$$x^2 > 7 \longleftarrow \{x \mid x^2 > 7\}$$

$$x^3 - x = 3 \longleftarrow \cap\ \{x \mid x^3 - x = 3\}$$

intersect $\cap$ union $\cup$ and $\wedge$ or $\vee$

$X = Y$ iff $X \subseteq Y$, $Y \subseteq X$

Review

$A = \{1, 2, 3\}$ $B = \{a, b\}$

$\langle 1, b \rangle \in A \times B$

$A \times B = \{ \langle 1, a \rangle, \overbrace{\langle 1, b \rangle}^{\text{ordered pair}}, \ldots$
$\langle 2, a \rangle, \ldots \}$

$\underline{\text{People} \times \text{Actors}} \ni \langle \text{Ian}, \text{CD} \rangle$

$\not\ni \langle \text{CD}, \underline{\text{Ian}} \rangle$

Set Representations:

{5, 7, 9}

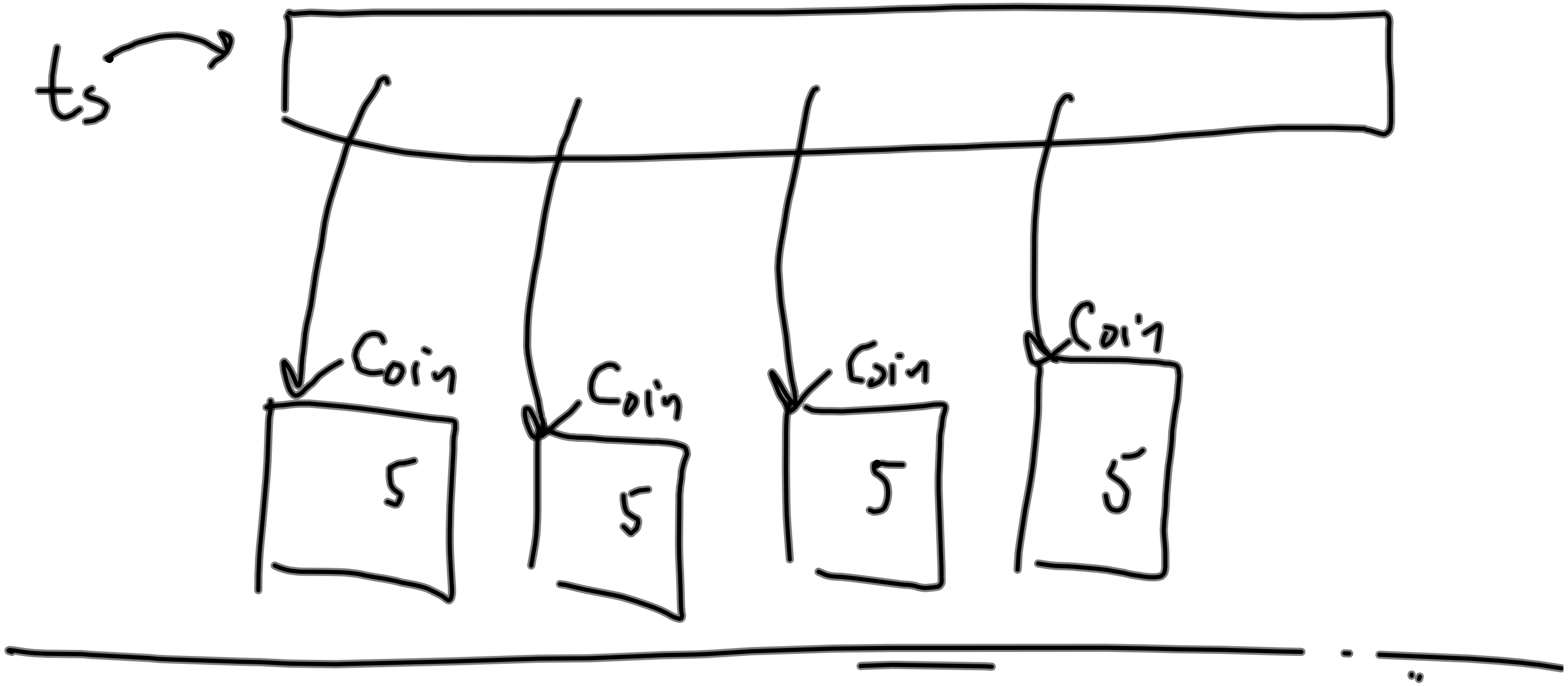
- List

```
Set union(Set a, Set b)
  Set r = new List();
  r.addAll(a)
  r.addAll(b)
  return r;
```

- HashSet

```
put(5, true)
get(5)
```

```
List ts;
 ts = new List();
 for i = 1 to 1000
       ts.add( new Coin(5)   )
```



"1111101111111111111111"

the j^{th} char is '1' iff $coin_j$ is in the set.

$A = 1101101$

$B = 1000111$

$A \cup B = 1101111$ $A \mid B$

$A \cap B = 1000101$ $A \& B$

$$M_5 = \{0, 5, 10, 15, \dots\}$$

$$= \{m \mid \exists k \in \mathbb{N}.\ m = 5k\}$$

$$M_7 = \{0, 7, 14, 21, \dots\}$$

$$M_2 \cup M_3 \cup M_5 \cup M_7 \cup M_{11} \cup \dots$$

$$= \boxed{\bigcup_{i \in \text{primes}} M_i} = \mathbb{N} - \{1\}$$

$$\bigcup_{p \in PPG} \{p.\mathrm{faveColor}()\}$$

---

boolean f(String s, int n)

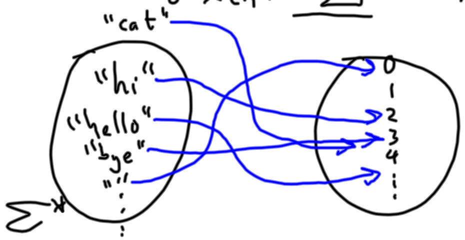
$$\text{sqrt} : \underbrace{\mathbb{R} \to \mathbb{R}}$$

$$\text{sqrt} \in (\mathbb{R} \to \mathbb{R})$$

$$\text{add1} \in (\underline{\mathbb{R}} \to \underline{\mathbb{R}})$$

A function is a mapping from one set (domain) to another (codomain).

Ex: strlen: $\Sigma^* \to \mathbb{N}$



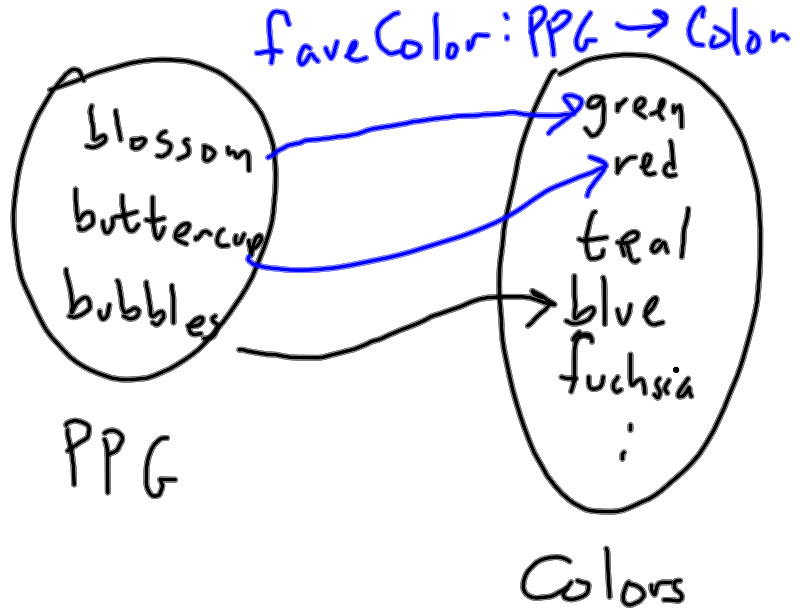
If something in codomain is mapped to by two different things in domain f is many-to-one.

Def'n: $f: A \to B$ is many-to-one iff $\exists b \in B. (\exists a_1 \exists a_2 . a_1 \neq a_2 \wedge a_1 \in A \wedge a_2 \in A \wedge f(a_1) = b \wedge f(a_2) = b)$

$\exists b \in B. \exists a_1 \in A \exists a_2 \in A . (a_1 \neq a_2 \wedge f(a_1) = b \wedge f(a_2) = b)$

Def'n: $f: A \to B$ is 1-to-1

$\forall a_1 \in A . \forall a_2 \in A . ((f(a_1) = f(a_2)) \to (a_1 = a_2))$



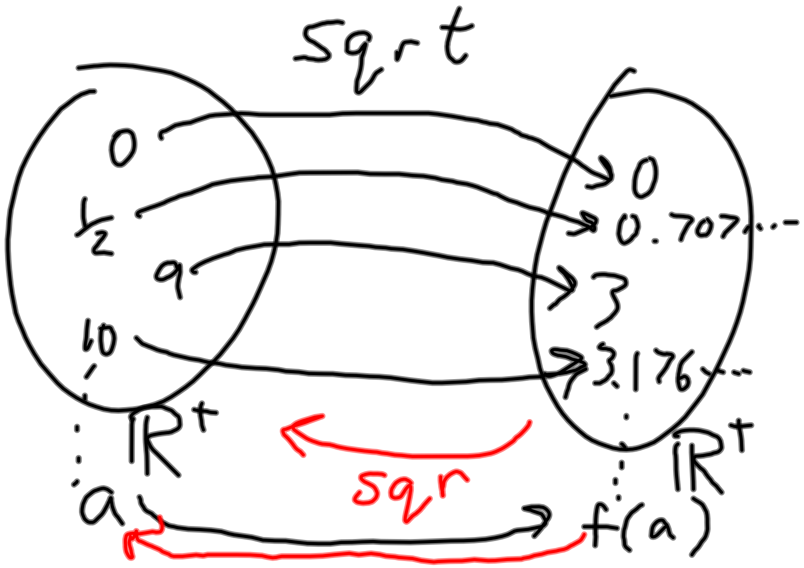
"faveColor is not onto"

Onto: "Everything b in co-domain is mapped to by something a in domain"

Def'n: $f : A \to B$ is onto iff

$$\forall b \in B. \exists a \in A. f(a) = b$$

Def'n: For $f: A \to B$,
$g: B \to A$ is an left inverse
of f if $\forall a \in A.\ g(f(a)) = a$



Th'm: f has a (left and right)
inverse iff f is onto and one-to-one.

Def'n

$f: A \to A$ is idempotent if

$$\forall a.\ f(f(a)) = f(a)$$

- upcase
- installer

New functions from old:

- $(f+g)(x)$ where $f(x) = x^2$, $g(x) = x+3$
  $= f(x)+g(x)$
- $(f*g)(x) = f(x)*g(x)$

Compose - $(f \circ g)(x) = f(g(x))$

Ex:
celcToFahr
kelvToCelc
kelvToFahr

celcToF $\circ$ kelvToCelc

Q: prove or disprove:
$f+g = g+f$

$= \forall x,\ (f+g)(x) = f(x)+g(x)$ (by def'n of f+g)
$= g(x)+f(x)$ + commutes
$= (g+f)(x)$ by def'n of f+g

Q: prove or disprove:
$\forall f,g$ $\quad f \circ g = g \circ f$

Counter example:
Take $f(x) = x+1$
$g(x) = 2x$.

$(f \circ g)(x) = 2x+1$

$(g \circ f)(x) = 2(x+1) = 2x+2$

$(f \circ g)(7) = 15$
$\neq 16 = (g \circ f)(7)$

Prove or disprove:

$\exists x,y \in \mathbb{R}$

x is irrational,

$\wedge$ $y \neq a$ is rational,

$\wedge$ $x \cdot y$ is rational.

$\neg \exists x. P(x)$

$= \forall x. \neg P(x)$

What is $\mathrm{sqrt}^*(\{4,16,2\})$

$= \{\mathrm{sqrt}(a) \mid a \in \{4,16,2\}\} = \{\sqrt{2}, 4, 2\}$

"map sqrt onto the set"

If $f : A \to B$

Then let $f^* : P(A) \to P(B)$

$f^*(S_A) = \{f(a) \mid a \in S_A\}$

such that

$\{x \mid x \text{ is prime}\}$

$* : P((A \to B)) \to P((P(A) \to P(B)))$