, and using the graph button to see their points on the coordinate plane and decide which function is represented.

For a class discussion the teacher can ask:

- "In what ways can we obtain data in order to plot points?"
- "Once we graph, how will we decide which function family is represented?"

- "How will we write the equation?"
- "How can we work together as a pair to complete the activity (if paired up)?"

Individual grouping can be used if the class is up to speed and doing well from the previous two days. Pairs should be used if the students are still unsure of the function families or if you would like them to discuss the properties together before making final decisions to write the equation.

By having the function/equation a mystery, it adds intrigue to the activity. Students can use their own data points to graph and have a puzzle or "game" mentality to find the answer while using prior mathematical knowledge.

Students will have to utilize prior knowledge of shape and properties for function families to decide which function to use and then apply the transformational approach to write the equation of the function in the activity.

Students will save their file and turn it into a dropbox for grading. Also, the discussion at the end of class can help students get quick feedback if their thought process was correct or not.

Student Exploration 1:

Individual or Small Group Work

Student/Teacher Actions:

Students will work either individually or in pairs on the Excel Activity-**Function Junction** (Appendix pages 56 – 56).

After going over the introduction sample with the class, the teacher should be floating around the room monitoring students to help them use the software and ask leading questions to allow students to figure out each function.

Questions to ask include: What shape does your graph have? Where is the middle? What does the curvature look like (extreme, moderate, flat)? What is the domain and range? What type of parent function does it remind you of?

Students can use a minimum of 5 coordinate points and up to ten to graph each function. Remind them to use more if they are still unsure. They could also graph on paper and add more coordinates.

Remind students to interact with their partner (if relevant). Have them take turns collecting data and trying to figure out function.

Monitoring Student Responses

Students should:

- communicate their thinking and their new knowledge through small group, class discussion, and journal reflections;
- work together and take turns using each part of the software program;

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The teacher should:

- monitor student progress and discussion to help with leading questions
- assist students who have difficulties; and
- extend the material for students that are ready to move forward.

Lesson Summary

- The teacher should bring the class back together about 10 minutes at the end of class to discuss the activity and the class findings.
- The students should then complete a journal reflection on their feelings about the activity and what they have learned.

Assessment

Questions

- o What is the hardest part about choosing a function family to use in your equations?
- o How did you go about choosing a function to use?

Journal/writing prompts

- o What did you like about this activity? What did you dislike?
- o What did you learn about functions and equations?
- o What did you learn about yourself?

Other

- o Students will turn in their Excel file in order to see their responses
- o Teachers will look for common misconceptions over closely related function graphs.

Extensions and Connections (for all students)

Lesson extensions- This lesson will be extended in the next 3 lessons by using the properties and identifying functions to write equations by transformation and find trendlines of data. Other connections will be in lessons 4-6 when we apply this knowledge and deduction technique to trendlines and predicting outcomes of real-world situations given data points.

Strategies for Differentiation

Ideas for addressing needs of a diverse population of students:

- For kinesthetic learners, pre-printed graphs can be used instead of having the computer graph coordinates, if needed to allow for physical involvement. Both the auditory and visual learner styles should be met through the lecture and software application use.
- Students with motor issues should not be hindered since all of the material is completed on Excel and the graph is constructed by the software.
- Teachers should work with the ELL teacher to help bridge the language gap for English language learners (ELLs).
- High-ability students can look at more involved mystery functions in order to keep them challenged.

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Trendlines in the Trenches

Strand
Algebra-Functions
Data Analysis and Probability

Mathematical Objective(s)
In this lesson, students will use real-world data from different subject areas to form scatter plots and graph an appropriate trendline equation by analyzing the shape and R^2 value. Once the trendline is established, students will use this information to model and predict information specific to the real-world situation. The focus of this lesson will be limited to linear and quadratic functions.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

Related SOL AII.6, AII.7

NCTM Standards List all applicable NCTM standards related to each lesson. Example:
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Draw reasonable conclusions about a situation being modeled
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Materials/Resources

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- Student Computers with Microsoft Excel (From Microsoft Office package version 2007 and higher)
- Smart Board/Computer & Projector for Teacher to demonstrate Excel application(From Microsoft Office package version 2007 and higher)
- Excel activity-See attached file
- Graphing Calculator

Assumption of Prior Knowledge

- Exposure to functions, function families and their properties
- Understanding of how equations and graphs are connected for functions
- Students should recognize the general shape of the functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students should know how to use Microsoft Office Products including Excel
- The teacher may have to spend time showing individual students how to use Excel

Introduction: Setting Up the Mathematical Task

- This lesson will take 1 – 90 minute block or 2 – 45 minute periods.
- In this lesson students will have to create a scatterplot and determine which type of trendline best fits the data. They will then use the equation of the trendline and/or graphing calculator to answer questions or make predictions.
- The teacher will demonstrate a problem using the interactive white board. (15 min.) The students will then work through the Excel activity. At least 15 minutes should be allotted for final discussion.
- Given a data set, students will create a scatterplot. Use the shape of the scatterplot to determine which type of trendline to create. Once students have the trendline (equation of the line), they will use it and their graphing calculator to answer the questions.
- What shape do the following parent functions have? Linear, quadratic, cubic, etc
- How do we find the trendline using Excel? How do we know if it is correct?
- Students will have to utilize prior knowledge of shape and properties for function families to decide which trendline to use.
- Students may work individually or in pairs
- The teacher will discuss the solutions with the students to see if their answers make sense based on the data.
- The students will turn in or display their scatterplots with the trendline through the points as well as the answers to the questions.

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Student Exploration 1:

Students will work either individually or in pairs on the Excel Activity **-Trendlines in the Trenches** (Appendix pages 57 – 59)
After going over the introduction sample with the class, the teacher should be moving around the room monitoring students to help them use the Excel and ask leading questions to allow students to figure out the shape of each scatterplot as it relates to the parent functions. Questions to ask include: "What shape does your scatterplot have?", "What parent functions does it remind you of?"
Remind students to check correlation to make sure the trendline is strong.
Remind students to interact with their partner (if relevant). Have them take turns trying to figure out each function.
At the conclusion of the lesson, use group discussion to make sure students' answers to prediction questions correlate to the data.
Have students present their problems to the class

Monitoring Student Responses

Students should:

- communicate their thinking and their new knowledge through small group, class discussion, and presentations
- work together and take turns using each part of the Excel program;

The teacher should:

- monitor student progress and discussion to help with leading questions
- assist students who have difficulties; and
- extend the material for students that are ready to move forward.

Lesson Summary

- The teacher should bring the class back together about 15 minutes at the end of class to discuss the activity and the class findings.
- The students should then describe one of their scatterplots and present their findings to the class.

Assessment

Questions

- o How did you decide which type of trendline to use?
- o Do your answers to the questions reflect the trends of the data?

Journal/writing prompts

- o Describe the advantages and/or disadvantages of using Excel versus a graphing calculator.

Other

- Accommodations for motor-skill issues can be made by pairing students
- Students will submit Excel files. They will be checked for:
    - accuracy of the graph
    - correct trendline equation
    - correct answers to questions

Extensions and Connections (for all students)

- This lesson will be continued with other types of functions in Real-world Regressions
- This lesson is connected directly with finding the curve of best fit
- This lesson may be linked to any subject based on the data sets used

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
    - Accommodations for motor-skill issues can be made by pairing students
    - Consult with ESL teacher for modifications if any student has trouble with the word problems.
    - Adjust the size of data sets to accommodate special needs students
    - For enrichment, have students research a topic of their choosing(physics, chemistry, economics etc.) and find data to create scatter plots and trendlines to make predictions.
    - Students may use a graphing calculator to compare results that they find.

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Real-World Regressions

Strand
Algebra-Functions
Data Analysis and Probability

Mathematical Objective(s)

This lesson is an extension of the previous lesson and will include polynomial, exponential and logarithmic functions. In this lesson, students will use real-world data from different subject areas to form scatter plots and graph an appropriate trendline equation by analyzing the shape and R^2 value. Once the trendline is established, students will use this information to model and predict information specific to the real-world situation.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

Related SOL AII.6, AII.7

NCTM Standards List all applicable NCTM standards related to each lesson. Example:
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Draw reasonable conclusions about a situation being modeled
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Materials/Resources

- Student Computers with Microsoft Excel (From Microsoft Office package version 2007 and higher)
- Smart Board/Computer & Projector for Teacher to demonstrate Excel application(From Microsoft Office package version 2007 and higher)
- Excel activity (Appendix pages 63 – 66)
- Graphing Calculator

Assumption of Prior Knowledge

- This is an extension of the previous lesson. Students will have modeled linear and quadratic functions.
- Exposure to functions, function families and their properties
- Understanding of how equations and graphs are connected for functions
- Students should recognize the general shape of the functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students should know how to use Microsoft Office Products including Excel

Introduction: Setting Up the Mathematical Task

In this lesson students will have to create a scatterplot and determine which type of trendline best fits the data. This lesson will include polynomial, exponential and logarithmic functions. They will then use the trendline equation and/or graphing calculator to answer questions or make predictions.

This lesson *may not take the entire class period* if all students have mastered the previous lesson. The students will work through the Excel activity. At least 15 minutes should be allotted for final discussion.

Given a data set, create a scatterplot. Use the shape of the scatterplot to determine which type of trendline to create. Once students have the trendline (equation of the line) that best suits the given data, they will use it to answer the questions.

What shape do the following parent functions have? Cubic, exponential, logarithmic etc.

How do we find the trendline using Excel? How do we know if it is correct?

Students will have to utilize prior knowledge of shape and properties for function families to decide which trendline to use.

Students may work individually or in pairs

The teacher will discuss the solutions with the students to see if their answers make sense based on the data.

The students will turn in or display their scatterplots with the trendline through the points as well as the answers to the questions.

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Student Exploration 1:

- The teacher should begin by asking the students what other types of functions can be modeled by real-world situations.
- Students will work either individually or in pairs on the Excel Activity **- Real-world Regressions.** (Appendix pages 63 – 66)
- The teacher should be moving around the room monitoring students to help them use the Excel and ask leading questions to allow students to figure out the shape of each scatterplot as it relates to the parent functions.
- Questions to ask include: What shape does your scatterplot have? What parent functions does it remind you of?
- Remind students to check correlation to make sure the trendline is strong.
- Remind students to interact with their partner (if relevant). Have them take turns trying to figure out each function.
- At the conclusion of the lesson, use group discussion to make sure students' answers to prediction questions correlate to the data.
- Have students present their problems to the class

Monitoring Student Responses

Students should:

- communicate their thinking and their new knowledge through small group, class discussion, and presentations
- work together and take turns using each part of the Excel program;

The teacher should:

- monitor student progress and discussion to help with leading questions
- assist students who have difficulties; and
- extend the material for students that are ready to move forward.

Lesson Summary

- The teacher should bring the class back together about 15 minutes at the end of class to discuss the activity and the class findings.
- The students should then describe one of their scatterplots and present their findings to the class

Assessment

Questions

- How did you decide which type of trendline to use?
- Do your answers to the questions reflect the trends of the data?

Journal/writing prompts

- Identify one real-world situation and describe the importance of using data to make predictions relating to it.

Other

- Accommodations for motor-skill issues can be made by pairing students
- Students will submit Excel files. They will be checked for:
  - accuracy of the graph
  - correct trendline equation
  - correct answers to questions

Extensions and Connections (for all students)

- This lesson is connected directly with finding the curve of best fit
- This lesson may be linked to any subject based on the data sets used

Strategies for Differentiation

- List ideas for addressing needs of a diverse population of students such as:
- Accommodations for motor-skill issues can be made by pairing students
- Consult with ESL teacher for modifications if any student has trouble with the word problems.
- Adjust the size of data sets to accommodate special needs students
- For enrichment, have students find 3 occupations where data is used to make predictions and list a specific example for each.
- Students may use a graphing calculator to compare results

Project & Presentation

Strand
Algebra-Functions
Data Analysis and Probability

Mathematical Objective(s)
The goal of this lesson is for students to research and choose a set of data that interests them in order to use that data to create a line of best fit and answer questions pertaining to that graph. During this lesson, students will use GeoGebra and Microsoft Excel to analyze their data, create a graph, find the best trendline, and answer questions using that trendline. Each student will work with a partner to complete this lesson throughout lesson 2 – 5, and then present to their class their findings.

Mathematics Performance Expectation(s)
MPE. 12 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

MPE. 13 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

Related SOL AII.6, AII.7

NCTM Standards List all applicable NCTM standards related to each lesson. Example:
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Draw reasonable conclusions about a situation being modeled
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Additional Objectives for Student Learning (include if relevant; may not be math-related):

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Depending on the students choice of what they wish to research will determine whether or not other content area is addressed.

Materials/Resources
- Computer
- GeoGebra software
- Microsoft Excel
- Choice of presentation method/materials (i.e. PowerPoint, poster board, etc.)
- Classroom set of graphing calculators

Assumption of Prior Knowledge
- Exposure to functions, function families, and their properties
- Understanding of how equations and graphs are connected for functions
- Students should be familiar with using a TI-83 plus (or higher) graphing calculator
- Students should know how to use Microsoft Office Products particularly Excel and PowerPoint Presentation
- The teacher may have to spend time showing individual students how to use the software or how to interpret their data

Introduction: Setting Up the Mathematical Task
In this lesson students will use all of their acquired skills from lessons 1 - 5 to research a set of data of their choice, and then use it to create a function and trendline. Then, students will use this equation to answer questions about their data and make predictions.
This lesson is designated for student presentations. Students will be given 2 - 90 minute blocks, or 4 - 45 minute periods, in order to present to their classmates.
Students will begin working on this project after learning lesson 1. Each pair of students should continue working on this project during lessons 2 - 5.

Student Exploration 1:
- Students should create an outline explaining the process they have taken throughout this unit to research their data, find the function and equation that represents their data best, and use this equation to answer questions about the data. This outline should be used to help create their presentation.
- Students can choose to create a PowerPoint presentation, video presentation, poster board, etc. to display their work. They will be presenting about their data and the process they took to discover the function that represents it best.
- Each group presentation should take about 10 minutes.

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- Students will answer questions about their data using the trendline they found. The student should also explain what type of function is the parent function to their function.

  Note: if this assignment is too difficult because it is so open-ended, then the teacher should realize this and modify this assignment. In other words, the teacher can provide data sets for the students similar to the data sets found on the worksheets in lesson 4 and 5. Then, the student pairs can choose which data set they would like to present on. If this is the case, the teacher should have each pair of students pick one of the given topics which were teacher provided, preventing groups from presenting on the same data sets.

Assessment

- Students will turn in their data, their (scratch) work to find the best trendline (excel file), and any work that pertains to their presentation.
- Students will be assessed through their presentations to the class. There will be a rubric completed to make sure all aspects of the presentation are covered and that all materials for the project were turned in.
- Rubric (Appendix page 70)

Strategies for Differentiation

- This lesson is naturally differentiated for a diverse range of ability students in that students can be as creative as they wish with how they choose their data set, how they use that data set to make predictions, and what method of presentation they choose. Furthermore, this lesson meets the needs of students that are gifted more technologically than mathematically since they are provided with the opportunity to demonstrate their skills using whatever presentation method they choose. The fact that students are choosing their own data sets, then using it to find the function and equation that best suits it, allows them to express their creativity and challenge themselves at a level that is best suited to their abilities.
- Students with disabilities will be given assistance using GeoGebra and/or Microsoft Excel to analyze their data.
- Ideas for addressing needs of a diverse population of students:
- For kinesthetic learners, they will be able to physically graph their function using GeoGebra and Microsoft Excel.
- Both the auditory and visual learner styles should be met through the presentation aspect of this lesson.
- Students with motor issues can utilize the GeoGebra software to graph their functions and print out copies in place of handwriting or drawing the function for their presentation.
- Teachers should work with the ELL teacher to help bridge the language gap for English language learners (ELLs).

Name: ______________________________

Appendix

Date: ______________________________

Function Family Reunion

Below is a listing of parent functions for several types of functions. Complete the information and draw a graph of the parent function. Remember that all members of a function type have the same characteristics as the parent.

1. Linear Function (identity)

Parent Graph Equation: y=x

Domain:______________________________

Range:______________________________

Intercepts:____________________________

Increasing Interval:___________________

Decreasing Interval:___________________

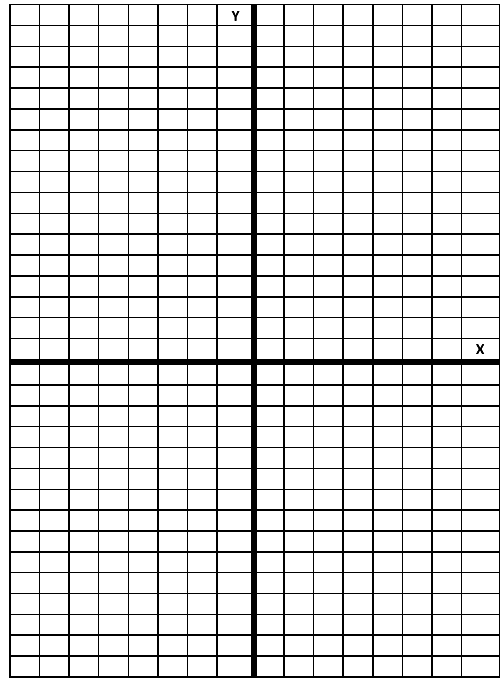
Asymptote:____________________________

End behavior: x→ ∞:__________________

x → -∞:_______________

Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



2. Quadratic Function (square)

Parent Graph Equation: $y=x^2$

Domain:______________________________

Range:______________________________

Intercepts:____________________________

Increasing Interval:___________________

Decreasing Interval:___________________

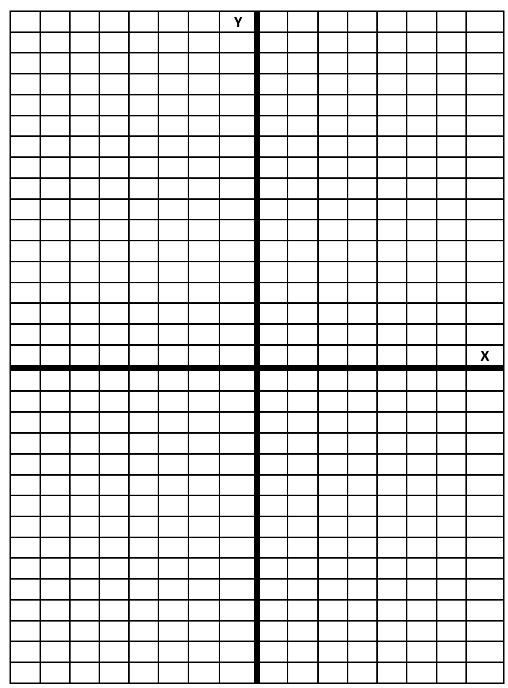
Asymptote:____________________________

End behavior: x→ ∞:__________________

x → -∞:_______________

Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



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3. Cubic Function (cube)

Parent Graph Equation: $y=x^3$

Domain:____________________________

Range:______________________________

Intercepts:__________________________

Increasing Interval:__________________

Decreasing Interval:__________________

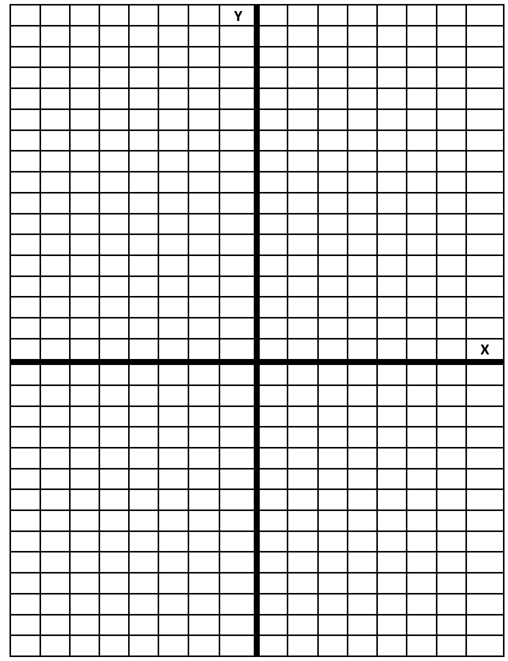
Asymptote:__________________________

End behavior: $x \to \infty$:_________________

$x \to -\infty$:_______________

Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



4. Square Root Function

Parent Graph Equation: $y = \sqrt{x}$

Domain:____________________________

Range:______________________________

Intercepts:__________________________

Increasing Interval:__________________

Decreasing Interval:__________________

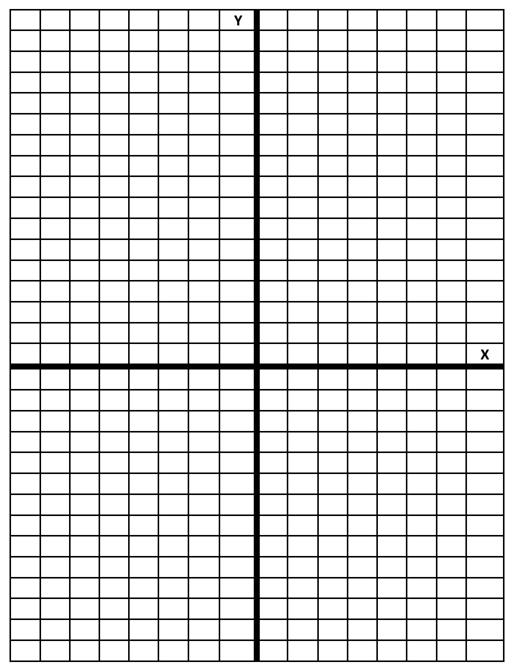
Asymptote:__________________________

End behavior: $x \to \infty$:_________________

$x \to -\infty$:_______________

Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



5. Absolute Value Function

Parent Graph Equation: $y = |x|$

Domain:____________________________

Range:_____________________________

Intercepts:__________________________

Increasing Interval:__________________

Decreasing Interval:__________________

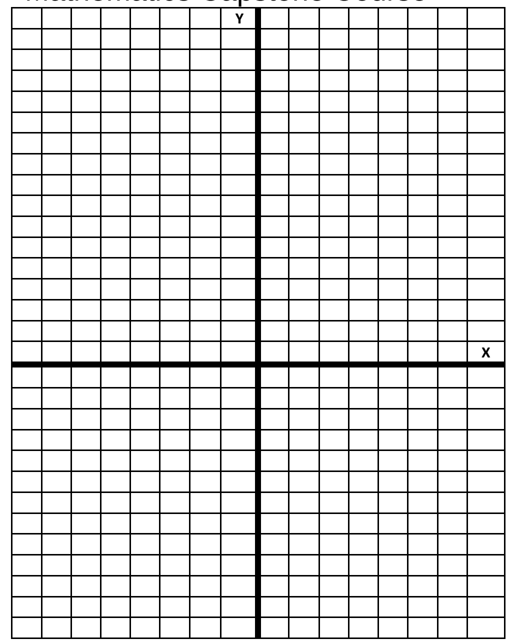
Asymptote:__________________________

End behavior: x→ ∞:_________________

x → -∞:_______________

Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



6. Reciprocal Function

Parent Graph Equation: $y = \frac{1}{x}$

Domain:____________________________

Range:_____________________________

Intercepts:__________________________

Increasing Interval:__________________

Decreasing Interval:__________________

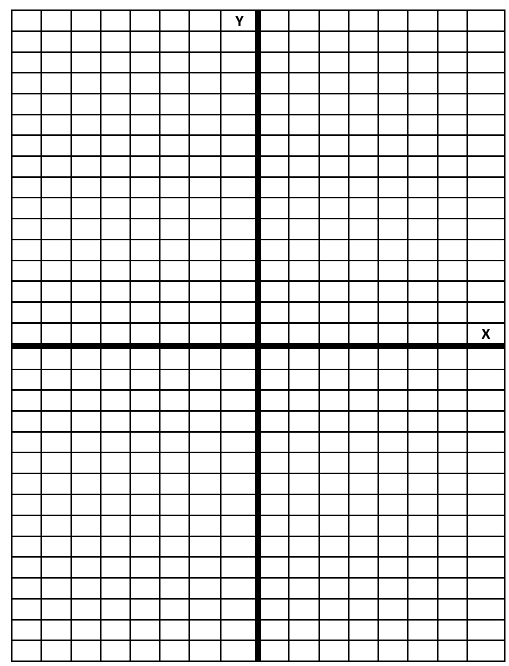
Asymptote:__________________________

End behavior: x→ ∞:_________________

x → -∞:_______________

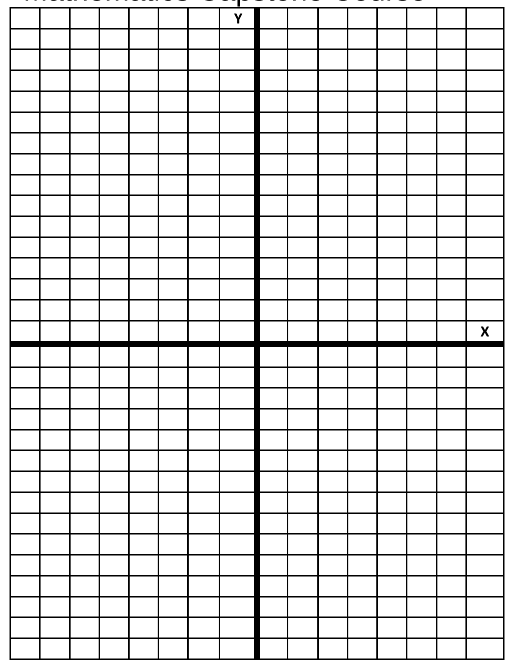
Even/Odd/Neither

X	Y
-2	
-1	
0	
1	
2	



7. Exponential Function

X	Y
-2	
-1	
0	
1	
2	



Parent Graph Equation: $y = a^x$

Domain:___________________________

Range:____________________________

Intercepts:_________________________

Increasing Interval:_________________

Decreasing Interval:_________________

Asymptote:_________________________

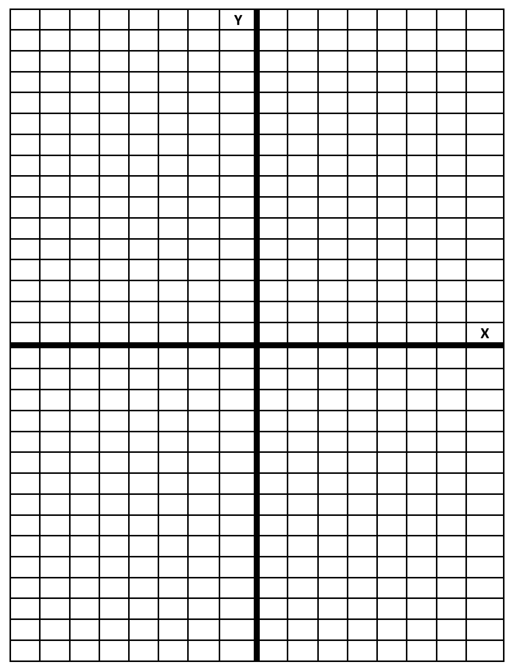
End behavior: x→ ∞:________________

x → -∞:______________

Even/Odd/Neither

8. Logarithmic Function

X	Y
-2	
-1	
0	
1	
2	



Parent Graph Equation: $y = log_a x$

Domain:___________________________

Range:____________________________

Intercepts:_________________________

Increasing Interval:_________________

Decreasing Interval:_________________

Asymptote:_________________________

End behavior: x→ ∞:________________

x → -∞:______________

Even/Odd/Neither

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Function Family Review Teacher Instructions:

Download GeoGebra Software from www.geogebra.org or use the online applet.

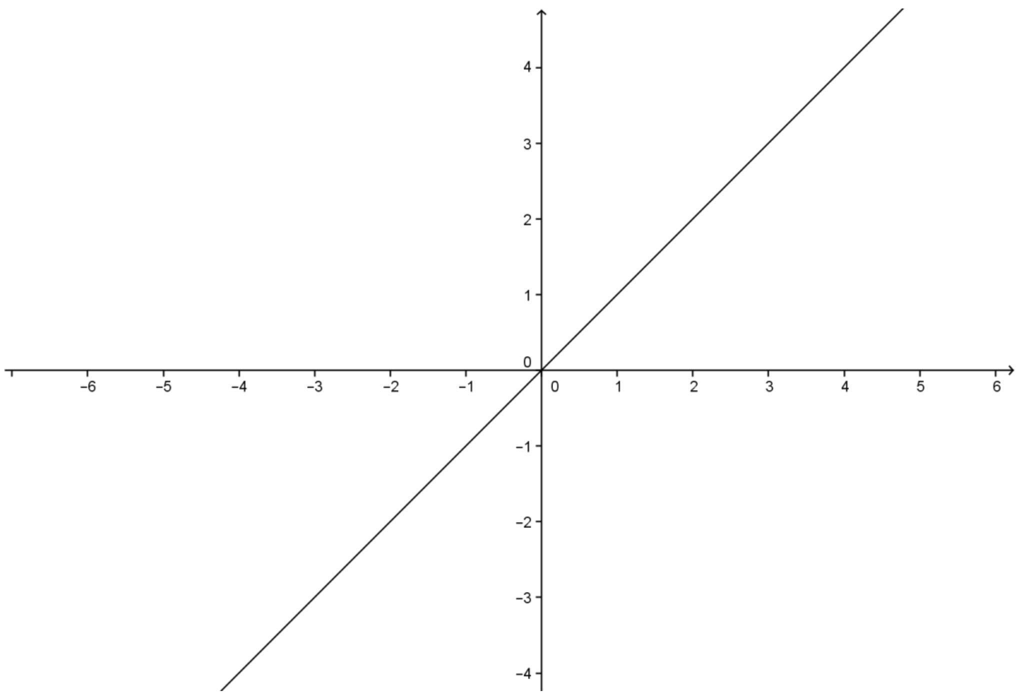
Have a completed form of the Function Family Reunion Sheet- Graph Key is on last page.

Use GeoGebra to help show the parent graph and discuss the properties of each: (there are more parents graphs that can be discussed- Use time wisely and choose the functions you feel are most important to cover).

To graph each function- Enter the parent equation into the "Input" field on GeoGebra

1. Linear

Parent Graph Equation: y=x



Domain:_ all real no.__________ Range:_ all real no.__________

Intercepts:__ (0,0)__________ Increasing Interval:__$(-\infty, \infty)$_____

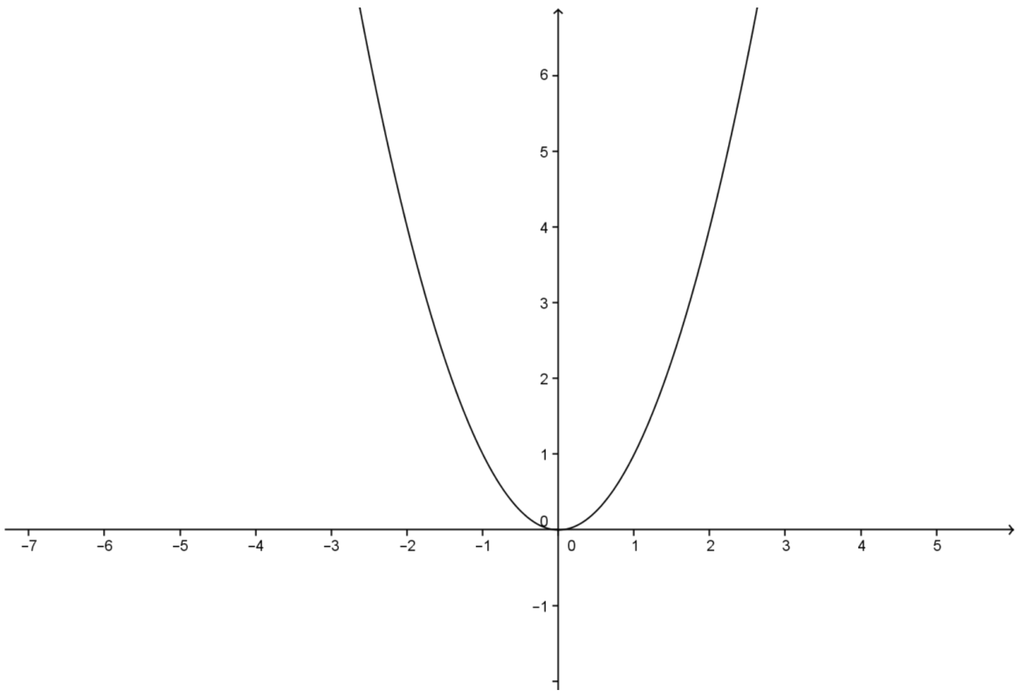
Decreasing Interval:__ NA________ Asymptote:___ NA________

End behavior: x→ ∞:_f(x)→ ∞:____ Odd Function

x → -∞:_f(x)→- ∞:____

2. Quadratic

Parent Graph Equation: $y=x^2$



Domain: all real no.

Range: $y \geq 0$

Intercepts: (0,0)

Increasing Interval: $(0, \infty)$

Decreasing Interval: $-\infty, 0$

Asymptote: NA

End behavior: $x \to \infty$: $f(x) \to \infty$:

$x \to -\infty$: $f(x) \to \infty$:

Even Function

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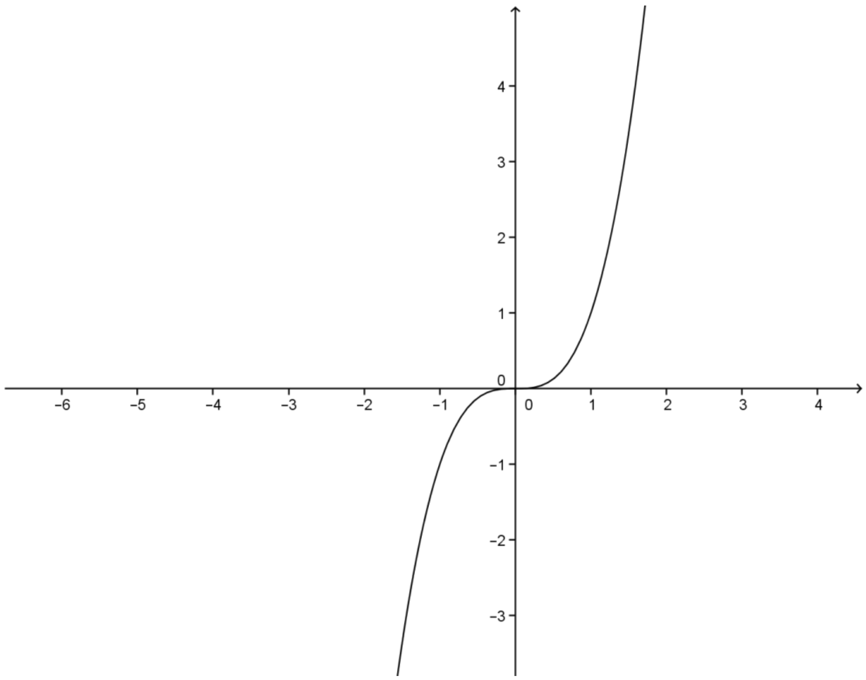
3. Cubic

Parent Graph Equation: $y=x^3$



Domain: all real no.

Range: all real numbers

Intercepts: (0,0)

Increasing Interval: $(-\infty, \infty)$

Decreasing Interval: NA

Asymptote: NA

End behavior: $x \rightarrow \infty$: $f(x) \rightarrow \infty$:

$x \rightarrow -\infty$: $f(x) \rightarrow -\infty$:

Odd Function

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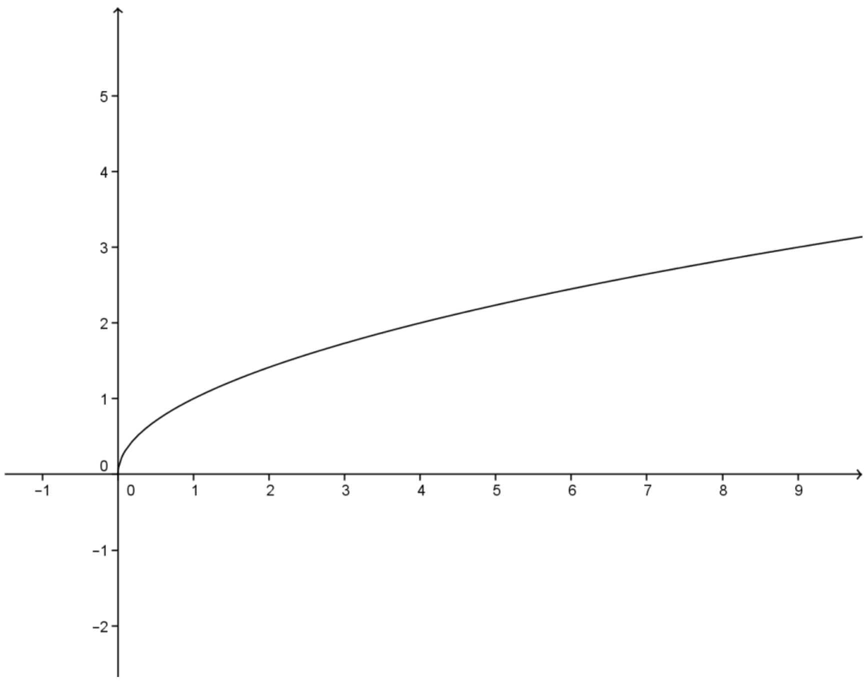
4. Square Root

Parent Graph Equation: $y = \sqrt{x}$ **(Remember: y=x^(1/2))**



Domain: x≥0

Range: y≥0

Intercepts: (0,0)

Increasing Interval: (0, ∞)

Decreasing Interval: NA

Asymptote: NA

End behavior: x→ ∞: f(x)→ ∞

x → 0+: f(x)→0

Neither Even nor Odd

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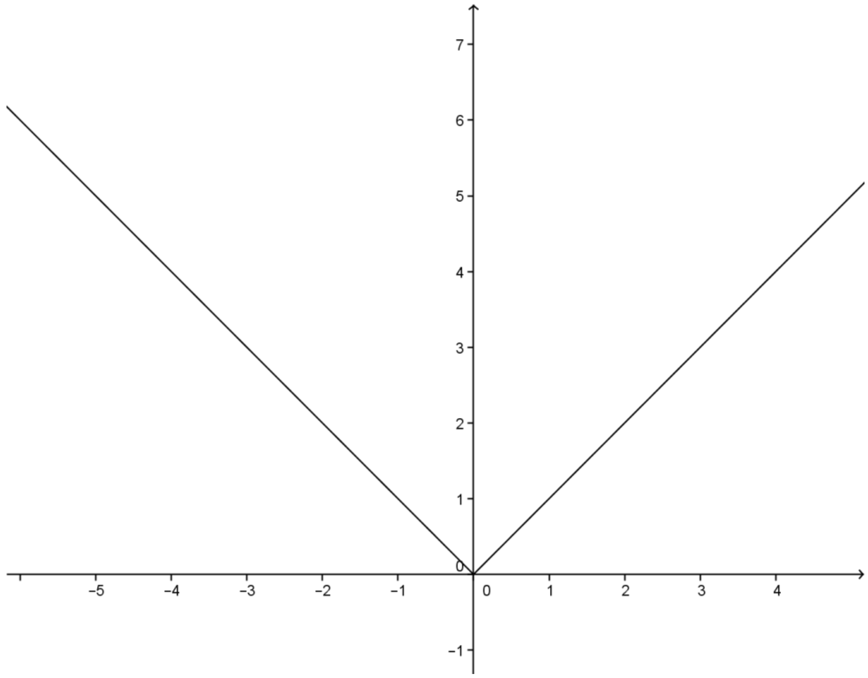
5. Absolute Value

Parent Graph Equation: $y = |x|$ **(Remember: y=abs(x))**



Domain: all real no.

Range: y≥0

Intercepts: (0,0)

Increasing Interval: $(0, \infty)$

Decreasing Interval: $(-\infty, 0)$

Asymptote: NA

End behavior: x→ ∞: f(x)→ ∞:
x → -∞: f(x)→∞:

Even Function

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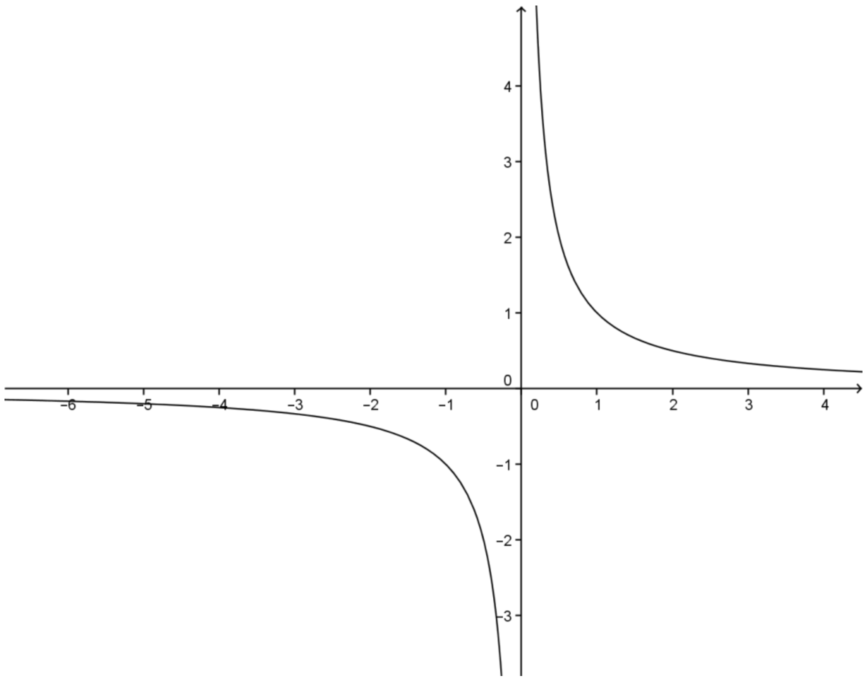
6. Reciprocal

Parent Graph Equation: $y = \frac{1}{x}$



Domain:__x≠0_________________
Range:__y≠0_________________
Intercepts:__None__________________
Increasing Interval:__NA____________
Decreasing Interval:__ $(-\infty, 0)$& $(0, \infty)$__
Asymptote:__x=0; y=0____________
End behavior: x→ ∞:_f(x)→0:______
x → -∞:_f(x)→0:_____
Odd Function

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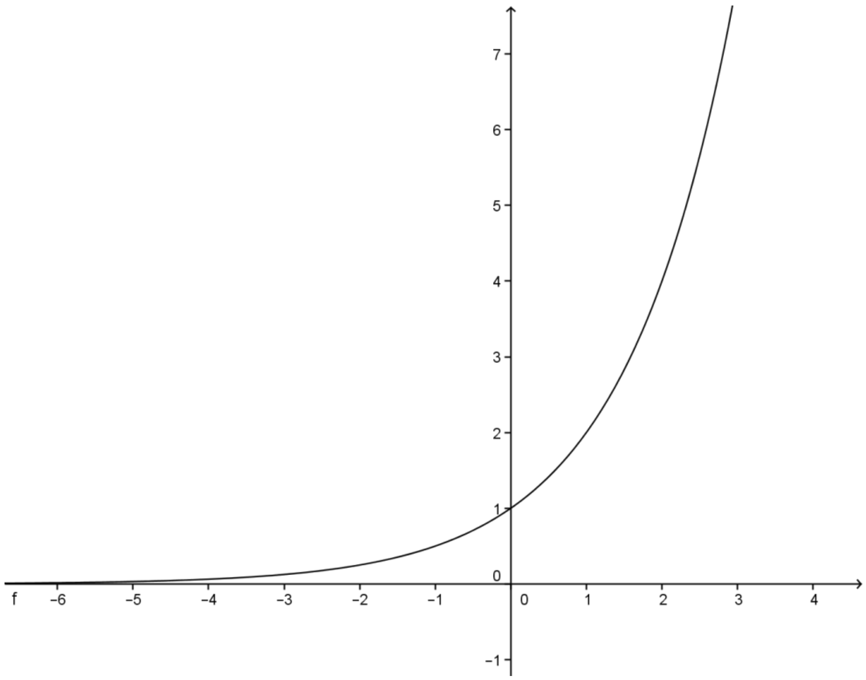
7. Exponential

Parent Graph Equation: $y = a^x$ **(Use y=2^x as parent or use e^x)**



Domain: all real no.

Range: y>0

Intercepts: (0,1)

Increasing Interval: (-∞, ∞)

Decreasing Interval: NA

Asymptote: y=0

End behavior: x→ ∞: f(x)→ ∞:

x → -∞: f(x)→0:

Neither Even nor Odd

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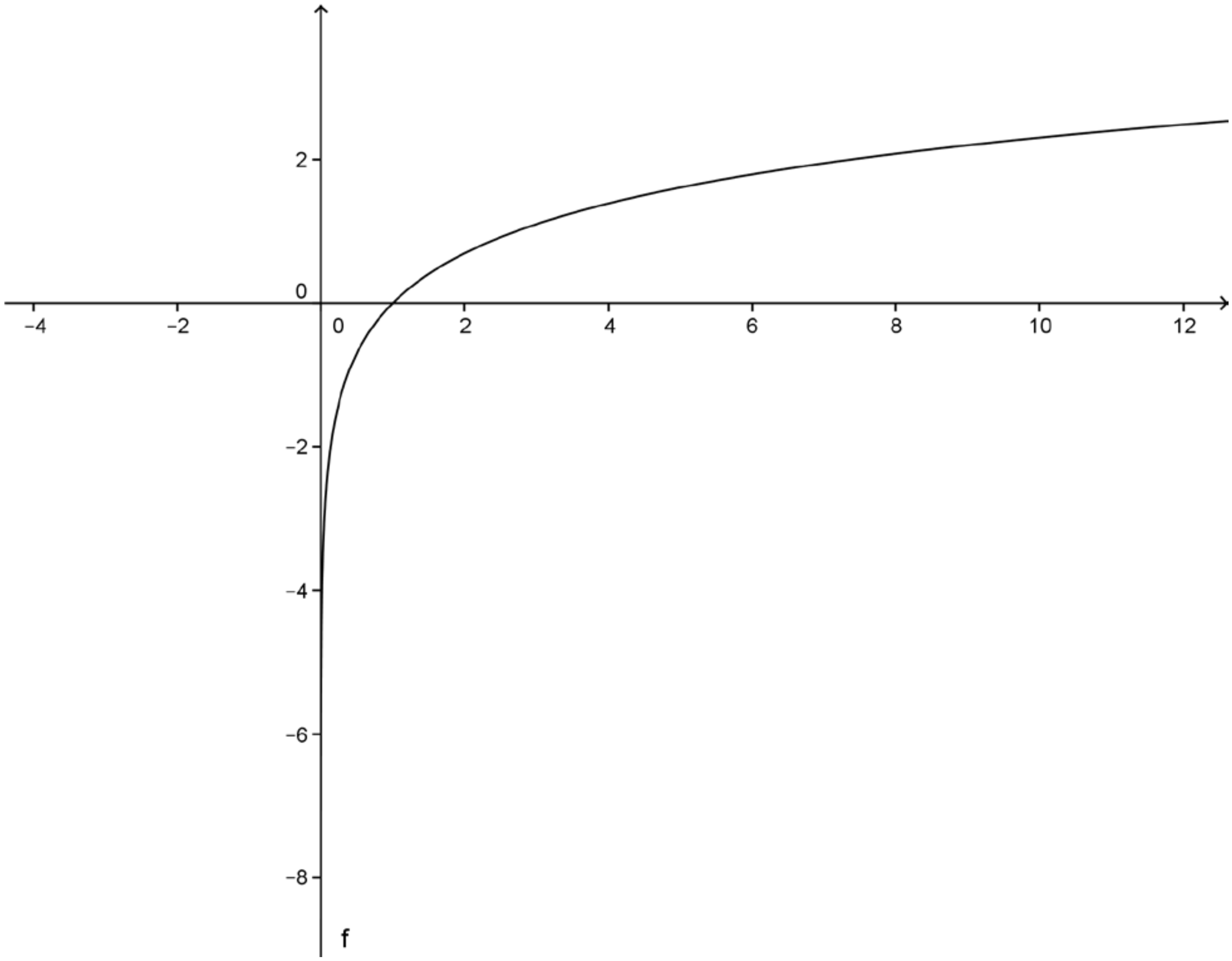
8. Logarithmic

Parent Graph Equation: $y = log_a\, x$ **(Remember: y=log(x) or use y = ln(x))**



Domain:__x>0____________________ Range:_ all real no.________________

Intercepts:__ (1,0)__________________ Increasing Interval:__$(0, \infty)$_____

Decreasing Interval:__ NA_____________ Asymptote:__ x=0________________

End behavior: x→ ∞:_f(x)→ ∞:_____ Neither Even nor Odd

x → 0-:_f(x)→−∞:____

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Name: ______________________________

Date: ______________________________

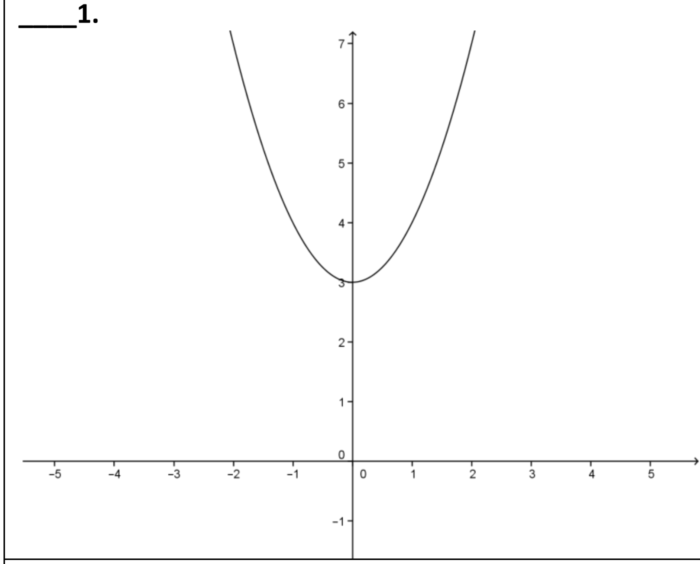
Help! Find my missing child!

Match each child graph with its parent function name.
Then describe the properties that are the same and different from the parent graph.

Choose from the parent function family below:

A. Quadratic B. Cubic C. Square root D. Absolute Value E. Reciprocal F. Exponential G. Logarithmic

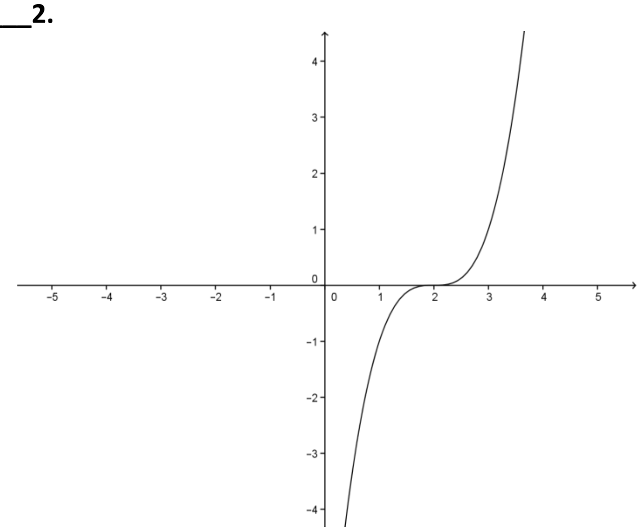
____1.



Same:

Different:

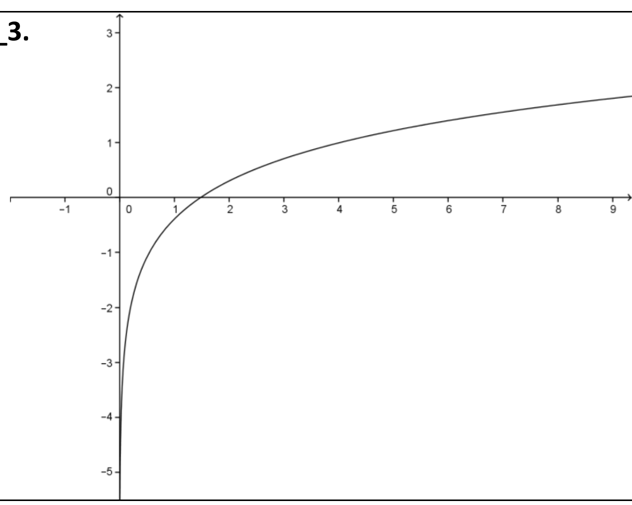
____2.



Same:

Different:

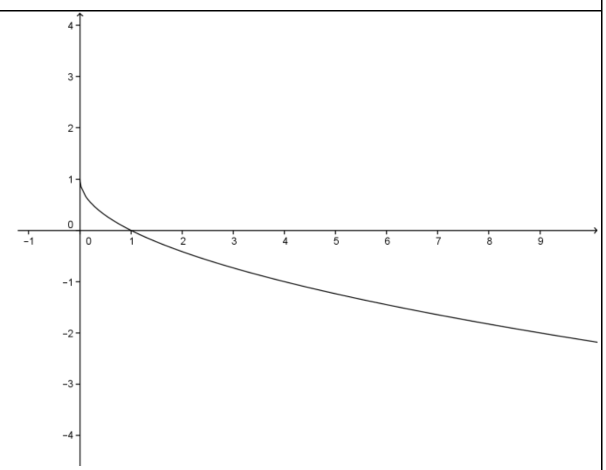
____3.



Same:

Different:

____ 4.



Same:

Different:

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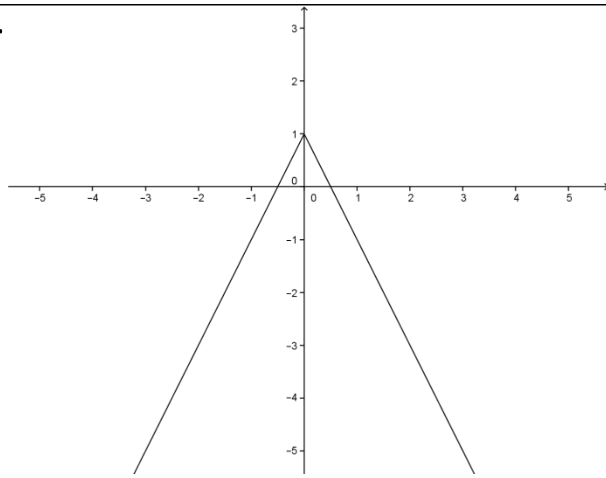
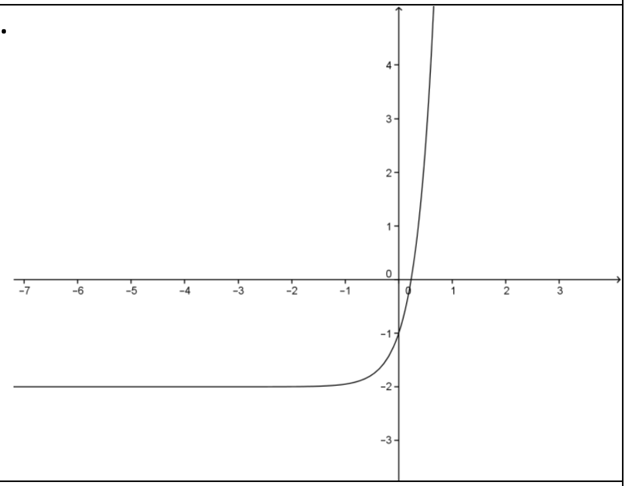
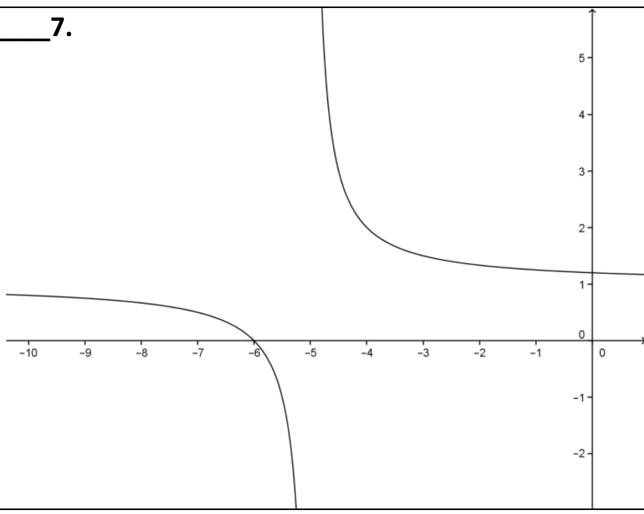
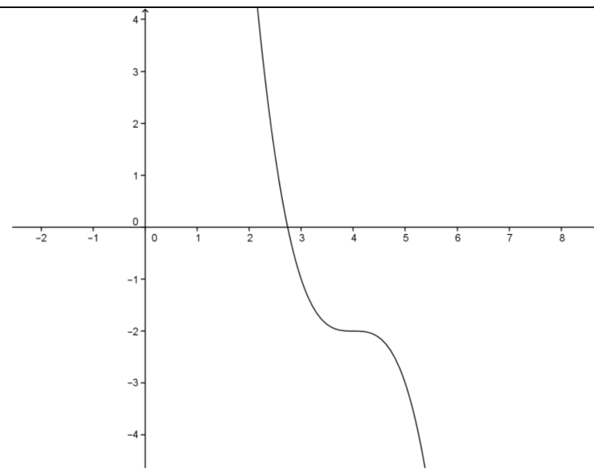
____5. 	**____6.** 
Same: **Different:**	**Same:** **Different:**
____7. 	**____8.** 
Same: **Different:**	**Same:** **Different:**

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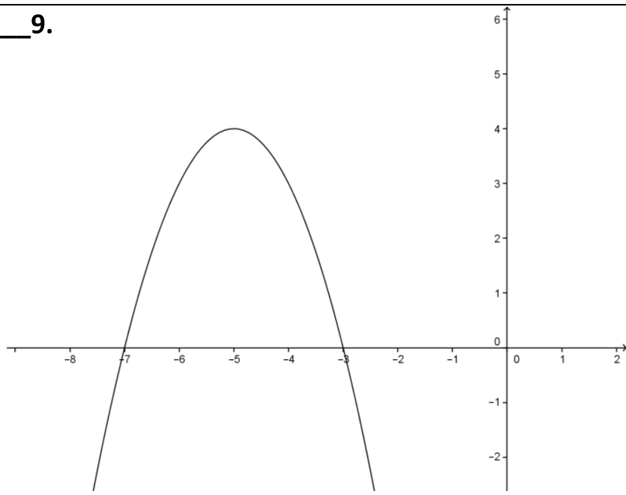
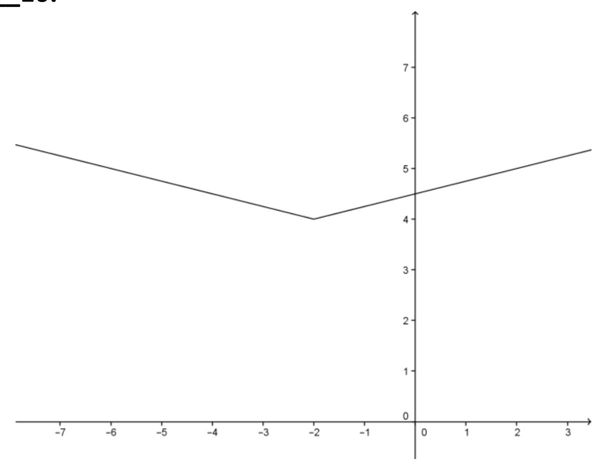
____9.	____10.
	
Same: **Different:**	**Same:** **Different:**

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Lesson 2 Student Exploration 1 Rubric Template

(Dependent on teacher specifications for number of functions and number of transformations required per function)

Criteria	Points Earned	Points Possible
Graphed 4 different parent functions using GeoGebra		10
Created 2 unique transformations for each parent function		10
Created a total of 12 different functions using GeoGebra (4 parent functions with 2 transformations each = 12 total graphs)		10
Creativity of transformations (i.e. horizontal shifts, vertical shifts, stretched or shrunk vertically, reflections, etc.)		20
Both group members interacted with the graphs and worked collaboratively		10
TOTALS:		**60**

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Name: ______________________________

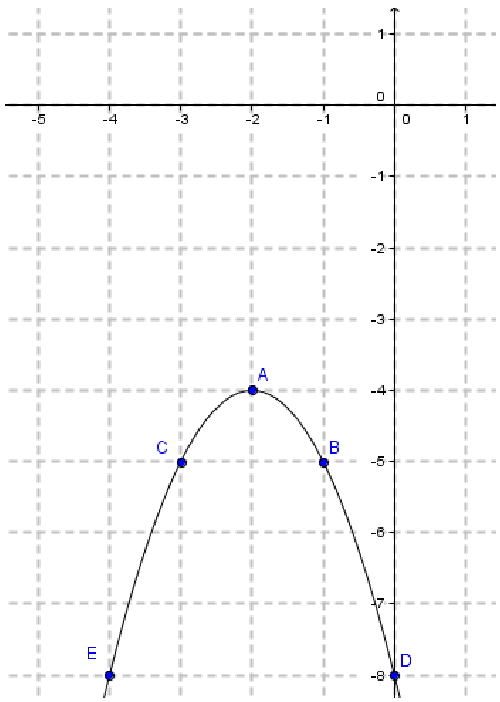
Date: ______________________________

Mathematics Capstone Course

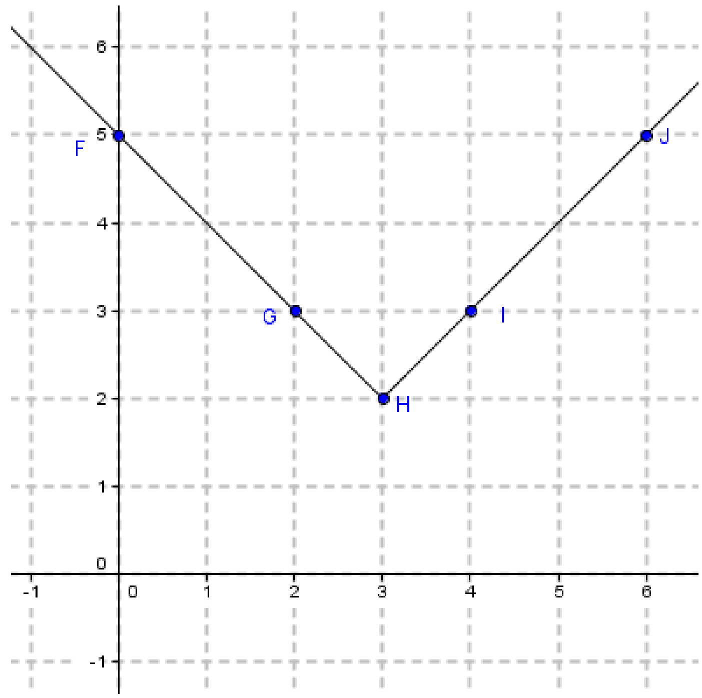
Transformation Techniques

For exercises 1 – 10, please write the equation that best fits the given graph in vertex form.

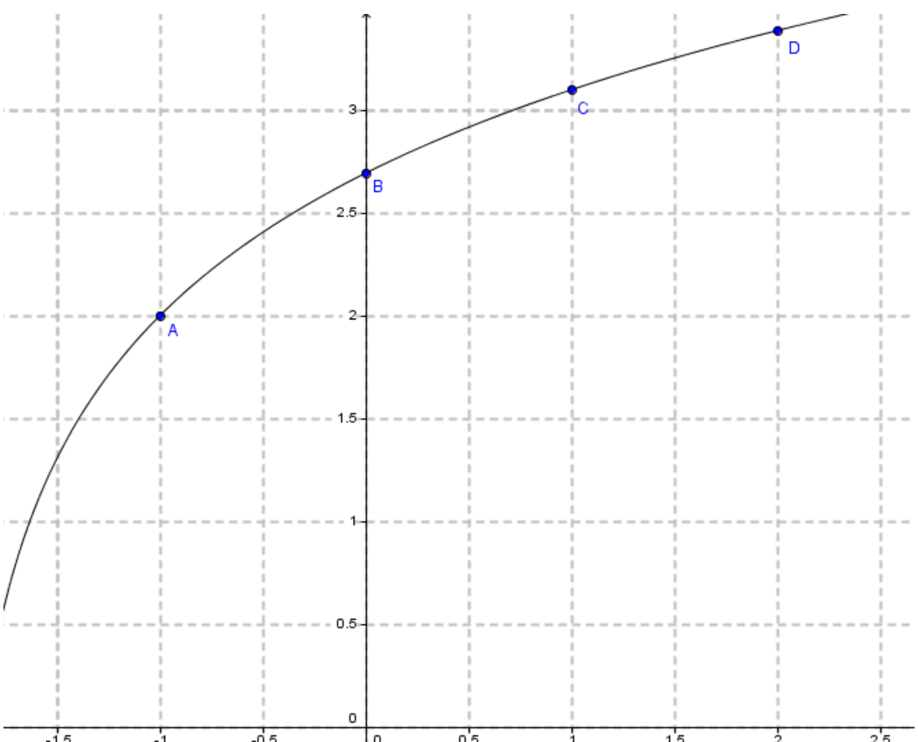
1. ____________________



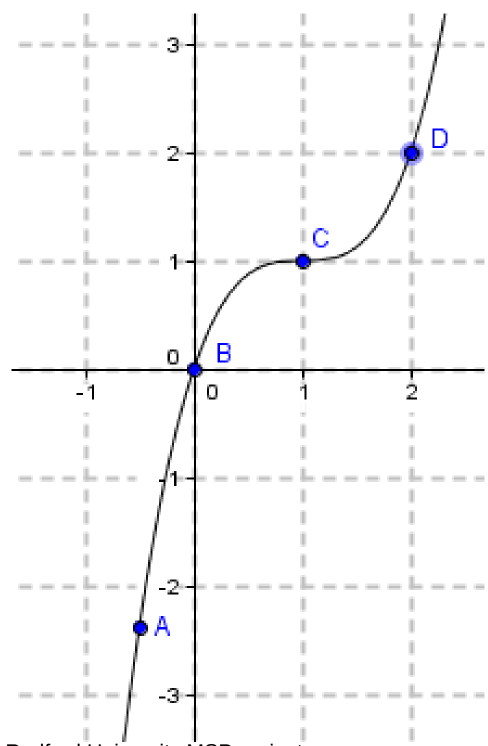
2. ____________________



3. ____________________



4. ____________________



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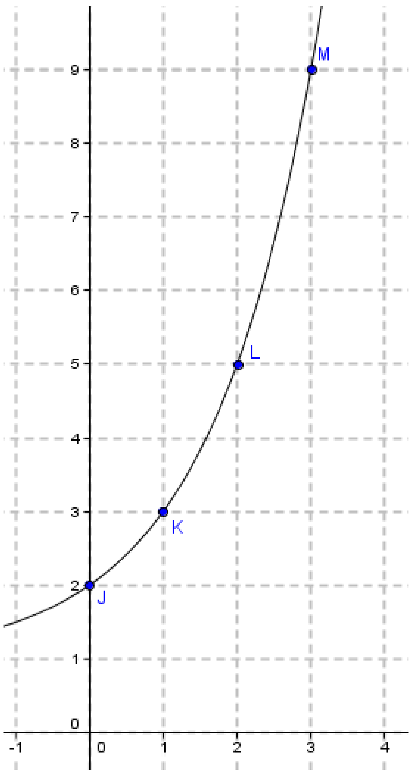
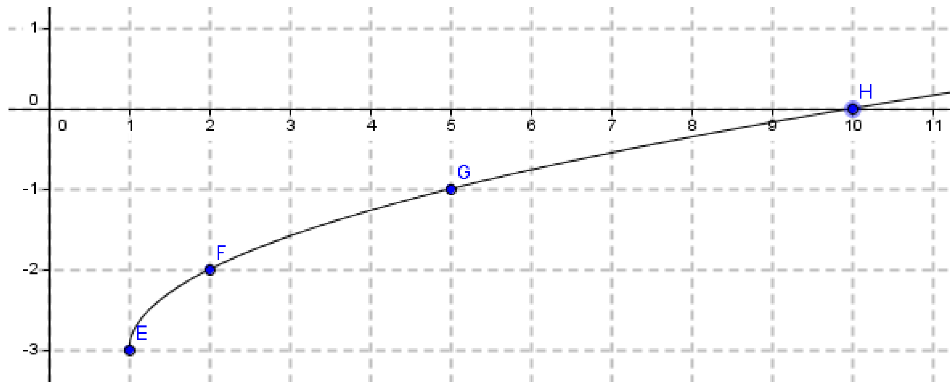
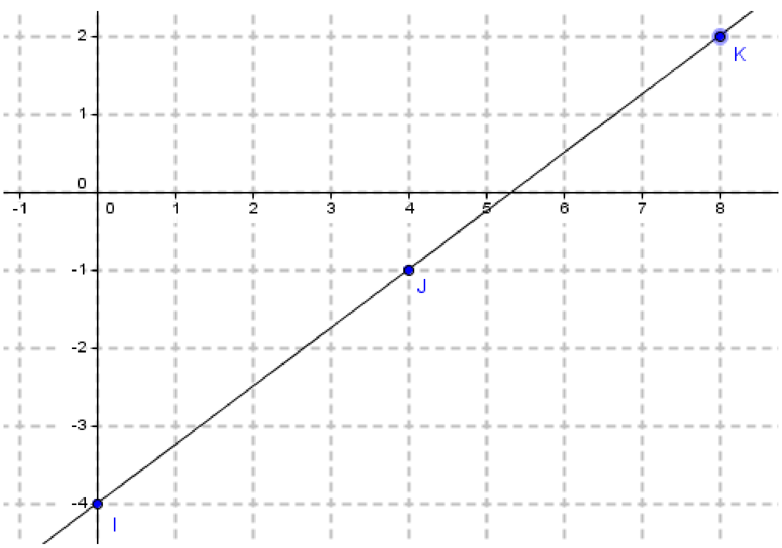
5. ____________________



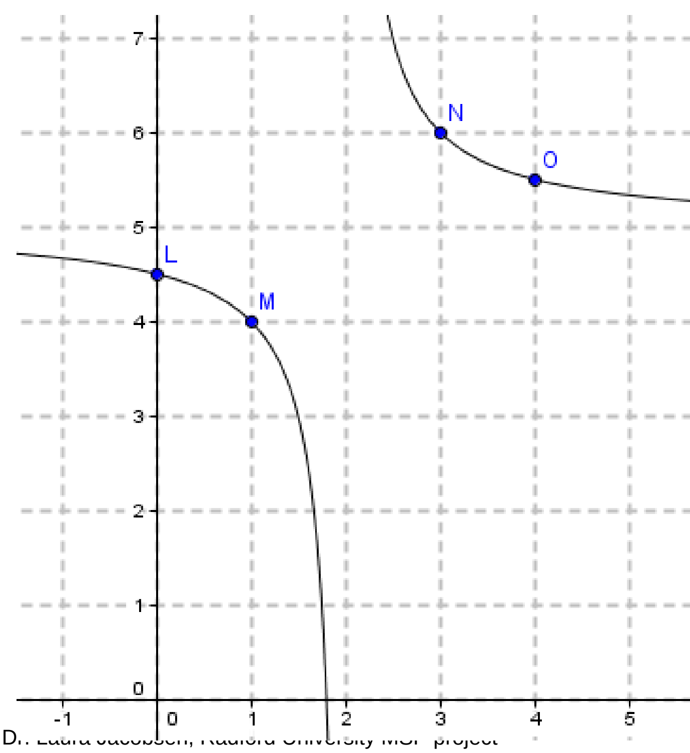
6. ____________________



7. ____________________



8. ____________________



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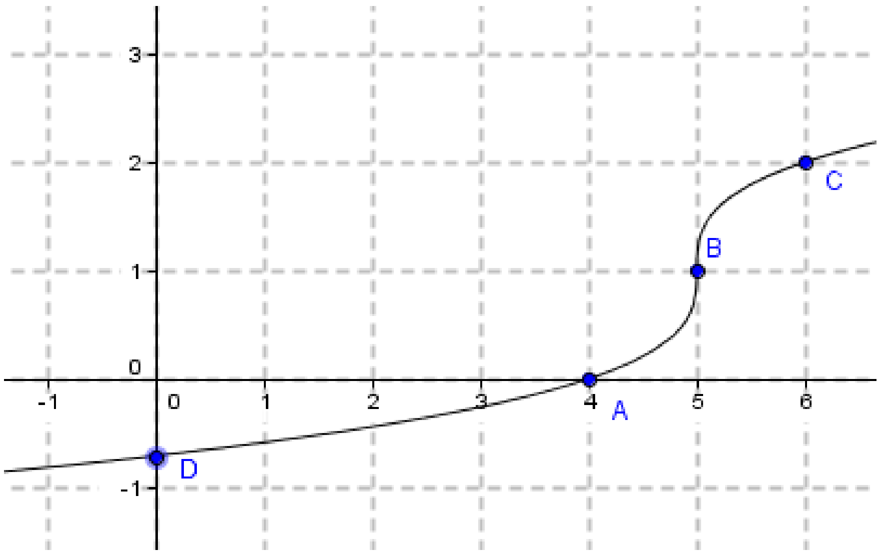
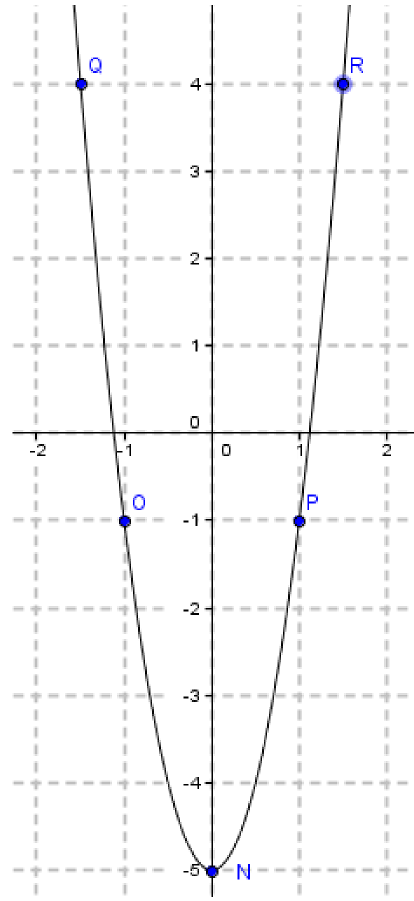
9. ____________________

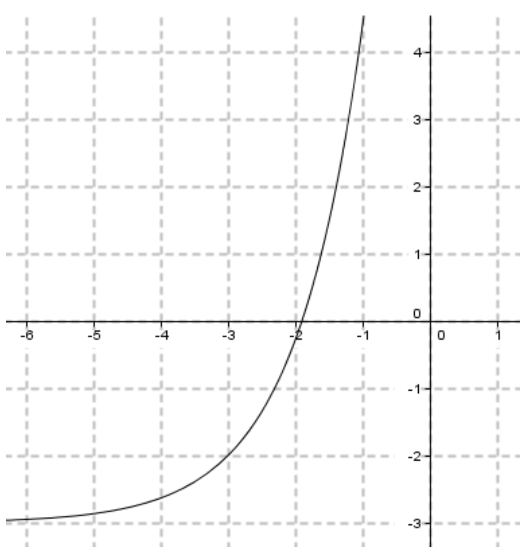


10. ____________________



For exercises 11 –15, determine which equation best represents the given graph. Write your answer on the blank provided.

11. ____________________



$$a(x) = e^{x+3} - 3$$

$$b(x) = e^{x+3} - 2$$

$$c(x) = e^{x-3} + 3$$

$$d(x) = e^{x-2} - 2$$

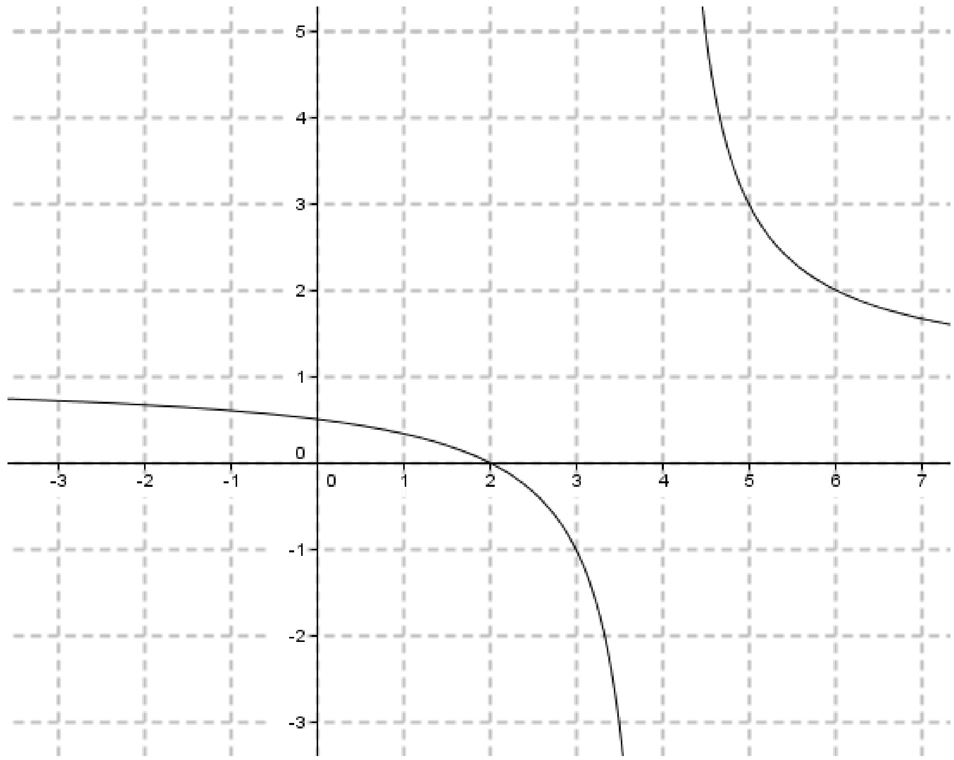
12. ____________________

$$a(x) = \frac{x}{x-4}$$

$$b(x) = 2 \cdot \frac{x}{x-4}$$

$$c(x) = \frac{x-2}{x-4}$$

$$d(x) = \frac{1}{x-4}$$



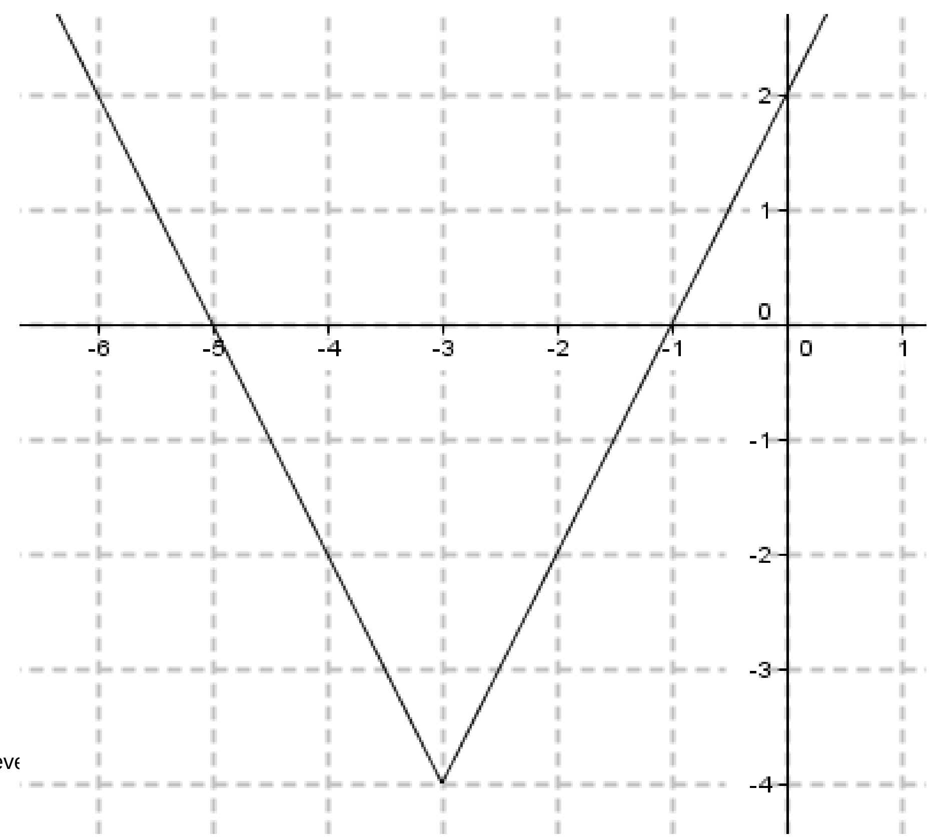
13. ____________________

$$a(x) = \frac{1}{2}\ |x-3| - 4$$

$$b(x) = \frac{1}{2}\ |x+3| - 4$$

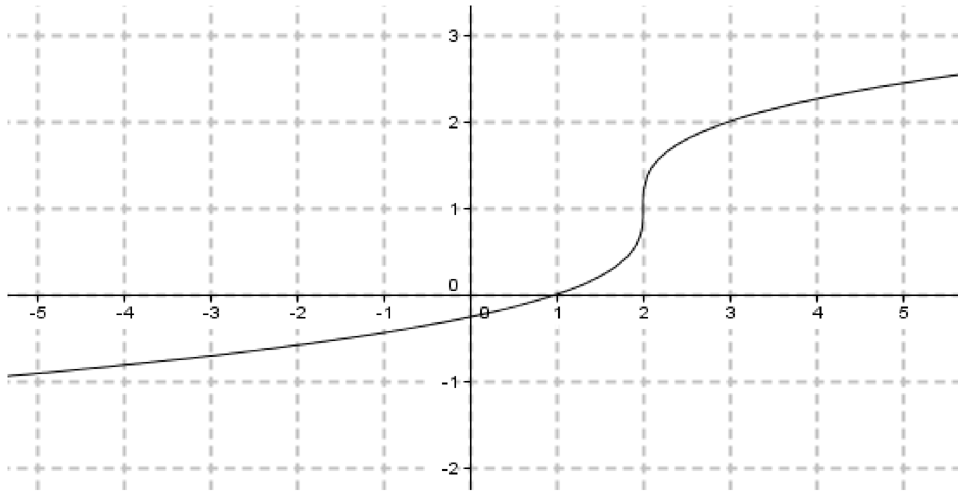
$$c(x) = 2\ |x-3| - 4$$

$$d(x) = 2\ |x+3| - 4$$



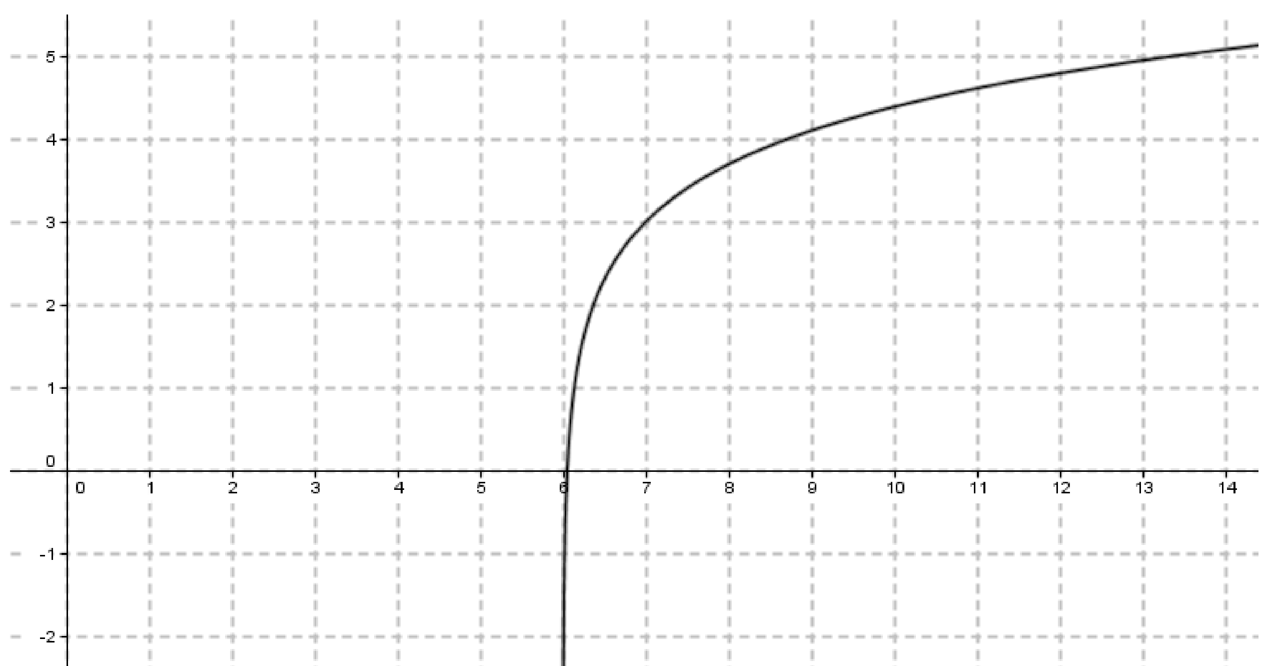
14. ____________________

$$a(x) = \sqrt[3]{x-2}$$
$$b(x) = \sqrt[3]{x-2} + 1$$
$$c(x) = \sqrt[3]{x-1}$$
$$d(x) = \sqrt[3]{x-1} + 2$$



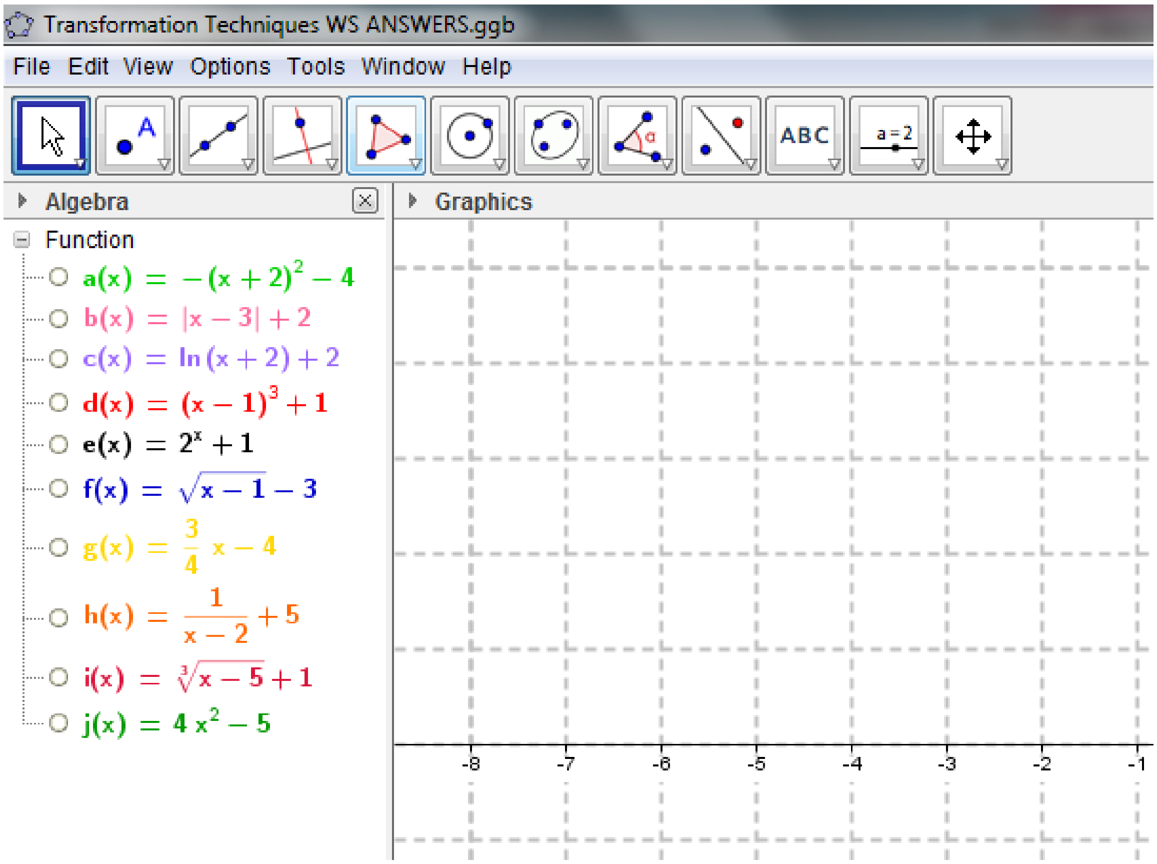
15. ____________________

$$a(x) = \ln(x-3) + 6$$
$$b(x) = \ln(x-3) - 6$$
$$c(x) = \ln(x-6) + 3$$
$$d(x) = \ln(x-6) - 3$$



Transformation Techniques Worksheet **ANSWERS**

#1 – 10



11. $a(x) = e^{x+3} - 3$

12. $c(x) = \dfrac{x-2}{x-4}$

13. $d(x) = 2\,|x+3| - 4$

14. $b(x) = \sqrt[3]{x-2} + 1$

15. $c(x) = \ln(x-6) + 3$

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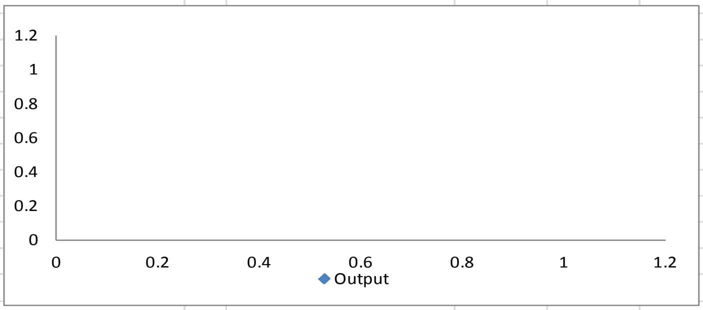
*****It is recommended to access these files on our website https://sites.google.com/site/functionfamilyfun/home/lesson-3 . We recommend this because you can save the Excel file titled "Student Activity," and then have your students solve it (as opposed to trying to use the files that are embedded below, however, they will work if you choose to use them instead).*****

Teacher Intro:

	Type your function here		Input	Output
Mystery Function 1:	y=		0	2

Real-World Application:
City streets are crowded and hard to navigate. There is usually little parking and a lot of people choose to utilize public transportation, like the bus.
This mystery function gives an example of what it may cost to ride the bus in the city.



Data Collected:	Input	Output
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

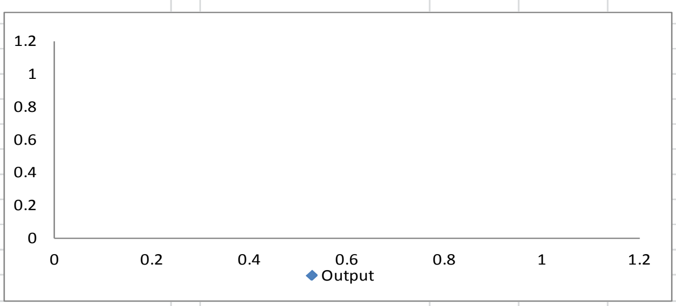
Describe what your function means:

Now Predict: How much will I have to pay if I need to take the bus to work which is 11 miles from my house?

Student Activity Sheet:

	Type your function here		Input	Output
Mystery Function 1:	y=		0	-12

Real-World Application: Julie bakes awesome pies!! In order to sell her pies at the carnival, Julie must pay an entry fee. Once at the carnival, she sells her pies for the same price.



Data Collected:	Input	Output
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Describe what your function mean?

Now Predict: If Julies bakes and sells 25 pies, what will her profit be?

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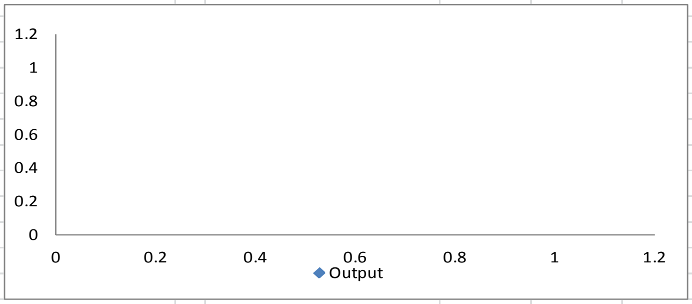
	Type your function here		Input	Output
Mystery Function 2:	y=		0	5

Data Collected:	Input	Output
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Real-World Application: Tony is 7 yrs old and got his first bow and arrow set, and he is getting good. He can fire his arrow up it will land almost in his neighbors yard. He is having trouble getting it over the fence.



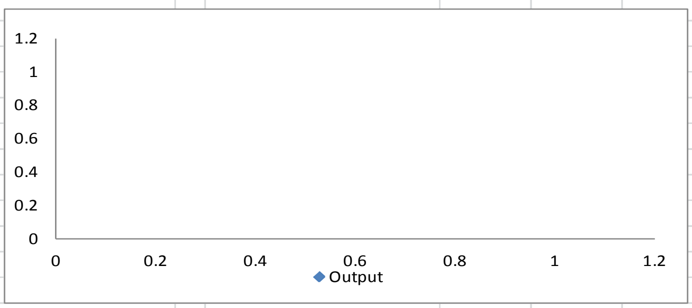
Describe what your function means:

Now Predict: Will Tony be able to get his arrow over the neighbor's fence if it is 5 foot tall? If yes, where will he have to stand to make sure it gets over?

	Type your function here		Input	Output
Mystery Function 3:	y=		0	5

Data Collected:	Input	Output	(hint: what is the doubling
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Real-World Application: Ever wonder why it is so loud outside in the summertime? Can you hear all the tree frogs chirping? Tree Frogs double in population every 3 weeks. In fact, there are probably 5 tree frogs still in your backyard right now.



Describe what your function means?

Now Predict: Since the doubling time is every 3 weeks, how many tree frogs could you have in the next year if they all stayed in your backyard?

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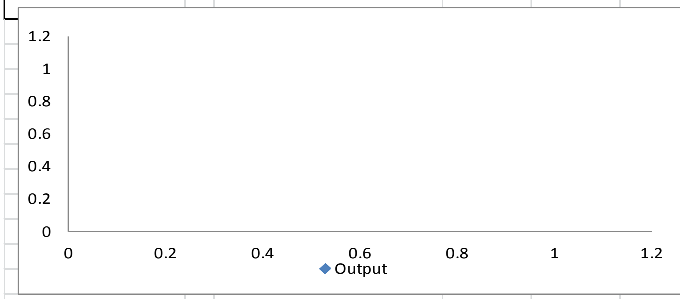
		Type your function here	Input	Output
Mystery Function 4:	y=		42	37.52332608

Real-World Application: In a natural history museum you see leg bones of dinosaurs and want to know how fast they walked. The maximum walking speed S (in feet per second) of an animal can be modeled by a specific equation using gravity, which is 32 ft/sec2 times the length (in feet) of the animal's leg. Dr. Math was at a dig and found a leg bone, but 2 feet of it seemed to be missing.
(Source: Discover-problem adapted from McDougal Littel text)



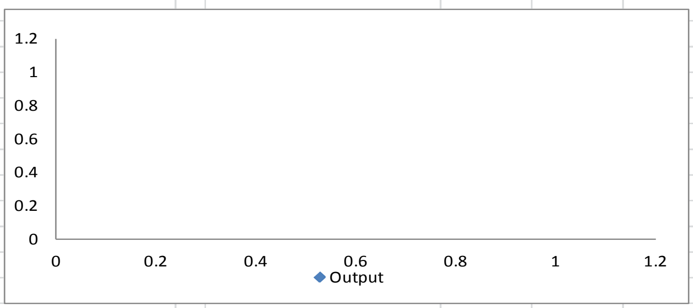
Data Collected:	Input	Output	(hint- use a large spread of no)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

Describe what your function means:

Now Predict: What if Dr. Math found a bone *in the same condition*, that measured 42 feet long. How fast would that dinosaur be able to walk?

		Type your function here	Input	Output
Mystery Function 5:	y=		2	0

Real-World Application: The loudness of sound is measured in units called decibels. Assuming an initial intensity of 1, the sound we wish to measure is assigned an intensity level, which is then converted to decibel by a function of the ratio and multiplied by 10. Jasmin went to a rock concert last night with friends. Jasmin's intensity level was described as a different number than the rest of the whole group (who agreed).



Data Collected:	Input	Output	hint:use decimal inputs)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

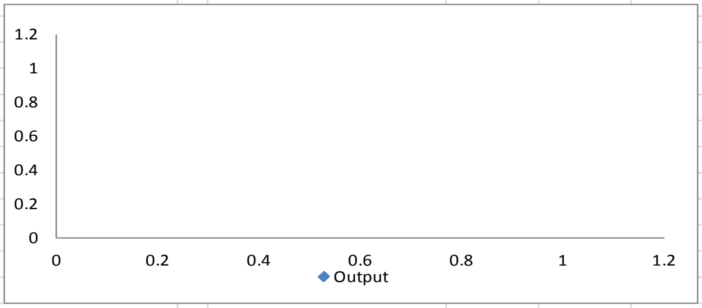
Describe what your function means:

Now Predict: The next weekend, Jasmin went to another concert and described the intensity as a 13. What would the decibel level be described as?

	Type your function here	Input	Output		
Mystery Function:	y=	0	100000		
CHALLENGE!!!					

Data Collected:	Input	Output
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Real-World Application: Romeo's grandfather left him a bank CD in his will. The CD was old and faded and all he could make out was the rate-8%. He went to the bank and they said that it was now worth a lot of money.



Describe what your function means:

Now Predict: The bank finally found out that the CD was bought about 57 years ago. What was the starting investment?

ANSWER KEY for student activities:

#	function equation	Describe	Predict	NOTES
Intro	y=x+2	The function is giving the bus fair which is based on $1 per mile plus $2	$13	Click on the red arrows on the Intro to see comments
1	y= 7x-12	To Calculate Julies profit is 7 times the number of pies less the $12 entry fee	$163	Students should be asked if the 7 represents the selling price-what is the sale price if she spends $3 to make each pie
2	y= 1/2 (x)^2 +5	The slope must be 1/2 the square of the distance away and the 5 ft is the fence height	3.16 ft	Distance found by solving for 0- (sqrt 10) ~ 3 ft way from fence
3	y = 5*2^x	The five is the beginning # of frogs, then exponential function for the doubling periods	807000 ft	x is the number of doubling periods-not # of weeks!!
4	y = Sqrt(32*(x+2))	The sqrt of gravity (32) * bone length (must add 2 because it was broken)	38 m/s2	comes from actual equation
5	y = 10*log(x-1)	the 10 is the multiplier time the log of Jasmin's intensity -1 (since she is overly sensitive)	11 db	This one is difficult and may have to be hinted around as the teacher roams the room
Challenge	y = 100000/(e^(.08x)	Inverse of A=Pert formula- the 100,000 is the new Amount divided by e^(rate*time)	$1,060	This is a reciprocal function- of the exponencial function to solve for the principal

Name: ______________________________

Date: ______________________________

Trendlines in the Trenches - Excel Activity

This lesson involves linear and quadratic functions. For each data set construct a graph using Excel. Include the trendline. Answer the questions about each data set.

1. One measure of form for a runner is stride rate, defined as the number of steps per second. A runner is considered to be efficient if the stride rate is close to optimum. The stride rate is related to speed; the greater the speed, the greater the stride rate.

In a study of 21 top female runners, researchers measured the stride rate for different speeds. The following table gives the average stride rate of these women versus the speed.

From the Exploring Data website - http://curriculum.qed.qld.gov.au/kla/eda/
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Source: R.C. Nelson, C.M. Brooks, and N.L. Pike, *Biomechanical comparison of male and female runners*, in P. Milvy (ed.), The Marathon: Physiological, Medical, Epistemiological, and Psychological Studies, New York Academy of Sciences, 1977, pp. 793-807.

Speed	15.86	16.88	17.50	18.62	19.97	21.06	22.11
Stride Rate	3.05	3.12	3.17	3.25	3.36	3.46	3.55

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) Predict the stride rate for a speed of 18 feet per second.

d) According to your equation, what is the stride rate for a speed of zero? Explain

e) What is your speed (to the nearest hundredth) if your stride rate is 3.20?

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2. Olympic Gold Medal Performances

The modern Olympic Games are a modified revival of the Greek Olympian Games that came to be largely through the efforts of the French sportsman and educator Baron Pierre de Coubertin. The Games are an international athletic competition that has been held at a different site every four years since their inauguration in 1896, with occasional interruptions in the times of world wars.

The data for the gold medal performances in long jump, high jump, and discus throw are given below (in inches). Year is coded to be zero in 1900.

a) Plot the data for each event on a separate scatterplot.

b) Describe the shape of the distribution.

c) Predict the Olympic performance in each event for the Sydney 2000 games.

Year	Long Jump	High Jump	Discus Throw
-4	249.75	71.25	1147.5
0	282.875	74.8	1418.9
4	289	71	1546.5
8	294.5	75	1610
12	299.25	76	1780
20	281.5	76.25	1759.25
24	293.125	78	1817.125
28	304.75	76.375	1863
32	300.75	77.625	1948.875
36	317.3125	79.9375	1987.375
48	308	78	2078
52	298	80.32	2166.85
56	308.25	83.25	2218.5
60	319.75	85	2330
64	317.75	85.75	2401.5
68	350.5	88.25	2550.5
72	324.5	87.75	2535
76	328.5	88.5	2657.4
80	336.25	92.75	2624
84	336.25	92.5	2622
88	343.25	93.5	2709.25
92	342.5	92	2563.75

3. A study compared the speed (in miles per hour) and the average fuel economy (in miles per gallon) for cars. The results are shown in the table.

Source – Transportation Energy Data Book

speed	20	30	40	45	50	55	60	65	70
economy	25.5	29.0	30.0	29.9	30.2	30.4	28.8	27.4	25.3

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) Estimate the fuel economy at a speed of 35 MPH.

d) Use your equation and calculator to find the optimum speed to get the best fuel economy

e) What does this type of function tell you about the relationship between speed and fuel economy?

4. The table lists the data of the number of new vehicles sold in the U.S. from the years 1982 – 1991. 1982 is year zero.

Source - Transportation Energy Data Book

year	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
vehicles	10,202	11,858	13,856	14,952	16,263	15,089	15,681	14,700	14,129	12,528

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) If this trend continued, how many vehicles would have been sold in 1995?

d) The actual number of vehicles sold in 1995 was 15,061. Explain some possible reasons why it is so different than your prediction.

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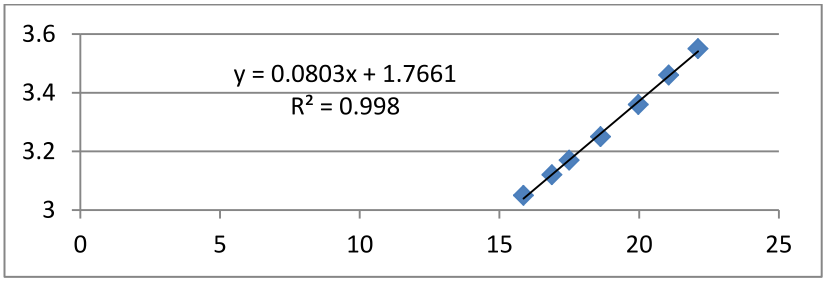
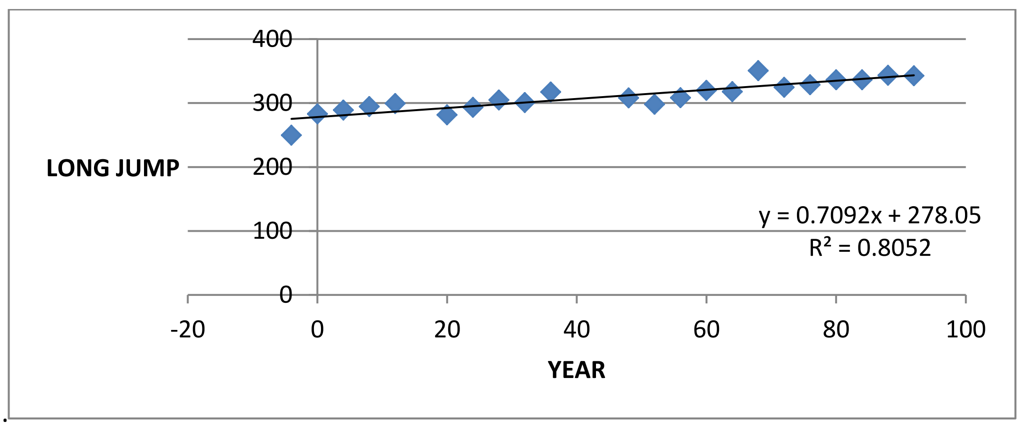
Trendlines in the Trenches – Answer Key

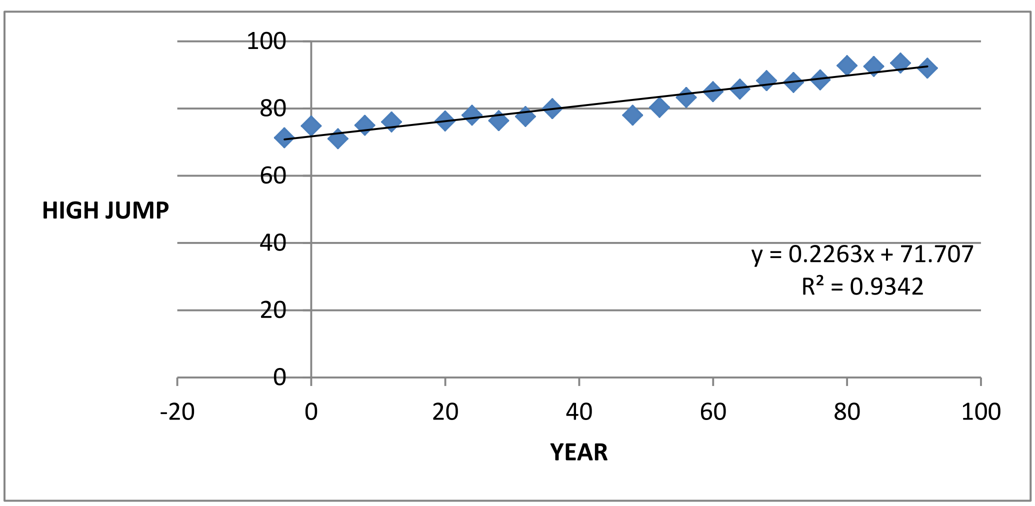
1. 

c) 3.21 steps per second

d) 1.7661 feet per second. The actual rate is zero. Answers will vary. Most runners are traveling at similar speeds. The equation will only be valid for values in this range.

e) 17.86 feet per second

2. 



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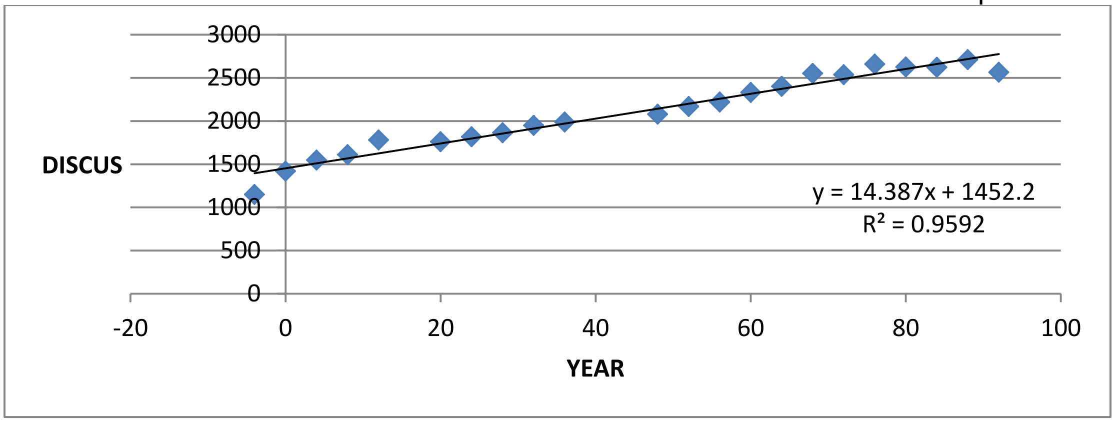
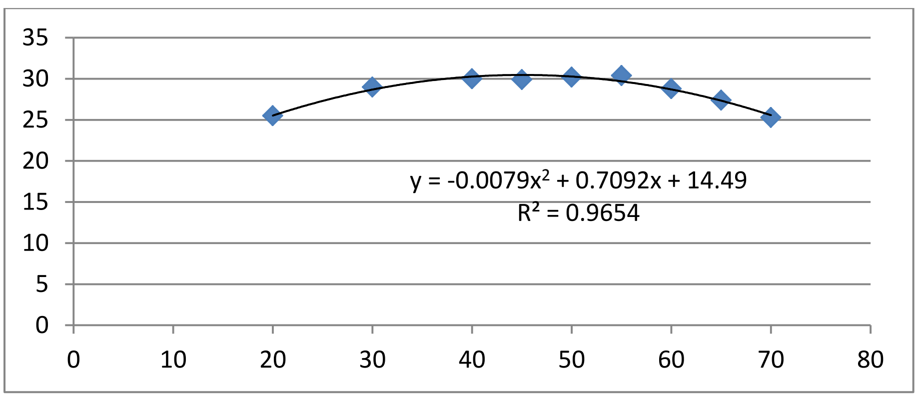


c) Long Jump – 348.97 inches High Jump – 93.7 inches Discus – 2890.9 inches

d) Possible extension:

Have students research to find results of one or more of the events to check the accuracy of their prediction. This will most likely involve conversion of units……meters to inches.



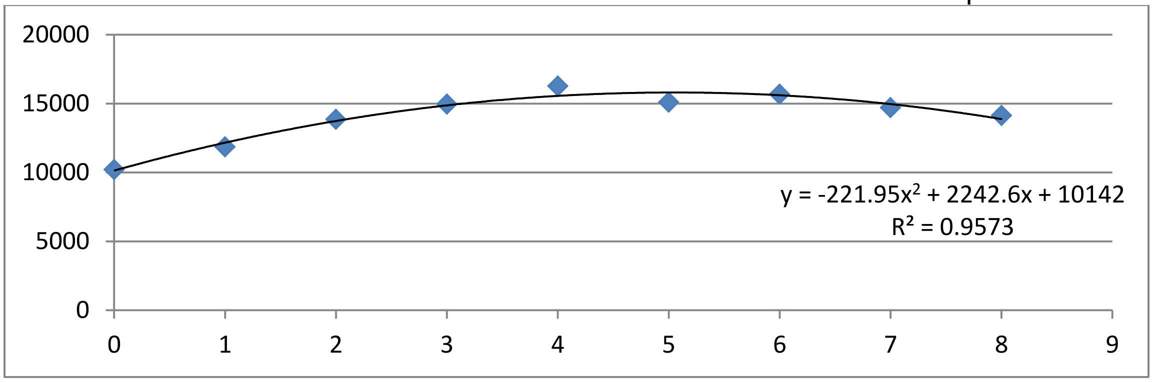
3.

c) 29.6 MPH

d) Students may use the - 2nd –TRACE 4:Maximum function to find the vertex of the parabola. This occurs at the point (45.4,30.4) which makes the best speed 45.4 MPH.

e) Answers may vary. Driving too fast or too slow both reduce fuel economy.

4.



c) 1786 vehicles

d) Answers will vary. The number of vehicles sold is a function of the economy. A quadratic function will continue to trend in one direction unlike the economy. It appears as if the economy was the strongest around 1986.

e) Extension:

Students could research data to see if the economy was better in 1986.

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Name: ______________________________

Date: ______________________________

Real-World Regressions - Excel Activity

For each data set, construct a scatterplot using Excel. Include the trendline. Limit your functions to cubic, exponential, or logarithmic. Then, answer the questions about the data set.

1. The data below show the average growth rates of 12 Weeping Higan cherry trees planted in Washington, D.C. At the time of planting, the trees were one year old and were all 6 feet in height.

Source - mathbits.com

Age of Tree (in years)	Height (in feet)
1	6
2	9.5
3	13
4	15
5	16.5
6	17.5
7	18.5
8	19
9	19.5
10	19.7
11	19.8

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) What was the average height of the trees at two and one-half years (to the nearest tenth of a foot)?

d) Estimate the average height of a tree at 20 years (nearest tenth of a foot).

e) Extension: Use the table function on your graphing calculator to estimate the age of a tree if the average height is ten feet.

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2. The following table gives the number of motor vehicle thefts (in thousands) in the U.S. for the years 1983 - 1993. x = 1 represents 1983.

Source - http://faculty.nwacc.edu/tobrien

year	1	2	3	4	5	6	7	8	9	10	11
thefts	1008	1032	1103	1224	1289	1433	1565	1636	1662	1611	1561

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) Predict the number of thefts in 1998 and 2010.

d) Is this reasonable? Explain.

3. The data shows the cooling temperatures of a freshly brewed cup of coffee after it is poured from the brewing pot into a serving cup. The brewing pot temperature is approximately 180° F.

Source - mathbits.com

Time (mins)	Temp (° F)
0	179.5
5	168.7
8	158.1
11	149.2
15	141.7
18	134.6
22	125.4
25	123.5
30	116.3
34	113.2
38	109.1
42	105.7
45	102.2
50	100.5

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) Based upon the equation, what was the initial temperature of the coffee?

d) What is the predicted temperature of the coffee after 1 hour?

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e) Extension: use the table function on your calculator to answer the following.

In 1992, a woman sued McDonald's for serving coffee at a temperature of 180° that caused her to be severely burned when the coffee spilled. An expert witness at the trial testified that liquids at 180° will cause a full thickness burn to human skin in two to seven seconds. It was stated that had the coffee been served at 155°, the liquid would have cooled and avoided the serious burns. The woman was awarded over 2.7 million dollars. As a result of this famous case, many restaurants now serve coffee at a temperature around 155°. How long should restaurants wait (after pouring the coffee from the pot) before serving coffee, to ensure that the coffee is not hotter than 155°?

4. In the years before the Civil War, the population of the United States grew rapidly, as shown in the following table from the U.S. Bureau of the Census. Assume 1790 is year 0.

Year	Population in Millions
1790	3.93
1800	5.31
1810	7.24
1820	9.64
1830	12.86
1840	17.07
1850	23.19
1860	31.44

a) What type of function does the data represent?

b) What is the best equation for the trendline?

c) Estimate the population in 1825.

d) Using your equation, predict the population in 1990. Is it close to the actual population of 314 million? Why or why not?

5. The following data shows the U.S. population (in millions) for the years 1950 to 2000. Construct a scatterplot using Excel.

Source - http://www.multpl.com/united-states-population/table

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995	2000
pop	152.3	169.9	180.7	194.3	205.1	216.0	227.2	237.9	249.6	266.3	282.2

a) How does the shape of this scatterplot differ from the one in problem 4?

b) What type of function does this data represent?

c) What is the best equation for the trendline?

d) Why do you think it is different from the one in problem 4?

e) Estimate the current population using your equation.

f) Is it close to the actual population? Why or why not?

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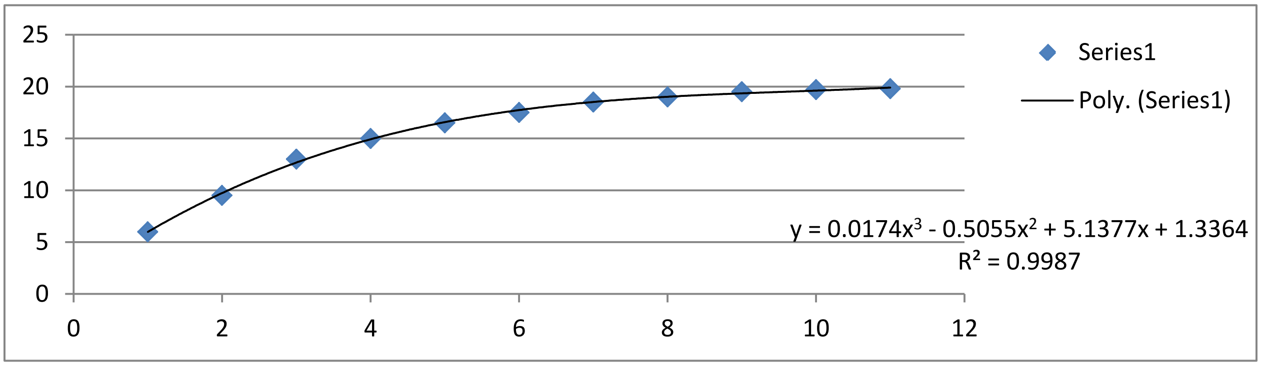
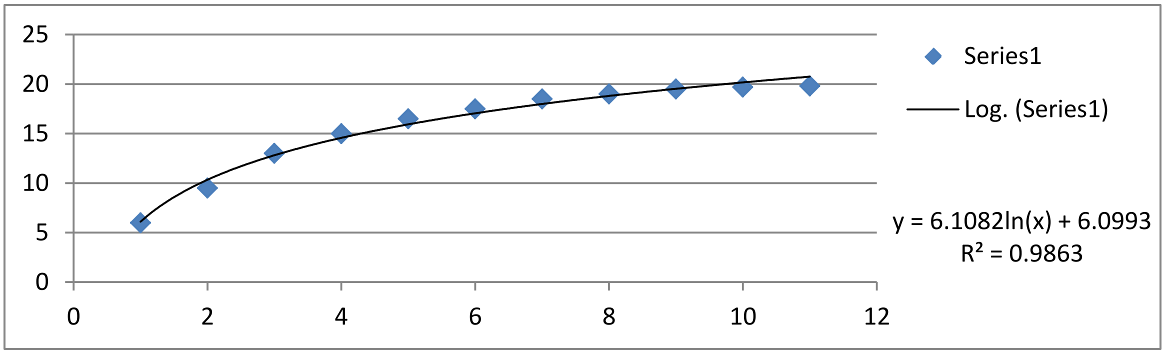
Real-World Regressions – **answer key**

1.



Both of these are pretty good with the cubic being slightly better. Given the shape, most students will likely chose log function but the r-value is better for cubic.

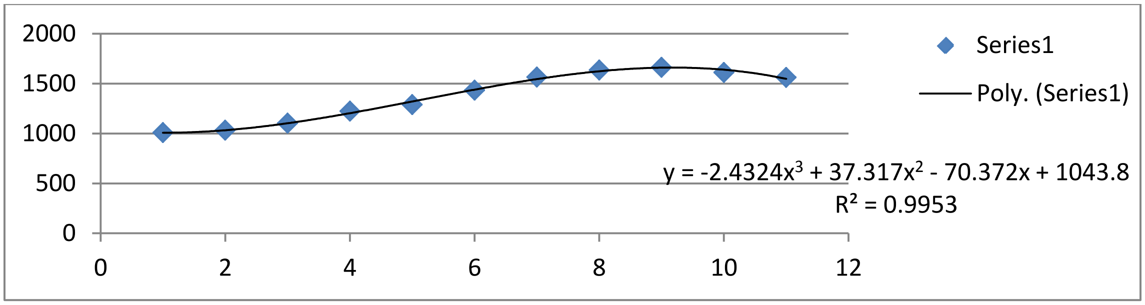


c) 11.3 ft cubic 11.7 ft log

d) This is where it gets interesting. If the students use the table function they will see a huge difference- 41.1 ft. cubic or 24.4 log. If they look further down the table, the cubic function increases at an unreasonable rate (over 500 feet @ 40 years). This is a great discussion point for the class. It should reveal that the log function is actually the better one to use.

e) Based on the log function, approximately 2 years old

2.



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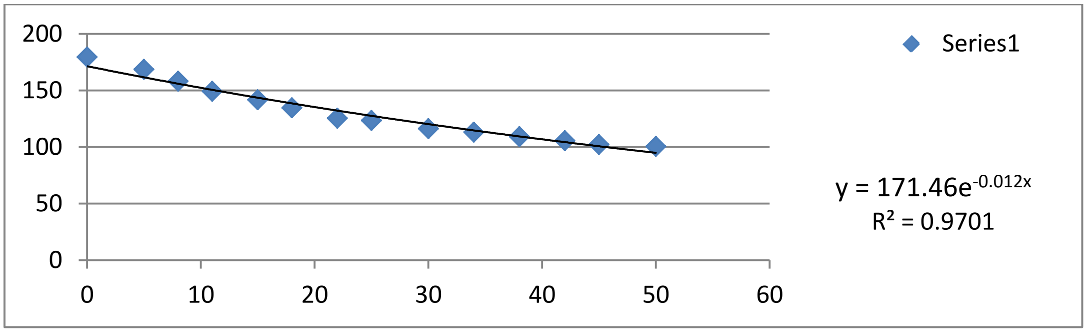
a) cubic

c) 1998 = 176.55 2010 = -21521

d) No the cubic function continues to trend downward when, in fact, the number of thefts will only vary slightly. The cubic function is not good for predictions far into the future.

3.



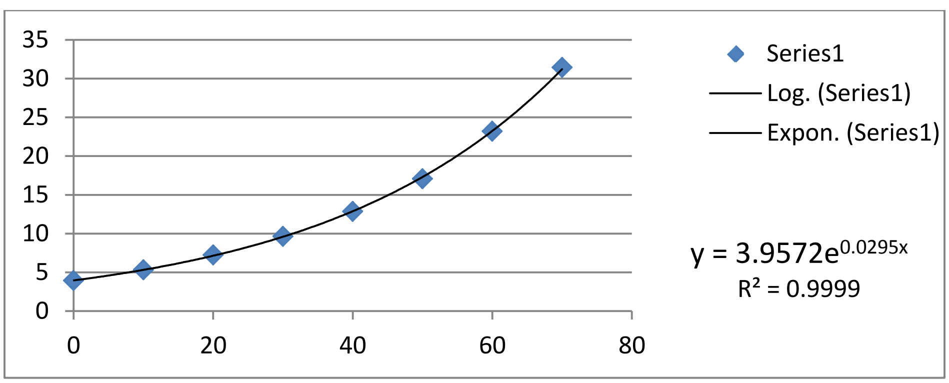
a) Exponential (decay)

c) 171.46 degrees based on the equation (not the table) – Discussion point!

d) Approximately 83 degrees

e) Approximately 8 minutes

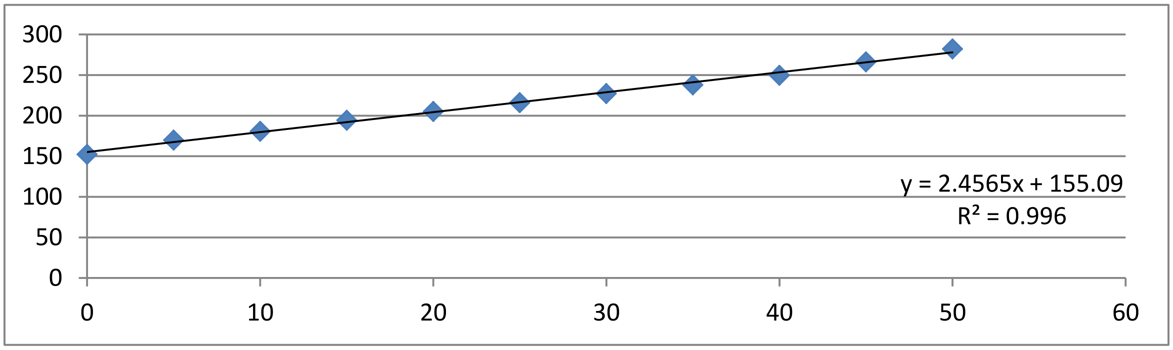
4.

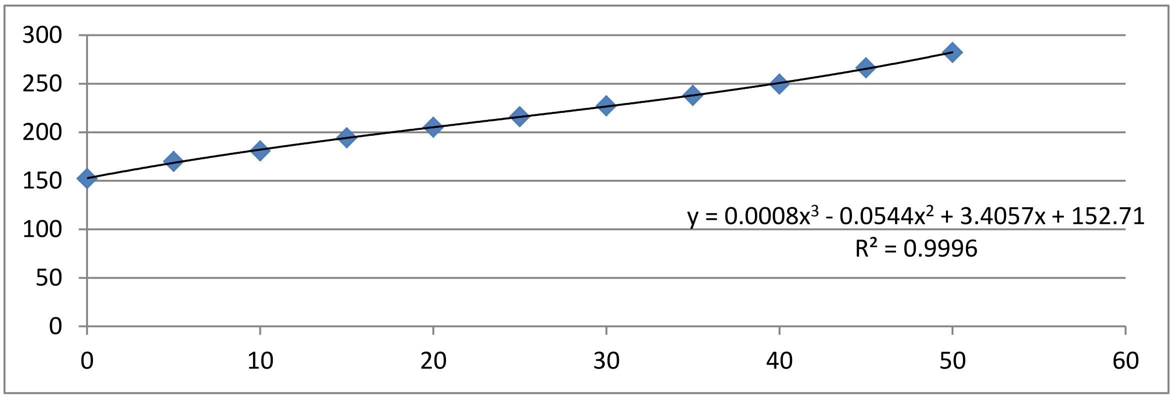


c) 11.11 milion

d) 1,444,500,000 - Not close to the actual population of 314,000,000. The U.S. was expanding at a much faster rate during this time period. After the industrial revolution and women getting jobs, the population increased at a much slower rate. (sample answer)

5.





a,b,c Answers will vary

d) The population growth rate has changed. See problem 4(d)

e) The linear equation gives the best estimate of the current population

Lesson 6

Project/Presentation Rubric

Criteria	Points Earned	Points Possible
Researched and found a set of data		10
Used data to create a scatter plot		10
Used scatter plot to find trendline		10
Type of function that best suits the data (parent) was indicated in presentation (i.e. linear, quadratic, cubic, exponential, etc.)		5
Used trendline to make predictions about data set		10
Turned in all work related to this project (Excel file, scratch work, etc.)		5
Fluidness in presenting (sufficient use of notecards, student did not continuously read from notecards or PowerPoint slides)		10
Sources Cited		5
Presentation Creativity and Design		5
TOTALS:		**70**

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