Performance Based Learning and Assessment Task

Population Project

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
In the Population Project, students will deepen their understanding of exponential growth and decay by researching and analyzing a country's population growth or decay every decade over a 100-year period. With this information, students will model the population graphically, make predictions about the country’s current and future population, create a poster or brochure describing the country and its population trends, present their findings, and take a brief quiz on a different country’s population.

II. UNIT AUTHOR:
Maggie Hughes, Hidden Valley High School, Roanoke County Public Schools

III. COURSE:
Algebra II

IV. CONTENT STRAND:
Functions

V. OBJECTIVES:
• The student will be able to collect, organize, analyze, and present population data using their knowledge of exponential growth and decay.
• The student will be able to model population by creating exponential equations and graphs.
• The student will be able to use exponential equations and graphs to make predictions.
• The student will be able to understand and describe exponential growth and decay.
• The student will be able to correctly use and explain the \( A = Pe^{rt} \) formula.
• The student will be able to make predictions using the information they’ve found and developed.
• The student will be able to analyze the process he/she uses to find their answers since their work is being recorded.

I. REFERENCE/RESOURCE MATERIALS:
• Attached Population Project handout (1 per student) (page 7)
• Attached Population Quiz (1 per student) (page 8)
• Attached Assessment & Rubric Sheets (pages 9 & 10)
• Poster board or brochure paper (1 per group)
• Laptops
• Library access for books on various countries
• Notes on exponential growth and decay
• Classroom set of graphing calculators

II. PRIMARY ASSESSMENT STRATEGIES:
Attached is the Assessment List and Rubric for the Population Project. Both focus on the student’s successful completion of the project, how they’ve presented their work, if they are able to discuss and explain what they’ve done, and the correctness of their graph and formula. Completing the task itself is important since the goal of this activity is to deepen students’ understanding of exponential growth and decay by modeling real population data. The understanding of the concept is checked during the presentation of their findings, with the visual representation of their findings, the mathematical work they turn in with their presentation, and the brief quiz that each student takes individually.
### III. EVALUATION CRITERIA:
Students will be evaluated on the correctness of the mathematical models (graph, equation, table) of their country’s population, their ability to meaningfully present their findings (both verbally and visually), and their performance on the individual quiz. Each of the criteria are further described in the attached Population Project Rubric. The students’ group presentations and models will be evaluated using the rubric. The quizzes will be evaluated using a key, created by the teacher.

### IV. INSTRUCTIONAL TIME:
The instructional time of the Population Project is three to four 90-minute blocks – one for research, one for organizing and creating their presentations, and one or two for presentations, depending on class size. The students will be required to finish their presentations at home, if they do not complete them during the research and organization days. Please note that students will have already had a lesson on exponential and logarithmic growth and decay prior to the project.
Population Project

Strand
Functions

Mathematical Objective(s)
• The student will be able to collect, organize, analyze, and present population data using their knowledge of exponential growth and decay.
• The student will be able to model population by creating exponential equations and graphs.
• The student will be able to use exponential equations and graphs to make predictions.
• The student will be able to understand and describe exponential growth and decay.
• The student will be able to correctly use and explain the \( A = Pe^{rt} \) formula.
• The student will be able to make predictions using the information they’ve found and developed.
• The student will be able to analyze the process he/she uses to find their answers since their work is being recorded.

The mathematical goal of this activity is for students to understand and explain the concept of exponential growth and decay. Students are using their knowledge of exponential growth and decay to research and model real population data. This project reinforces their ability to use, understand, and explain exponential growth and decay and the \( A = Pe^{rt} \) formula, effectively work with other students, research and organize data, and correctly answer questions using data, formulas, and graphs.

Related SOLs
All. 6 (shape of exponential and logarithmic function graphs, conversion between graphical and symbolic representations)
All. 7 (analyze exponential functions both graphically and symbolically)
All.9 (collect, organize, and compare data to model real world situations)

NCTM Standards

Algebra:
Students will:
• Generalize patterns using explicitly defined functions.
• Use symbolic algebra to represent and explain mathematical relationships.
• Use mathematical models to represent and understand quantitative relationships.
• Approximate and interpret rates of change from graphical and numerical data.

Data Analysis:
Students will:
• Identify trends in bivariate data and find functions that model the data.

Problem Solving:
Students will:
• Solve problems that arise in mathematical and other contexts.

Communication
Students will:
• Organize and consolidate their mathematical thinking through communication.
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
• Use the language of mathematics to express mathematical ideas precisely.

Connections:
Students will:
• Recognize and apply mathematics in contexts outside of mathematics.

Representation:
Students will:
• Create and use representations to organize, record, and communicate mathematical ideas.
• Select, apply, and translate among mathematical representations.
• Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources
• Attached Population Project worksheet (1 per student)
• Attached Population Quiz (1 per student)
• Attached Assessment & Rubric Sheets
• Poster board or brochure paper (1 per group)
• Laptops
• Internet access
• Library access for books on various countries
• Notes on exponential growth and decay
• Classroom set of graphing calculators

Prior to assigning the project, the instructor must be sure to sign up for library use on the research day of the activity.

Assumption of Prior Knowledge
• Students should have an understanding of families of functions, especially exponential and logarithmic functions. They should know and understand the characteristics of these functions and their graphs. Students should understand the number e and the $A = Pe^{rt}$ formula. Students should understand population and what a census is.
• Student should be operating on Level 3 Abstraction on the Van Hiele scale with respect to exponential functions and models, graphing these models, and writing equations.
• Students will begin to express ideas of exponential growth and decay in the real-world context of population and how many real-life situations are modeled with exponential growth and decay. Students will discuss these mathematical ideas in terms of modeling situations, not creating situations to fit information that they already know.
• Students may have trouble creating the exponential formula to model their population data, specifically keeping A and t variables and estimating an overall r-value.
• Students should have already explored and discussed exponential and logarithmic functions and their graphs.
• Students should take this project and consider the information gained from a census and how population growth or decay affects civilizations and natural resources. Additionally, they should try to think about other situations that are usually modeled with exponential growth and decay.
Introduction: Setting Up the Mathematical Task

- Discuss exponential functions briefly as a reminder of what they’ve previously learned. Ask them about the $A = Pe^{rt}$ formula – do you remember this? What does each variable stand for? Then, ask students for situations that may be modeled using this formula. Ask why these situations would be modeled exponentially. Write their ideas on the board as a brainstorming activity. *(approximately 15 minutes)*

- Handout Population Project Handout, read through it, and introduce the assignment.
  - “In this project, you will research and analyze a country’s population growth or decay. The country will be assigned to you. You will use exponential equations and graphs to correctly model the population and present your findings. You will also be quizzed on the mathematics behind the concept to ensure you fully understood and participated in the project.”
  - Let them know that today (day 1) is a research day in the library. Day 2 is a work day to organize their findings meaningfully and prepare their presentation and visual. Day(s) 3 (and 4) are for class presentations.
  - Make sure students know that they will likely want to work outside of school to finish their visuals and presentations. Advise them to schedule a time with their partner to work outside of class.
  - Take questions about the handout, timeline, and purpose of the project.
  - See Population Project Handout for further details about the task.

- Assign partners and a country to each set.

Student Exploration

Country and Population Research – Day 1 *(approximately 70 minutes)*

- Take students to the library. The librarian will have books about countries already pulled and ready for them to use.
- Have students work with their partner and use the books and their laptops to look up information about their country, importantly the populations at the specified dates.
- Walk around and help students navigate through all of the information they are finding. Guide them in their decisions of which information to use in their project and which not to include.
- To integrate technology, students will be researching on their laptops and citing their sources. Additionally, they will be graphing their data and using a calculator to help them do so and check their work.

Presentation Preparation – Day 2 *(90 minutes)*

- For more details on what the students are doing, what they should be thinking about, and how the project is done, see the attached Population Project Handout (page 7).
- Have students get out the research they’ve done and any other materials they will need. If students are doing a brochure, they will need to be able to print. If students are doing a poster board visual, have poster board options available for students in case they did not bring their own.
- Remind students that their equation needs to have two variables in it – independent and dependent. Discuss which variables those should be, as a class. Discuss the variables they will use in the equation they create, especially how students will choose an r-value for their equation – will they find the r-value between 1910 and 2010? Will they use the most recent r-value? Or will they take an average of all of the r-values between each decade?
- Students will work with their partner to make decisions on their presentation.
  - What information will they include?
How will they organize their information?
What will they say during their presentation?

- The teacher will need to move around to each group and help them figure out their equations and graphs. The teacher will make sure students understand the concept and guide them to making the correct mathematical models of their data.
- If students finish making their visual, they can begin practicing their presentation. Most students will likely need to do work at home in order to present the next class.

Presentations and Quiz – Day(s) 3 (and 4) (90 to 180 minutes)
- Have students get out their visuals and have them ready for their turn to present.
- Each set of partners will present their findings and their visual. They will also take questions from the teacher and their peers.
- Students will take a short quiz to check their mathematical understanding.

Monitoring Student Responses
- Students are expected to communicate their thinking and new knowledge by writing down their findings, keeping their work, developing models of their country’s population, and presenting all of their information orally and visually.
- Students will effectively communicate to their partner by equally participating in the project and discussing what they are doing at each step. They are to ask each other questions and make decisions about their presentations together.
- The teacher will introduce the project and how their knowledge of exponential functions will be utilized. Additionally, the teacher will facilitate the understanding of these ideas by giving hints and asking probing questions to the partner sets as they complete their project on days one and two.
- For students that are ready to move forward, the teacher will have additional questions about population growth or decay. What would the model look like if there was a famine? What were some of the reasons the population changed the way it did?

Summary (approximately 5 to 10 minutes):
- After quizzes have been graded, hand them back to students. Then, discuss what students wrote as their response to the last question about the census. What ideas did we have as a whole? What were some ideas that we didn’t think about? Why is it important to be correct in the models of population? How does it affect civilizations?

Assessment List and Benchmarks
- Assessment List, Rubric, and Benchmarks are attached.
- Assessments for each task and activity are provided within each day of the described task. Additional guidance in the activity is shown in the Population Project Handout.
- English Language Learners may need someone to read to them and help translate any instructions, questions, ideas they may have, and the research materials.
Population Project Handout

This project is in two parts. Together, both parts shall constitute one test grade (60% project, 40% quiz).

Part 1: You shall present your results in class as a poster or brochure (NON-electronic). This part of your project will be graded using the included rubric and will be 60% of the overall test grade. DO NOT just recite the numbers! Tell us about general trends and discuss reasons for significant changes.

Part 2: There will be an in-class assessment that will be graded on the correctness of your mathematical understanding of population growth and decay. This will be 40% of the overall test grade.

You and your partner will work together to research and analyze the population at specific times of your assigned country. Your country is ___________________. Then, you will organize your information, create a table, graph, and equation modeling your data, and present these findings to the class. After presentations are completed, you will each take a quiz on your own, given information about another country.

You will conduct appropriate research to find the population as of:


Guidelines (these things should be included in your presentation and on your visual, unless stated otherwise):

- Give a brief background of your country (location, dominant language, claim to fame).
- Use the skills taught in class to develop the proper exponential equation that accurately describes the population growth or decay from 1910 to 2010. Use discussions in class to help make the correct decisions to model your data.
- Find the r-value between each decade and have that either on your visual or with your work that you will turn in. Answer these questions on the back of your project or on a separate sheet of paper: What is the significance of the r-value? How did you find it to use in your model?
- Create a table of your data (you could include the r-values here if you would like).
- Create a graphical representation of your data. Create a graph of the exponential equation you developed. These graphs may be separate or on the same one. Make sure to label your data points, axes, scale, and each graph appropriately.
- Predict your country’s current population and the population at the end of the decade. Be able to discuss how you made this prediction.
- Consider the importance of monitoring populations and taking a census. How does population change affect the country and the world? Will the model you used continue in the same pattern or will it change? Why or why not?
- For the quiz (not to be included in presentation), be sure you can use given data and write an equation modeling it, graph the data, make predictions, and write about the importance of monitoring population change.
Population Project Part 2  
(Grade is 40% of overall test grade)

The population of the United States by decade:

1960: 179,323,175 people  
1970: 203,392,031 people  
1980: 226,545,805 people  
1990: 248,709,873 people  
2000: 281,421,906 people  
2010: 308,745,538 people

1. Develop an equation modeling the United States’ population change from 1960 to 2010. You may need to determine the curve of best fit in order to model this appropriately.

2. Develop an equation modeling the United States’ population change from 2000 to 2010.

3. Use both models to predict what the current population of the United States is.

4. Using the 2nd model, predict what the population will be in 2020. Sketch a graph of the second model with the predicted data point included.

5. Based on what you have learned from this project, discuss why taking the census every 10 years is important.
<table>
<thead>
<tr>
<th></th>
<th>Element</th>
<th>Point Value</th>
<th>Self</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student equally participated in project research, organization, and presentation.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Student turned in mathematical work not included on visual.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Student created a paper brochure or poster board.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Student included an accurate table of the data they found.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Student included the correct graphical models of data and equation.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Student created a correct equation model of their data.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Student’s presentation was easily understandable.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Student’s oral and visual presentation was well organized.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Student’s work is well organized.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Student’s work is neat.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Student can explain their reasoning for their ideas, formulas, and work.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Student correctly/meaningfully answered questions asked by peers and teacher.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Student’s presentation was engaging.</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Population Project Presentation Rubric

<table>
<thead>
<tr>
<th>#</th>
<th>Element</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student equally participated in project research, organization, and presentation.</td>
<td>Student did not participate.</td>
<td>Student contributed less than their half.</td>
<td>Student contributed their half.</td>
</tr>
<tr>
<td>2</td>
<td>Student turned in mathematical work not included on visual.</td>
<td>No work provided.</td>
<td>Part of work provided.</td>
<td>All work provided.</td>
</tr>
<tr>
<td>3</td>
<td>Student created a paper brochure or poster board.</td>
<td>No visual created.</td>
<td>Visual partially complete with some information.</td>
<td>Visual complete with all information.</td>
</tr>
<tr>
<td>4</td>
<td>Student included an accurate table of the data they found.</td>
<td>No table of data.</td>
<td>Table includes some data.</td>
<td>Table includes all data.</td>
</tr>
<tr>
<td>5</td>
<td>Student included the correct graphical models of data and equation.</td>
<td>No graphs create.</td>
<td>Graphs partially complete or correct.</td>
<td>Complete and correct graphs created.</td>
</tr>
<tr>
<td>6</td>
<td>Student created a correct equation model of their data.</td>
<td>No equation developed.</td>
<td>Equation developed but is incorrect.</td>
<td>Correct equation developed.</td>
</tr>
<tr>
<td>7</td>
<td>Student’s presentation was easily understandable.</td>
<td>Presentation not understood.</td>
<td>Presentation was partially difficult to understand.</td>
<td>Presentation was easily understandable.</td>
</tr>
<tr>
<td>8</td>
<td>Student’s oral and visual presentation was well organized.</td>
<td>Presentation not organized.</td>
<td>Presentation was difficult to follow – poorly organized.</td>
<td>Presentation was easy to follow and organized well</td>
</tr>
<tr>
<td>9</td>
<td>Student’s work is well organized.</td>
<td>Work unorganized.</td>
<td>Work partially organized.</td>
<td>Work fully organized and easy to follow.</td>
</tr>
<tr>
<td>10</td>
<td>Student’s work is neat.</td>
<td>Lacks neatness.</td>
<td>Needs improvement.</td>
<td>Neat and legible.</td>
</tr>
<tr>
<td>11</td>
<td>Student can explain their reasoning for their ideas, formulas, and work.</td>
<td>Student provides no explanation.</td>
<td>Explanation provided with logical flow, is mostly incorrect.</td>
<td>Explanation provided with logical flow and is mostly correct.</td>
</tr>
<tr>
<td>12</td>
<td>Student correctly/meaningfully answered questions asked by peers and teacher.</td>
<td>Does not answer questions.</td>
<td>Answers questions poorly or incorrectly.</td>
<td>Correctly answers questions.</td>
</tr>
<tr>
<td>13</td>
<td>Student’s presentation was engaging.</td>
<td>Presentation not engaging.</td>
<td>Presentation sometimes engaging.</td>
<td>Presentation always engaging.</td>
</tr>
</tbody>
</table>
Benchmark

Since all work is submitted electronically, I’ve provided images of what a few visuals look like. Notice that they are each different in how students organized and presented their findings – all of these representations were excellent. Part of the activity is to allow students to make the decisions on the best way to present their models.
Population and Growth in Denmark

A = Por (Formula for Exponential Growth)

• Average Exponential Rate for 20th Century, Early 20th °C to 1970°C
• Projected Exponential Rate for 2021 = 0.34°C

Exponential Growth in Denmark 1900-2021

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1900</td>
<td>3,500,000</td>
</tr>
<tr>
<td>1950</td>
<td>4,000,000</td>
</tr>
<tr>
<td>2000</td>
<td>5,000,000</td>
</tr>
<tr>
<td>2021</td>
<td>5,300,000</td>
</tr>
</tbody>
</table>
## How Will You Invest? Project

### I. ASSESSMENT TASK OVERVIEW & PURPOSE:
In the How Will You Invest? Project, students will deepen their understanding of exponential growth by researching, comparing, and making decisions about cash and real estate investments in hopes of planning for their future. With the information they find, students will mathematically show how their investments would manifest over time, decide which investment is best for them, and make a presentation of their findings. Then, students will interview an adult in their life about investments and write a paper showing what they learned from the interview.

### II. UNIT AUTHOR:
Maggie Hughes, Hidden Valley High School, Roanoke County Public Schools

### III. COURSE:
Algebra II, AFDA

### IV. CONTENT STRAND:
Functions (AII), Algebra and Functions (AFDA)

### V. OBJECTIVES:
- The student will be able to collect, organize, analyze, and present investment information and findings using their knowledge of exponential functions and models.
- The student will be able to model investment situations using exponential functions and graphs.
- The student will be able to use exponential equations to make predictions about their future investments.
- The student will be able to understand and describe exponential growth.
- The student will be able to correctly use and explain the $A = Pe^{rt}$ formula.
- The student will be able to analyze the process he/she uses to find their answers since their work is being recorded.

### VI. REFERENCE/RESOURCE MATERIALS:
- Attached How Will You Invest? Project handout (*1 per student*) (pages 20 & 21)
- Attached Assessment & Rubric Sheets (pages 22 & 23)
- Poster board or PowerPoint Presentation (*1 per student*)
- Laptops
- Notes on exponential growth, $A = Pe^{rt}$ formula
- Classroom set of graphing calculators

### VII. PRIMARY ASSESSMENT STRATEGIES:
Attached is the Assessment List and Rubric for the How Will You Invest? Project. Both focus on the student’s successful completion of the project, how they’ve presented their work, if they are able to discuss and explain what they’ve done, and the correctness of their formulas and mathematical findings. Completing the task itself is important since the goal of this activity is to deepen students’ understanding of exponential growth by modeling real investment situations. The understanding of the concept is checked during the presentation of their findings, with the visual representation of their findings, the mathematical work they turn in with their presentation, and the paper about their interview.

### VI. EVALUATION CRITERIA:
Students will be evaluated on the correctness of the mathematical models (equations and answers) of the investments, their ability to meaningfully present their findings (both verbally and visually), and their description of their interview and decisions. Each of the criteria are further described in the How Will You Invest? Project Rubric attached. The students’ group presentations and models will be evaluated using the rubric. Students will also be quizzed on material.

VII. INSTRUCTIONAL TIME:
The instructional time of the How Will You Invest? Project is three to four 90-minute blocks – two for their mathematical problem solving and organizing and creating their presentations, and two or three for presentations, depending on class size. The students will be required to interview, write their reflection, and finish their presentations at home, if they do not complete them during the research and organization days. Please note that students will have already had a lesson on exponential and logarithmic growth and decay prior to the project.
How Will You Invest? Project

Strand
Functions (AII)
Algebra and Functions (AFDA)

Mathematical Objective(s)
- The student will be able to collect, organize, analyze, and present investment information and findings using their knowledge of exponential functions and models.
- The student will be able to model investment situations using exponential functions and graphs.
- The student will be able to use exponential equations to make predictions about their future investments.
- The student will be able to understand and describe exponential growth.
- The student will be able to correctly use and explain the \( A = Pe^{rt} \) formula.
- The student will be able to analyze the process he/she uses to find their answers since their work is being recorded.

The mathematical goal of this activity is for students to understand and explain the concept of exponential growth in terms of real life investment opportunities. Students are using their knowledge of exponential growth to solve problems and make decisions about these investment opportunities presented to them. This project reinforces their ability to use, understand, and explain exponential growth and the \( A = Pe^{rt} \) formula, organize mathematical findings, and correctly analyze investment situations, and make decisions about their future using these findings.

Related SOLs
AII. 6 (shape of exponential functions – graphically and symbolically)
AII. 7 (investigate and analyze exponential functions)
AFDA.1 (investigate and analyze exponential functions)
AFDA.2 (using knowledge of transformations for graphical representation of cash investments)
AFDA.4 (transfer between and analyze variables, graph, and equations of investments)

NCTM Standards
Algebra:
Students will:
- Generalize patterns using explicitly defined functions.
- Use symbolic algebra to represent and explain mathematical relationships.
- Use mathematical models to represent and understand quantitative relationships.
- Approximate and interpret rates of change from graphical and numerical data.

Problem Solving:
Students will:
- Solve problems that arise in mathematical and other contexts.

Communication
Students will:
- Organize and consolidate their mathematical thinking through communication.
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
• Use the language of mathematics to express mathematical ideas precisely.

Connections:
Students will:
• Recognize and apply mathematics in contexts outside of mathematics.

Representation:
Students will:
• Create and use representations to organize, record, and communicate mathematical ideas.
• Select, apply, and translate among mathematical representations.
• Use representations to model and interpret physical, social, and mathematical phenomena.

Materials/Resources
• Attached How Will You Invest? Project handout (1 per student)
• Attached Assessment & Rubric Sheets
• Poster board or PowerPoint Presentation (1 per student)
• Laptops
• Notes on exponential growth, \( A = Pe^{rt} \) formula
• Classroom set of graphing calculators

Prior to the lesson, the teacher should work through the investment opportunities to know the correct answers to guide the students. They also need to collect all materials.

Assumption of Prior Knowledge
• Students should have an understanding of families and functions, especially exponential functions. They should know and understand the characteristics of these functions and their graphs. Students should understand the number \( e \) and the \( A = Pe^{rt} \) formula.
• Student should be operating on Level 3 Abstraction on the Van Hiele scale with respect to exponential functions and models, graphing these models, and writing equations.
• Students will express ideas of exponential growth in the real-world context of investing money and how many real-life situations are modeled with exponential growth and decay. Students will discuss these mathematical ideas in terms of modeling situations.
• Students may have trouble creating the exponential formula to model the investment information given, specifically assigning variables appropriately.
• Students should have already explored and discussed exponential and logarithmic functions and their graphs.
• Students should take this project and consider the information they’ve found about investment opportunities presented to them in their lives. They should discuss how investments may benefit their futures and their plans for investing money.

Introduction: Setting Up the Mathematical Task
• Put students in small groups of 3-5. Give them a potential investment opportunity that could present itself for them in their future. Have students work through the problem, labeling variables in the \( A = Pe^{rt} \) formula and mathematical solving for an unknown. Have the group read the question,
write their work on the board, and talk to the class about the variables, the problem, and the answer they found. (approximately 15 minutes)

- Handout How Will You Invest? Project Handout, read through it, and introduce the assignment.
  - “In this project, you will research potential investment opportunities and the potential exponential growth of your money. You will use exponential equations and graphs to correctly model these opportunities and present your findings. You will also be required to interview an adult in your life about their investments then reflect on the interview.”
  - Let them know that today (day 1) and next class (day 2) are for modeling each situation mathematically, finding the unknown values, then organizing their work in their presentations. Day(s) 3 (and 4) are for class presentations.
  - Make sure students know that they are required to interview and reflect outside of class. In addition to likely working outside of school to finish their presentations.
  - Take questions about the handout, timeline, and purpose of the project.
  - See How Will You Invest? Project Handout for further details about the task.

**Student Exploration**

**Investment Opportunities Research and Problems – Days 1 & 2 (approximately 160 minutes)**

- For more details on what the students are doing, what they should be thinking about, and how the project is done, see the attached How Will You Invest? Project Handout.
- Students will be working individually to go through the handout, research the investment possibilities, and then mathematically solve for the missing values.
- Students will likely need to do research on the investments. Many will not know what some of the cash investments are and how they work. They will need to fully understand these types of investments in order to make an informed decision about their own money.
- The teacher will walk around and help students by giving hints on how to set up their equations and what they will find. Remind them that the assigning of variables correctly is very important.
- Students will work to make decisions about investments and how to present their ideas.
  - Which investment makes the most sense for them? Why? What is your decision based on?
  - What is the importance of investments? Why does this matter to me?
  - How should I organize my work? Is there a way that is easier to follow?
- To integrate technology, students will be researching on their laptops and citing their sources. Additionally, they will be graphing their findings on a calculator.

**Presentations – Day(s) 3 (and 4) (90 to 180 minutes)**

- Have students get out their presentations and papers ready for their turn to present.
- Each student will present their findings and their ideas. They will discuss which investment they will make and why. Then, they will talk about who their interviewed, what they discussed, and how their interview changed their ideas or influenced their decisions about investing their money.
- Students will submit PowerPoint presentations and electronic versions of their reflection paper via email or Blackboard. Students will turn in poster boards and hard copies of their reflection, by hand.

**Monitoring Student Responses**

- Students are expected to communicate their thinking and new knowledge by writing down their findings, keeping their work, developing models of the investments, and presenting all of their
information orally and visually. Students will not be required to explain all of their work during their presentation but should be able to answer questions about it, if needed.

- The teacher will introduce the project and how their knowledge of exponential functions will be utilized. Additionally, the teacher will facilitate the understanding of these ideas by giving hints and asking probing questions to the partner sets as they complete their project on days one and two.
- For students that are ready to move forward, the teacher will have additional questions about monetary growth. What would the models look like if you changed your initial investment? What happens if you need to get the money sooner than five years?

- **Summary (approximately 5 to 10 minutes):**
  - After students discuss their findings and decisions, have a classroom discussion about how others’ presentations have changed your mind, if at all. Why or why not?

**Assessment List and Benchmarks**
- Assessment List, Rubric, and Benchmarks are attached.
- Assessments for each task and activity are provided within each day of the described task. Additional guidance in the activity is shown in the How Will You Invest? Project Handout.
- English Language Learners may need someone to read to them and help translate any instructions, questions, ideas they may have, and the research materials.
How Will You Invest? Project Handout

This project is worth one test grade for the current grading period.

You shall present your results in class as a poster or PowerPoint Presentation. This part of your project will be graded using the included rubric. DO NOT just recite the numbers during your presentation! Tell us about your overall findings and discuss which investment you have chosen to make. All mathematical work must be included on the poster board or in the PowerPoint, but you do not need to take us through all of your steps, unless asked to do so.

You will research and analyze the investment opportunities, mathematically solve the problems presented to you, then make a decision about which investment you would like to make. Then, you will organize your information, create a graph comparing the cash investments (only the initial situation of $20,000 and a five-year period – you will need to graph data for each year in this period), create equations modeling each investment, and present these findings to the class. After your presentation is complete, you will turn this in.

Additionally, you will interview a trusted adult (family member, teacher, member of the community, etc). In the interview, you must ask them about the investments they have or have not made in their lifetime. Ask why they made or didn’t make those investments, what influenced their decision, and how their decisions have played out and affected their life. Do they suggest that you make investments and in what? Why or why not? Use this conversation to write a 1.5 to 2 page reflection (double-spaced, 12 pt. Times New Roman font, 1 inch margins) reflection. Discuss what you talked about and how it influenced your ideas and decisions about investments.

Cash Investments

We are assuming you have inherited $20,000 and are looking at different investments to decide which gives the best rate of return. All investments are over a five-year period. Use the appropriate formula(s) to mathematically work out the problems and solve any problems presented.

➢ Invest in a savings account at 1% APR.
  o What is a savings account? How does one work?
  o What is APR?
  o What happens if you only invest half of your money? How does that affect your account over the 5 years?

➢ Invest in a five year CD (Certificate of Deposit) at 2.75% APR.
  o What is a CD? How does one work?
  o What happens if you inherited $25,000? $5000 is a significant amount of money. Does it significantly impact your investment, in comparison to a $20,000 investment?

➢ Invest in a mutual fund at 8% APR.
  o What is a mutual? How does one work?
  o In addition to looking at you 5-year investment, find out what a 10-year investment would do for you. Why does adding 5 years to your investment change the

➢ Invest in a stock portfolio at 10% APR.
  o What is a stock portfolio? How does one work?
What happens if you double the amount you invest? How does this affect the amount of money you’ve made in 5 years?

Choose which cash investment you think is best for you and be able to discuss why you chose that investment.

Real Estate Investments

Invest in an $180,000 home, taking out a $160,000 mortgage for 30 years at 5.75%. Find the monthly payments, determine the total paid over the life of the mortgage, and the total interest paid. Why are you paying so much for an $180,000 home? Is it worth it to get a mortgage if you could pay outright?

Calculate the value of the home after 30 years assuming a 5% increase in value every year.

Calculate the value of the home and the mortgage after 5 years. Find the profit you will make if you sell the house at market value. Calculate your percent return.

Repeat finding the monthly payments, total paid over the life of the mortgage, and the total interest paid for a 15-year mortgage at 6%. Would you rather take out a longer mortgage at a lower interest percentage or a shorter mortgage with a higher percentage? Why?

Guidelines for oral aspect of presentation:

• You presentation should be about 5 minutes long.
• You will need to show us the visual you’ve created.
• Give a brief overview of your findings. Discuss the pros and cons of each investment opportunity, in your opinion.
• Which investment(s) do you chose to make (at the $20,000 and 5-year period). Why? What influenced your decision? Is this decision both realistic and reasonable for you?
• All mathematical work must be included on the poster board or in the PowerPoint, but you do not need to take us through all of your steps, unless asked to do so.
Benchmark
A student’s PowerPoint Presentation and Reflection Paper are below.
AA&T Investment Project

Cash Investments
- Savings Account at 1% APR - $1,025.42
- CD at 2.75% APR - $3,846.03
- Mutual Fund at 8% APR - $9,836.49
- Stock Portfolio at 10% APR - $12,974.43

Cash Investments (work)
- Savings Account - $20,000
  \[ A = 20,000 \times \left(1 + \frac{0.01}{12}\right)^{12 \times 8} \]
  \[ A = 22,983.49 \]
- CD - $20,000
  \[ A = 20,000 \times \left(1 + \frac{0.0275}{12}\right)^{12 \times 8} \]
  \[ A = 22,983.49 \]
- Mutual Fund - $20,000
  \[ A = 20,000 \times \left(1 + \frac{0.08}{12}\right)^{12 \times 8} \]
  \[ A = 29,836.49 \]
- Stock Portfolio - $30,000
  \[ A = 30,000 \times \left(1 + \frac{0.10}{12}\right)^{12 \times 8} \]
  \[ A = 32,974.43 \]

Real Estate Investment
- 30 Years
  - Monthly payments - $3,694.46
  - Total paid over life of mortgage - $888,003.36
  - Total interest paid - $738,003.36
  - Value of home with 5% increase every year - $906,704.63
  - Amount to be made - $91,290.33
Real Estate Investment (math)

30 Years
A=180,000\text{, initial}
A=199,003.36
Monthly payment = 899,003.36/360(months)
Interest paid = 899,003.36-180,000
Value of house:
A=180,000\text{, initial}
A=600.954.33

Real Estate Investment

5 Years
- Monthly payments = $3,694.91
- Total paid over life of mortgage = $213,294.49
- Total interest paid = $53,294
- Value of home with a 5% increase every year = $231,124.56
- Amount to be made = $17,850.66

Real Estate Investment (math)

5 Years
A=180,000\text{, initial}
A=213,294.49
Monthly payment = 213,294.49/60(months)
Interest paid = 213,294.49-180,000
Value of house:
A=180,000\text{, initial}
A=253,124.59
Amount to be made:
231,124.56
253,124.59
17,850.66

Real Estate Investment

15 Year Mortgage at 6%
- Monthly payments = $2186.31
- Total paid over life of mortgage = $393,636.60
- Total interest paid = $223,636.60
- Value of house with a 5% increase every year = $393,636.60
- Amount to be made = $14,763.60
Real Estate Investment (math)

15 Years

\[ A = A \times (1 + r)^n \]
\[ A = 100,000 \times (1 + 0.01)^{15} \]
\[ A = 164,000 \]

Monthly payment: \[ 253,036.50 \times \frac{180}{180} \] months

Interest paid: \[ 253,036.50 \times 180,000 \]

Value of house:
\[ A = 164,000 \times (1 + r)^{15} \]
\[ A = 164,000 \times (1 + 0.01)^{15} \]
\[ A = 199,000 \]

\[ \text{142,000 - 50,000} = 92,000 \]
Planning For Your Future

I spoke with my uncle about investments. He has been investing for many years now, so I thought he would be good to talk to. He does have a full time job, so he does investments on the side (as most people do). My uncle has invested in many different things, many different ways. He has bought stock, bought houses, and car. The reason for buying the houses and car is that he got a good deal. Once getting the good deal, he could sell if for more than he had paid. A lot of people say that buying and selling is not investing, but you still invest money and time into it. Buying and selling things such as cars or houses is the simplest and easiest form of investing. My uncle told me this because, like stocks, a car or house cannot loose tons of value in a blink of an eye (not considering crashes or fires). If you buy smart, you can make your money back and then some. He went on to explain his experiences with stocks. He has not bought many, but just a little here and there. He has not been involved in bubbles or such. He mainly has invested in things that will slowly go up over the years. Some of the things he has invested in are alcohol and tobacco companies. People will always what them, simply because they are addictive. As the company grows the value of your stock increases. What he did was buy small amounts of stock and he still has them. They have slowly increased in value and will continue to increase. Unless something is happening to the company, he will not sell them till he is near retirement.

I asked him “why have you made these investments?” He said “To make money”. Investments are risky. He has lost money by making investments before. But, luckily for him, making investment has affected his life in a very positive way. As I said, he has a full time job but that pays the bills and gets groceries. Money he uses for fun and has in his savings account is form investing. He strongly suggested that I invest when I get older. He discussed a simple plan that will help me out a lot. Start out with things that would get me a quick return. Starting out is not the time to make big/long term investments. After I have built up enough to make investment how I want, I should slowly start to make the big/long term investment. I shouldn’t put all of my money into them because I still need to do the quick return.
investments. As the quick return investments continue to make me money, I can continue to make the bigger investments with the money I’m getting from the quick investments. This is the example he gave me, you have a few apple trees and you pick all of the apples off. You need the money so you sell all of them. The next time you pick the apples, you use a sixth of that picking to plant more trees and sell the rest. You can do this since you sold all of the first picking and you have enough money to get by. As you do more pickings, you use some of the apples for planting and you sell the rest. He said that this wasn’t the best example, but I think that it was alright.

I have thought a lot about what he said. It is a good plan, yet it sounds very simple. I’m sure it is very complicated. Finding what would be a good quick return may not be easy depending on the market. Also choosing a long term investment wisely could be very difficult. My uncle also suggested that I have a savings account and put a small percent of each profit in it. And after I have a good amount saved up, spend it all on bonds. It is an easy was to raise your savings in a 10 year period. Having all you your savings invested in bonds is a good thing. You get a guaranteed profit. Instead of just having the money siting in the bank, buying bonds is the smart, simile thing to do.

I already knew a lot of what he had told me, but the conversation was still useful. I will use it in my future investing. And hopefully I use it for the best.
## How Will You Invest? Project Assessment List

<table>
<thead>
<tr>
<th>#</th>
<th>Element</th>
<th>Point Value</th>
<th>Earned Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student completed every task in the project.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Student turned in mathematical work within the visual.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Student created a poster board or PowerPoint.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Student included a correct graph comparing the cash investments.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Student included the correct mathematical work of each investment.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Student created correct equations for each investment.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Student’s presentation was easily understandable.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Student’s oral and visual presentation was well organized.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Student’s work is well organized.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Student’s work is neat.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Student can explain their reasoning for their ideas and decisions about investing.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Student correctly/meaningfully answered questions asked by peers and teacher.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Student’s presentation was engaging.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Student wrote reflection following all guidelines.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Student’s reflection was written well and made sense.</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>#</td>
<td>Element</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>----</td>
<td>------------------------------------------------------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>1</td>
<td>Student completed every task in the project.</td>
<td>Student did not complete any tasks.</td>
<td>Student completed half of tasks.</td>
</tr>
<tr>
<td>2</td>
<td>Student turned in mathematical work within the visual.</td>
<td>No work provided.</td>
<td>Part of work provided.</td>
</tr>
<tr>
<td>3</td>
<td>Student created a poster board or PowerPoint.</td>
<td>No visual created.</td>
<td>Visual partially complete with some information.</td>
</tr>
<tr>
<td>4</td>
<td>Student included a correct graph comparing the cash investments.</td>
<td>No graph.</td>
<td>Graph incorrect and/or includes some of cash investments.</td>
</tr>
<tr>
<td>5</td>
<td>Student included the correct mathematical work of each investment.</td>
<td>No work is correct.</td>
<td>Some of work is correct.</td>
</tr>
<tr>
<td>6</td>
<td>Student created correct equations for each investment.</td>
<td>No equations developed.</td>
<td>Equations developed but are incorrect.</td>
</tr>
<tr>
<td>7</td>
<td>Student’s presentation was easily understandable.</td>
<td>Presentation not understood.</td>
<td>Presentation was partially difficult to understand.</td>
</tr>
<tr>
<td>8</td>
<td>Student’s oral and visual presentation was well organized.</td>
<td>Presentation not organized.</td>
<td>Presentation was difficult to follow – poorly organized.</td>
</tr>
<tr>
<td>9</td>
<td>Student’s work is well organized.</td>
<td>Work unorganized.</td>
<td>Work partially organized.</td>
</tr>
<tr>
<td>10</td>
<td>Student’s work is neat.</td>
<td>Lacks neatness.</td>
<td>Needs improvement.</td>
</tr>
<tr>
<td>11</td>
<td>Student can explain their reasoning for their ideas and decisions about investing.</td>
<td>Student provides no explanation.</td>
<td>Explanation provided with logical flow, is mostly incorrect.</td>
</tr>
<tr>
<td>12</td>
<td>Student correctly/meaningfully answered questions asked by peers and teacher.</td>
<td>Does not answer questions.</td>
<td>Answers questions poorly or incorrectly.</td>
</tr>
<tr>
<td>13</td>
<td>Student’s presentation was engaging.</td>
<td>Presentation not engaging.</td>
<td>Presentation sometimes engaging.</td>
</tr>
<tr>
<td>14</td>
<td>Student wrote reflection following all guidelines.</td>
<td>No reflection.</td>
<td>Reflection does not meet all guidelines.</td>
</tr>
<tr>
<td>15</td>
<td>Student’s reflection was written well and made sense.</td>
<td>Reflection is never easy to follow now understand.</td>
<td>Reflection sometimes easy to follow and understand.</td>
</tr>
</tbody>
</table>