Projectile Motion and Quadratic Functions

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:
The student will examine the path of a projectile and explain the motion using a quadratic function. Neglecting air resistance, projectiles follow the path of a parabola in nature.

II. UNIT AUTHOR:
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III. COURSE:
Algebra I

IV. CONTENT STRAND:
Equations and Inequalities, Functions

V. OBJECTIVES:
The student will be able to:

- solve quadratic equations algebraically and graphically
- solve real-world problems involving equations and systems of equations
- investigate and analyze quadratic functions both algebraically and graphically
- make connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

VI. REFERENCE/RESOURCE MATERIALS:
Calculator, stopwatch, tennis ball, computer access.

VII. PRIMARY ASSESSMENT STRATEGIES:
The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The assessment for student activity 1 and student activity 2 are attached.

VIII. EVALUATION CRITERIA:
Assessment list for Activity 1 and 2, corresponding rubrics.

IX. INSTRUCTIONAL TIME:
The two combined activities are designed to take 2-3 90 minute blocks.

Projectile Motion and Quadratic Functions

Strand
Equations and Inequalities, Functions.

Mathematical Objective(s)
The student will be able to:

- solve quadratic equations algebraically and graphically
- solve real-world problems involving equations and systems of equations
- investigate and analyze quadratic functions both algebraically and graphically
- make connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

Related SOL

A.4 (quadratic equations, systems of equations),
A.7 (quadratic functions)

NCTM Standards

- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- use symbolic algebra to represent and explain mathematical relationships
- build new mathematical knowledge through problem solving
- recognize and apply mathematics in contexts outside of mathematics

Materials/Resources
- Graphing Calculators
- Graph Paper
- Stopwatch
- Tennis Ball
- Computer Lab or Projector for Videos

Assumption of Prior Knowledge

Students should have basic knowledge of conducting experiments, collecting data, and analyzing results. Students should be able to operate a stopwatch.

Students should have a basic understanding of the symmetry of quadratic functions and understand the concept of roots of quadratic functions. Students should also know how to find a quadratic function in vertex form and a knowledge of solving systems of equations in two variables.

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the relationship between the path traveled by projectiles and quadratic functions. In nature, projectiles such as bullets, balls being thrown, cannons, etc. follow a parabolic path. This path can be explained mathematically by a quadratic function. Students will work in groups of three to conduct an experiment that involves launching/bouncing a tennis ball an unknown distance and determining the quadratic function that describes the path of their ball knowing only how long it took. The quadratic function will be found in two different ways and the results will be compared to each other to see how closely they resemble. For ease of hand calculations, some rounding will take

place in our analysis, but the point of the activity will not be lost. The time allotted for this activity is 2-3 90 minute blocks.

Student Exploration

Optional: Review the roots of quadratic equations and the concept of symmetry of parabolas.

As an entire class or individually in a computer lab, have students view the following videos:

http://www.youtube.com/watch?v=H-Y4PcV_mto

http://www.youtube.com/watch?v=hlW6hZkgmkA&list=PLE65DF8D0F83FF77E&index=3

http://www.youtube.com/watch?v=N0H-rv9XFHk&list=PLE65DF8D0F83FF77E&index=4

http://www.youtube.com/watch?v=LT8K9r4FnJo&list=PLE65DF8D0F83FF77E&index=5

Student Activity 1: The Experiment

Ask the students to briefly summarize in their journals what they have viewed in the videos. Have students answer the following questions in their journals: (questions also included on worksheet)

1. What is the path of a projectile in nature?
2. How many different components of velocity does a projectile have at any point in its path?
3. Which component of velocity remains constant?
4. Which component of velocity is always changing?
5. What force causes one of the components of velocity to change?
6. What is the vertical component of velocity when the projectile is at its peak?

Divide the students into groups of three. Each group will need a stopwatch and a tennis ball. Outside or in a gymnasium, have students complete the following task.

Student 1 throws ball to student 2 while student 3 uses the stopwatch to time how long it takes the ball to travel from student 1 to student 2. Encourage students to throw the ball high enough so that there is an extended period of time that the ball remains in the air. Student 3 will record the time it takes in seconds for the ball to travel from student 1 to student 2. The first question on "data collection" handout provides room to record data and draw the path of the ball.

Have the students summarize in their journals the results of the experiment.

After completion of the task, bring the students back together for whole class discussion. The teacher can facilitate the discussion by moving the students toward the following conclusions:

- What was the shape of the motion that the ball took in flight from student 1 to student 2? Parabola
- If the hypothetical x-axis is the horizontal line from the point where student 1 threw the ball to the point where student 2 caught the ball, what are the x intercepts of the ball's flight? (0, 0) and (A, 0) with "A" being the time in seconds the ball was in the air.
- At what point along the hypothetical x-axis would you expect to find the vertex of the parabola? Half the time the ball was in the air.

- Knowing that the entire path from student 1 to student 2 took "s" seconds, how long do you estimate it took for the ball to fall from the peak height at the vertex to the hands of student 2? Half the total time.
- Do you know the height of the ball at the vertex? Not yet

At this point, the teacher should remind students that the vertical velocity of a projectile at its peak is 0. Explain to students that this essentially creates a free-fall situation from the vertical peak of the ball to the hands of student 2. For the purposes of this activity, assume the use of customary units. ***(Depending on the preference of the teacher, this may be modified to use metric units. We are also assuming no air resistance.)***

Explain to students that the formula for free fall for any projectile is as follows:

$d = \frac{1}{2}gt^2$, *where d=distance, g=32 feet/second^2, and t=time in seconds.*

Have students calculate the vertex of their parabola using the formula. Ask which variable in the formula represents the y-value of the vertex? If students have trouble, you can explain that the "d" represents the y value of the vertex.

Homework: Have students summarize the information they now know about their parabola in their journals. Given that they know the vertex and the roots, ask them to begin thinking about how they will find the quadratic function that represents ball flight.

Student Activity 2: Finding the Function

The teacher should begin with a summation of the data and how they now know the vertex and the roots of the path of ball flight. Explain that they will be finding the quadratic function that describes the ball flight using two different methods.

Method 1: System of three equations using elimination or substitution. For ease of calculation, it may be beneficial for students to round their roots and vertex to the nearest whole numbers.

Individual work with group data:

Have each student write a system of three equations and solve the system of equations using elimination or substitution. The general form of a quadratic equation is $y = ax^2 + bx + c$. Below is a computation using sample data.

Vertex: (1.4, 33)

x-intercepts: (0, 0), (2.8, 0)

$a(1.4)^2+b(1.4)+c=33$ $\quad\quad$ $1.96a+1.4b+c=33$

$a(0)^2+b(0)+c=0$ $\quad\quad$ $c=0$

$a(2.8)^2+b(2.8)+c=0$ $7.84a+2.8b+c=0$

Since c=0, we can rewrite a system of two equations as:

7.84a+2.8b=0

1.96a+1.4b=33,

Multiply equation 2 by 2:

7.84a+2.8b=0

3.92a+2.8b=66,

Subtract the two equations:

3.92a=-66

a=-16.84,

Substitute a into any equation,

1.96(-16.84)+1.4b=33

b=47.14

The equation for the parabola becomes $y = -16.84x^2 + 47.14x$

Ask students what was represented by x and y in the quadratic equation they have created. When they determine that y represented height and x represented time in seconds, have the students rewrite the equation as a quadratic function where height is a function of time using h(t) for y and t for x.

$$h(t) = -16.84t^2 + 47.14t$$

Using the function, have students create a table of values for the flight of their tennis ball every .25 seconds and graph the points on a coordinate plane. (See worksheet)

Method 2: Vertex Form of a parabola

Write the formula for the vertex form of a parabola on the board:

$$y = a(x-h)^2 + k, where\ (h,k) is\ the\ vertex\ of\ the\ parabola$$

Use the formula to find the quadratic equation for your ball flight and compare it to the formula derived in method 1.

Vertex (1.4, 33)

$y=a(x-1.4)^2+33$

$y=a(x^2-2.8x+1.96)+33$

Use an x-intercept to solve for a, (0, 0)

$0=a(0^2-2.8(0)+1.96)+33$

$-33=1.96a$

a=-16.83673, so $y = -16.84(x - 1.4)^2 + 33$

or height as a function of time, $h(t) = -16.84(t - 1.4)^2 + 33$

As in activity 1, have students complete a table of values for every .25 seconds of their ball's flight. Have the students record how the values compare.

Note: (There may be some discrepancies between the heights in the two student tables. This will be due to rounding)

Student Activity 1: Data Collection

Draw a picture of the ball toss. Be sure to label the height and time of the ball on the axes of your drawing.

Student 1 Student 2

Time for the ball to travel from student 1 to student 2: _____

Questions: (Journal)

1. What is the path of a projectile in nature?
2. How many different components of velocity does a projectile have at any point in its path?
3. Which component of velocity remains constant?
4. Which component of velocity is always changing?
5. What force causes one of the components of velocity to change?
6. What is the vertical component of velocity when the projectile is at its peak?

Free Fall: $d = \frac{1}{2}gt^2$, *where d=distance, g=32 feet/second^2, and t=time in seconds.*

Calculate the vertex of your ball. Which variable in the formula represents the y-value of the vertex?

Homework: Write down thoughts or suggestions for how you plan to find the quadratic function that describes your ball flight.

Student Activity 2: Finding the Function

Directions: Find the function that represents the path of the ball in two ways. Make a table of values for the function found using each method to determine the height of the ball in .25 second increments.

Coordinates of Vertex of ball flight (round second to nearest tenth and height to nearest whole number):_____

x-intercepts of ball flight:_____

Method 1 System of Equations

t	h(t)

Height as a function of time for tennis ball:

Method 2: Vertex Form

$$y = a(x-h)^2 + k, where\ (h,k) is\ the\ vertex\ of\ the\ parabola$$

Vertex (__,__)

x-intercepts :

t	h(t)

Height as a function of time for tennis ball:

Do you notice any difference in the tables of values for your two functions?

What could explain these differences?

Do you think the two functions are equal? If so, show your work.

Assessment List and Benchmarks

Assessment List for Activity 1: Data Collection

Number	Element	Point Value	Self	Teacher
1	The student was a willing and able participant in the group	2		
2	The student completed a picture that was appropriately labeled detailing the experiment.	2		
3	The student answered the questions necessary for understanding of the material.	2		
4	The student was able to use the data to arrive at a vertex representing the peak of ball flight.	2		
5	The student journal was maintained in the course of the exploration.	2		
	Total	10		

Rubric for Activity 1: Data Collection

Number	Element	0	1	2
1	Group Work	The student did not participate in the activity.	The student participated but was disruptive in the completion of the activity.	The student participated in the experiment with good behavior and helped the team complete the activity
2	Picture	The student did not draw a picture or picture was not labeled appropriately for either height or time.	Student picture was drawn and either height or time was labeled appropriately	Student picture was drawn and both height and time was labeled appropriately
3	Questions	The student correctly completed 1 or 0 questions	The student correctly completed 2, 3, or 4 questions.	The student correctly completed 5 or 6 questions.
4	Vertex	The student did not do the necessary work to find the vertex of the tennis ball.	The student attempted to find the vertex of the tennis ball but algebraic mistake led to wrong answer.	The student correctly found the vertex of the tennis ball.
5	Journal	The student did not keep a journal.	The student had 1 or 2 elements in a journal to include video summaries, experiment summation, and homework summation of what they now know about path of projectiles.	The student had all 3 elements in a journal to include video summaries, experiment summation, and homework summation of what they now know about path of projectiles.

Assessment List for Activity 2: Finding the Function

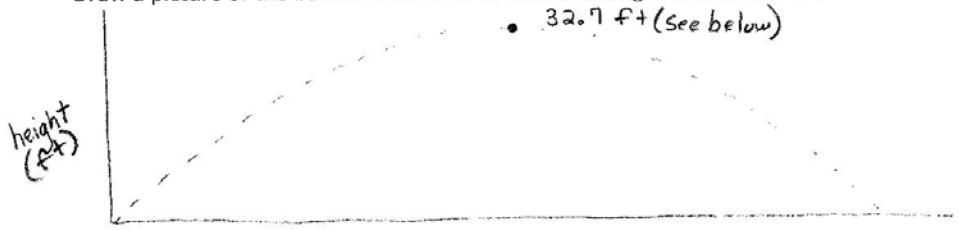
Number	Element	Point Value	Self	Teacher
1	Prerequisite Data	2		
2	System of Equations	2		
3	Vertex Form	2		
4	Quality	2		
5	Functions	2		
	Total	10		

Rubric for Activity 2: Finding the Function

Number	Element	0	1	2
1	Prerequisite Data	The student did not transcribe the necessary prerequisite data for calculations.	The prerequisite data was transcribed but not rounded according to instructions.	The prerequisite data was transcribed and rounded according to instructions.
2	System of Equations	The student did not write a proper system of equations in three variables.	The student wrote a proper system of equations but errors in algebra lead to wrong coefficients and misrepresented table.	The student correctly solved a system of equations and was able to correctly complete the table.
3	Vertex Form	The student did not correctly identify the vertex or x-intercepts.	The student correctly labeled the vertex and x-intercept but errors in algebra lead to wrong coefficients and misrepresented table.	The student correctly found the vertex form of the equation and correctly completed the table.
4	Quality of work	My five year old could do better.	Satisfactory	Einstein would be jealous.
5	Functions	No functions are given with proper notation.	One function is given with proper notation.	Both functions are given with proper notation.

Student Activity 1: Data Collection

Draw a picture of the ball toss. Be sure to label the height and time of the ball on the axes of your drawing.



Student 1 Student 2

Time for the ball to travel from student 1 to student 2: 2.86 seconds

Questions: (Journal) Answers also in journal

1. What is the path of a projectile in nature? parabola
2. How many different components of velocity does a projectile have at any point in its path? 2: x-horizontal, y-vertical
3. Which component of velocity remains constant? x-component: horizontal
4. Which component of velocity is always changing? y-component: vertical
5. What force causes one of the components of velocity to change? gravity causes y to change
6. What is the vertical component of velocity when the projectile is at its peak? $V_y = 0$

Free Fall: $d = \frac{1}{2}gt^2$, *where d=distance, g=32 feet/second², and t=time in seconds.*

Calculate the vertex of your ball. Which variable in the formula represents the y-value of the vertex?

t = half the total time

$d = \frac{1}{2}gt^2$

$d = \frac{1}{2}(32)(1.43)^2$

$d = 32.7$ ft.

Homework: Write down thoughts or suggestions for how you plan to find the quadratic function that describes your ball flight.

see journal

I will use the vertex to help find "a" in the vertex form. I am not sure about the system of equations. I will start constructing a system using my vertex and x-intercepts and try to solve for a, b, and c in standard form.

Student Activity 2: Finding the Function

Directions: Find the function that represents the path of the ball in two ways. Make a table of values for the function found using each method to determine the height of the ball in .25 second increments.

Coordinates of Vertex of ball flight (round second to nearest tenth and height to nearest whole number): (1.4, 33)

x-intercepts of ball flight: (0,0) (2.8, 0)

Method 1 System of Equations

$ax^2 + bx + c = y$

$a(1.4)^2 + b(1.4) + c = 33$

$a(0)^2 + b(0) + c = 0$ $\Rightarrow$

$a(2.8)^2 + b(2.8) + c = 0$

$1.96a + 1.4b + c = 33$

$c = 0$

$7.84a + 2.8b + c = 0$

$7.84a + 2.8b = 0$

$2(1.96a + 1.4b = 33)$ $\Rightarrow$

$7.84a + 2.8b = 0$

$3.92a + 2.8b = 66$

$3.92a = -66$

$a = -16.84$

$1.96(-16.84) + 1.4b = 33$

$b = 47.14$

t	h(t)
0	0
.25	10.7
.5	19.4
.75	25.9
1	30.3
1.25	32.6
1.5	32.8
1.75	30.9
2	26.9

Height as a function of time for tennis ball:

$$h(t) = -16.84t^2 + 47.14t$$

Method 2: Vertex Form

$$y = a(x-h)^2 + k, where\ (h,k) is\ the\ vertex\ of\ the\ parabola$$

Vertex (1.4, 33)

x-intercepts : (2.8, 0)(0, 0)

$y = a(x-1.4)^2 + 33$

$0 = a(0-1.4)^2 + 33$

$0 = 1.96a + 33$

$-33 = 1.96a$

$-16.84 = a$

t	h(t)
0	0
.25	10.7
.5	19.4
.75	25.9
1	30.3
1.25	32.6
1.5	32.8
1.75	30.9
2	26.9

Height as a function of time for tennis ball:

$$h(t) = -16.84(t-1.4)^2 + 33$$

Do you notice any difference in the tables of values for your two functions? No difference

What could explain these differences? I think rounding could make a difference if one was observed.

Do you think the two functions are equal? If so, show your work.

$-16.84t^2 + 47.14t = -16.84(t-1.4)^2 + 33$

$= -16.84(t^2 - 2.8t + 1.96) + 33$

$= -16.84t^2 + 47.152t - 33 + 33$

$= -16.84t^2 + 47.152t$

9 Very close