Linear Programming: Sports Shoes

**Strand**
Algebra, Functions and Data Analysis: Algebra and Functions

**Mathematical Objective(s)**
1. Students will create their own feasible region and use it to find the maximum and minimum values.
2. Students will use linear program to solve real life scenarios.

**Related SOL**

AFDA.5

The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

**NCTM Standards**

- Represent and analyze mathematical situations and structures using algebraic symbols

**Materials/Resources**

- assessment, and assessment list,

**Assumption of Prior Knowledge**

**Algebra**

- Students should know how to find the equation of a line using appropriate formulas or using the calculator.
- Students should be familiar graphing inequalities in two variables.
- Students should know how to plot points on a coordinate plane.
- Students should know how to identify vertices.
- Students should know how to graph a line with an undefined slope and write an equation of the line.
- Students should know how to solve system of equations algebraically and graphically.

**Misconceptions/difficulties:**

- Solving system of equations algebraically.
- Using appropriate inequality sign to represent a certain shaded region.
Introduction: Setting Up the Mathematical Task

- In this task, AFDA students will create and investigate system of inequalities to find the maximum and minimum values of a function.
- For the students to complete the assessment it will take one class period.
- The students will be given a real life scenario to solve using linear programming.
- To help the students with the task, ask the following questions:
  - Identify the two values being compared. How can you relate these two values? How do you check the answers to a system of equations and a system of inequalities? Why is using the linear programming process useful?

Student Exploration

Student/Teacher Actions:

- The sports store makes and sells the latest brands of shoes, Pie Airs and Radicals. It takes 4 lbs. of leather and 10 feet of string to make a dozen Pie Airs. To make Radicals it takes 20 lbs. of leather and 30 feet of string. The store makes a profit of $15.00 for Pie Airs and $45.50 for Radicals. The store has 600 lbs. of leather and 1200 feet of string. Your goal is to maximize the profit based on the amount of leather and string based on these constraints.
- The teacher will walk around and help students with questions they may have. The teacher is to direct the students and not give answers.

Monitoring Student Responses

- Students will communicate their new knowledge with the class by presenting their findings.
- If students are have difficulty expressing their thoughts the teacher and students will ask clarifying questions.

Summary

- After completing the assessment the teacher will led the students through a series of questions to pull everything together.

Assessment List and Benchmarks

- Assessment List, Rubric and Benchmarks attached.
- Questions:
  - What constraints would need to be included with the constraint \( y < -x +4 \) to ensure that the feasible region is bounded in the first quadrant?

Journal/writing prompts:

- Explain the how to identify the objective function and how to use the function
The sports store makes and sells the latest bands of shoes, Pie Airs and Radicals. It takes 4 lbs. of leather and 10 feet of string to make a dozen Pie Airs. To make Radicals it takes 20 lbs. of leather and 30 feet of string. The store makes a profit of $15.00 for Pie Airs and $45.50 for Radicals. The store has 600 lbs. of leather and 1200 feet of string.

1. Identify the objective function.

2. Identify the constraints.

3. Graph the feasible region based on the constraints.
4. How many of each shoe should be made to maximize your profit? Include vertices and calculations using the function objectives

5. What is the maximum profit?
# Rubric for Linear Programming

<table>
<thead>
<tr>
<th>#</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Appropriately set up the objective function.</td>
<td>Incorrectly identified one component of the objective function.</td>
<td>Incorrectly identified more than one component of the objective function.</td>
</tr>
<tr>
<td>2.</td>
<td>Appropriately set up the constraints.</td>
<td>Appropriately set up at least two out of the 4 constraints.</td>
<td>Incorrectly identified 3 or more of the constraints.</td>
</tr>
<tr>
<td>3.</td>
<td>Graphed the appropriate feasible region.</td>
<td>Appropriately graphed at least two out of the 4 constraints.</td>
<td>Graphed more than two constraints incorrectly.</td>
</tr>
<tr>
<td>4.</td>
<td>Appropriately calculated how many of each shoe needs to be made to maximize profit. (Include vertices and calculations using the function objectives)</td>
<td>Calculated how many of each shoe needs to be made to maximize profit without work.</td>
<td>Did not solve correctly.</td>
</tr>
<tr>
<td>5.</td>
<td>Appropriately calculated the maximum profit</td>
<td>Calculated a maximum profit.</td>
<td>Did not calculate maximum profit.</td>
</tr>
</tbody>
</table>
### Assessment List

<table>
<thead>
<tr>
<th>Element</th>
<th>Points Possible</th>
<th>Earned Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Set up the appropriate objective function.</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2. Set up the appropriate constraints.</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3. Graphed the appropriate feasible region.</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4. Identified how many of each shoe needs to be made to maximize profit. (included vertices)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5. Calculated the maximum profit</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Linear Programming Assessment

1. The sports store makes and sells the latest bands of shoes, Pie Airs and Radicals. It takes 4 lbs. of leather and 10 feet of string to make a dozen Pie Airs. To make Radicals it takes 20 lbs. of leather and 30 feet of string. The store makes a profit of $15.00 for Pie Airs and $45.50 for Radicals. The store has 600 lbs. of leather and 1200 feet of string.

   a. Identify the objective function.

   Function Objective: \( P = 15x + 45.50y \)

   b. Identify the constraints?

   \[ x \rightarrow \text{Pie Airs} \]
   \[ y \rightarrow \text{Radicals} \]
   \[ 4x + 20y \leq 600 \]
   \[ 10x + 30y \leq 1200 \]
   \[ x \geq 0 \]
   \[ y \geq 0 \]

   c. Graph the feasible region based on the constraints.
d. How many of each shoe should be made to maximize your profit? Include vertices and calculations using the function objectives

<table>
<thead>
<tr>
<th>vertices</th>
<th>15x + 45.50y</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(75, 15)</td>
<td>15(75)+45.50(15)</td>
<td>$1807.50</td>
</tr>
<tr>
<td>(0, 30)</td>
<td>15(0) + 45.50(30)</td>
<td>$1365</td>
</tr>
<tr>
<td>(120, 0)</td>
<td>15(120) + 45.5(0)</td>
<td>$1800</td>
</tr>
</tbody>
</table>

The company should make 75 Pie Airs and 15 Radicals.


c. What is the maximum profit?

The maximum profit is $1807.50