Transformational Graphing in the Real World

I. UNIT OVERVIEW & PURPOSE:

This unit specifically addresses the concept of transformational graphing. Absolute value, polynomial, and square root functions will be examined.

II. UNIT AUTHOR:

Donna Deplazes
Craig County Public Schools

III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Algebra

V. OBJECTIVES:

- To recognize and translate the graph of an absolute value function.
- To recognize and translate the graph of polynomial (specifically quadratic) functions.
- To establish a pattern for easy recognition of higher order polynomials.
- To recognize and translate the graph of square root function.
- To expose students to possible real world situations involving transformational graphing.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors or these functions.

VII. CONTENT:

This unit specifically addresses the topic of transformation graphing and general recognition of absolute value, polynomial (specifically quadratic), and exponential functions. All lessons are discussed in the context of a real world application.

VIII. REFERENCE/RESOURCE MATERIALS:

Graphing calculators will be required. Student Exploration Worksheets and Exit Slip Assessments will be needed for all three lessons. Computers to access GeoGebra would also be beneficial when presenting if you find or make your own animation applets to help students visualize the concepts.

IX. PRIMARY ASSESSMENT STRATEGIES:

Please see Exit Slip Assessments attached to each lesson. Upon completion of more functions a unit assessment on the transformation of functions will be given.

X. EVALUATION CRITERIA:

Students will complete a five question exit slip at the end of the lesson. Documents will be attached to each individual lesson. Students will earn a classwork grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

XI. INSTRUCTIONAL TIME:

Three 90-minute class periods.

Building Bridges

Strand

Algebra

Mathematical Objective(s)

Functions, Quadratic Functions, Transformational Graphing

In this lesson students will discuss transformational graphing by examining the construction of suspension bridges. Students will explore multiple representations of quadratic functions. They will solve given problems by transforming the graph of a quadratic function. Tables, graphs, and equations will be used by students to aide in finding the necessary solutions.

Mathematics Performance Expectation(s)

MPE.2: Collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE.12: Transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Select and use appropriate representations for analysis, interpretation, and prediction.

MPE.14: Recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

AII.6 The student will recognize the general shape of function (quadratic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AFDA.3 The student will collect data and generate an equation for the curve (quadratic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

Please note that transformation graphing will be applied in all lessons within the unit. The type of function addressed will change daily. It is intended for students to use basic transformation graphing techniques to help them on a day-to-day basis.

NCTM Standards

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.

Materials/Resources

- Classroom set of graphing calculators.
- Students need to be familiar with the structure of suspension bridges. Below are a couple of the famous American suspension bridges.
    - Golden Gate Bridge
    - Brooklyn Bridge
- SmartBoard or LCD Projector
- Building Bridges – Student Exploration WS #1
- Building Bridges – Student Exploration WS #2
- Building Bridges – Exit Slip Assessment

Assumption of Prior Knowledge

- Students should have completed Algebra II.
- Students should have experience using a graphing calculator, specifically finding a specific viewing window.
- Students might find it difficult to realize that horizontal transformations are represented "reversely" from graphic to symbolic representation.

- The relevant real life context in this problem is related to the construction of a suspension bridge.

Introduction: Setting Up the Mathematical Task

In this lesson, students will investigate how vertical and horizontal translations affect the symbolic representation of a quadratic function.

Introduction – 20 minutes
Student Exploration #1 – 20 minutes
Discussion of SE #1 – 10 minutes
Student Exploration #2 –20 minutes
Discussion of SE #2 – 10 minutes
CW Assignment: Exit Slip Assessment – 10 minutes

- Have students discuss the similarities and differences between the graphs of absolute value functions and quadratic functions. Students will be reminded about the symmetric characteristics of the graph of a quadratic function. It is important to note that only three points are needed to model a quadratic function. The TI-84+ will give a symbolic representation found through regression that is in standard form. However, if you use the symmetry of the parabola you can find the symbolic representation in vertex form.
    - First use the parent function $y = x^2$ to introduce students to quadratic functions.
    - Then use $y = 2(x-4)^2 + 7$ for an in class example. Have students examine the symmetry (centered around the vertex) in the table that represents the function.
    - Have students find the quadratic regression using their calculators and these given points: $(4, 7), (2,\ 15), (6,\ 15)$.
    - Then have the students verify that $y = 2(x-4)^2 + 7$ is equal to $y = 2x^2 - 16x + 39$.
- Students will work the exploration activities and then the class will discuss their conclusions. Each group will be asked to answer discussion questions on each exploration worksheet and comment on the answers provided by other groups. All students will have the correct answers before the end of class.
- Students are asked to explore transformational graphing of quadratic functions in the two exploration worksheets attached.
- The figure given in the exploration activities is designed to assist students who struggle to "picture" the actual situation.

Student Exploration 1:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction to complete a worksheet (Building Bridges – Student Exploration #1) that asks students to explain vertical and horizontal translations using the vertex form of a quadratic function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
- Students should realize that the vertex of the quadratic in the given situation is $(200,\ 15)$.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=150

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Student Exploration 2:

Group Work (groups of 2 or 3)

Student/Teacher Actions:

- Students should use the information from the introduction and the discussion of the first student exploration to complete a worksheet (Building Bridges – Student Exploration #2) that addresses horizontal and vertical translations as well as stretches and compressions of a given quadratic function.
- Teacher will be guiding students as needed if questions/problems arise, but will not answer the questions for the students.
    - Students should realize that they only need to change the location of the two towers (leaving the distance between the two towers and the height of the towers the same).
    - Students should realize that they need to change the height of the towers and the height of the cable above the bridge surface to form a vertical translation.

    - Changing the distance between the towers or just the height of the towers would result in a stretch or compression.
- If time allows and you have access to a lab or a classroom set of computers… The dynamic Explore Learning activity gizmo allows students to change $a, h,$ and k and see how the graph changes as well. This website is not free – a subscription is required. You may view free for five minutes.
  http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=150

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then discuss the questions as a class at the end of the activity.
- At the end of the second exploration students are asked to make generalizations about how horizontal and vertical shifts (translations) affect the function rule.
- The teacher will assist students who have difficulties and extend the material (add a step of difficulty) for students that are ready to move forward.

Assessment

Students will complete a five question exit slip at the end of the lesson. Please see attached document. Students will earn a CW grade out of 10 points – 5 of these points are from the answers to the exit slip, 2 points are for each group's participation in the class discussion, and 3 points are for each student's participation in their group.

Extensions and Connections (for all students)

The concept of vertical and horizontal stretching and compressing was brought up in the last exploration question and will be discussed in a classroom setting. The topic of transformational graphing will continue in subsequent days' lessons.

Strategies for Differentiation

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the graphical representations and the ability to dynamic exploration allowed within Explore Learning as well as the graphing calculator.

- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the involvement in classroom presentation.
- High ability students may start to begin to compare groups for similarities or differences and offer opinions to lead into tomorrow's lessons.

NAME: _____ DATE: _____

Building Bridges – Student Exploration #2

The figure shows a suspension bridge. We are going to find the quadratic function to symbolize the path of the support cable between the two towers that are 100 feet tall. There is a distance of 400 feet between the two towers. The cable reaches its lowest point at the middle of the span at a height of 15 feet above the bridge's surface. Below is a scale drawing on a coordinate plane of what the situation looks like that should help you.

Based on what was learned from Building Bridges – Student Exploration #1 use the same situation to answer the following questions.

1. What number or numbers in the situation would you change to move the quadratic function horizontally only (no stretching or compressing the actual shape)? Explain why or how your change(s) would move the function horizontally.

2. What number or numbers in the situation would you change to move the quadratic function vertically only (no stretching or compressing the actual shape)? Explain why or how your change(s) would move the function horizontally.

3. What number or numbers in the situation would you change to stretch the function horizontally? Explain why or how your change(s) would stretch the function horizontally.

4. What number or numbers in the situation would you change to compress the function horizontally? Explain why or how your change(s) would compress the function horizontally.

5. What number or numbers in the situation would you change to stretch the function vertically? Explain why or how your change(s) would stretch the function vertically.

6. What number or numbers in the situation would you change to compress the function vertically? Explain why or how your change(s) would compress the function vertically.

NAME: _____ DATE: _____

Building Bridges – Exit Slip

1. Draw a graph on a coordinate plane to symbolize the path of the support cable of a suspension bridge. The height of the two towers is 80 feet. There is a distance of 350 feet between the two towers. The cable reaches its lowest point at the middle of the span at a height of 22 feet above the bridge's surface. Be sure to label and divide your axis using an appropriate scale. Give the coordinates of three keys points of interest on the graph.

2. Using the three points identified in the graph use your graphing calculator to find the quadratic regression to represent the situation.

3. Now using the symmetry of the graph, develop the quadratic function that represents the situation in vertex form.

4. Verify that the regression equation in question 2 is equivalent to the function you found in question 3.

5. Discuss how this quadratic function has been transformed (horizontally or vertically translated, compressed or stretched) compared to the quadratic function from exploration activity #1 – $y = \frac{17}{8000}(x - 200)^2 + 15$.