Section 2.1

Secant Lines and Tangent Lines
Example 4 (Page 97 #2)

A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent lines represents the heart rate in beats per minute.

<table>
<thead>
<tr>
<th>t (min)</th>
<th>36</th>
<th>38</th>
<th>40</th>
<th>42</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heartbeats</td>
<td>2530</td>
<td>2661</td>
<td>2806</td>
<td>2948</td>
<td>3080</td>
</tr>
</tbody>
</table>

Use the data to estimate the patient’s heart rate after 42 minutes using the secant line between the points with the given values of t.

a) $t = 36$ and $t = 42$

$$m = \frac{2948 - 2530}{42 - 36} = \frac{418}{6} = 69.7$$

b) $t = 38$ and $t = 42$

$$m = \frac{2948 - 2661}{42 - 38} = \frac{287}{4} = 71.75$$

c) $t = 40$ and $t = 42$

$$m = \frac{2948 - 2806}{42 - 40} = \frac{142}{2} = 71$$

d) $t = 42$ and $t = 44$

$$m = \frac{3080 - 2948}{44 - 42} = \frac{142}{2} = 71$$
Example 5

The point P(3,1) lies on the curve $y = \sqrt{x - 2}$

a) If Q is the point $(x, \sqrt{x - 2})$, use your calculator the slope of the secant line PQ for the following values of x:

i) 2.5

$y = \sqrt{x - 2} = \sqrt{2.5 - 2} = \sqrt{5} = .707107 \Rightarrow (2.5,.707107)$

$m = \frac{1-.707107}{3 - 2.5} = \frac{.292893}{.5} = 585786$

ii) 2.9

$y = \sqrt{x - 2} = \sqrt{2.9 - 2} = \sqrt{.9} = .948683 \Rightarrow (2.9,.948683)$

$m = \frac{1-.948683}{3 - 2.9} = \frac{.051317}{.1} = .51317$
iii) 2.99
\[ y = \sqrt{x - 2} = \sqrt{2.99 - 2} = \sqrt{.99} = .994987 \Rightarrow (2.9, .994987) \]
\[ m = \frac{1 - .994987}{3 - 2.99} = .5013 \]

iv) 2.999
\[ y = \sqrt{x - 2} = \sqrt{2.999 - 2} = \sqrt{.999} = .999500 \Rightarrow (2.9, .999500) \]
\[ m = \frac{1 - .9995}{3 - 2.99} = .5 \]

v) 3.5
\[ y = \sqrt{x - 2} = \sqrt{3.5 - 2} = \sqrt{1.5} = 1.22474 \Rightarrow (3.5, 1.22474) \]
\[ m = \frac{1.22474 - 1}{3.5 - 3} = \frac{.22474}{.5} = .44948 \]

vi) 3.1
\[ y = \sqrt{x - 2} = \sqrt{3.1 - 2} = \sqrt{1.1} = 1.048809 \Rightarrow (3.1, 1.048809) \]
\[ m = \frac{1.048809 - 1}{3.1 - 3} = \frac{.048809}{.1} = .48809 \]

vii) 3.01
\[ y = \sqrt{x - 2} = \sqrt{3.01 - 2} = \sqrt{1.01} = 1.004988 \Rightarrow (3.01, 1.004988) \]
\[ m = \frac{1.004988 - 1}{3.01 - 3} = .4988 \]

viii) 3.001
\[ y = \sqrt{x - 2} = \sqrt{3.001 - 2} = \sqrt{1.001} = 1.000500 \Rightarrow (3.001, 1.000500) \]
\[ m = \frac{1.000500}{3.001} = .5 \]

b) Using the values of part a), guess the value of the slope of the tangent line to the curve of \( P(3,1) \)
\[ m = \frac{1}{2} \]
c) Using the slope from part b), find the equation of the line passing \( P(3,1) \)

\[
y - y_i = m(x - x_i)
\]

\[
y - 1 = \frac{1}{2}(x - 3)
\]

\[
y - 1 = \frac{1}{2}x + \frac{3}{2}
\]

\[
y = \frac{1}{2}x + \frac{3}{2}
\]

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**Example 6**

If an arrow is shot upward on the moon with a velocity of \( 55 \frac{m}{s} \) its velocity is given by the model \( h = 55t - .83t^2 \)

a) Find the average velocity over the given time intervals

\( i) [1,2] \)

\[
t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17
\]

\[
t = 2: h = 55(2) - .83(2)^2 = 110 - 3.32 = 106.68
\]

\[
m = \frac{106.68 - 54.17}{2-1} = \frac{52.51}{1} = 52.51
\]

\( ii) [1,1.5] \)

\[
t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17
\]

\[
t = 1.5: h = 55(1.5) - .83(1.5)^2 = 82.5 - 1.87 = 80.63
\]

\[
m = \frac{80.63 - 54.17}{1.5-1} = \frac{26.46}{.5} = 52.92
\]

\( iii) [1,1.1] \)

\[
t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17
\]

\[
t = 1.1: h = 55(1.1) - .83(1.1)^2 = 59.50
\]

\[
m = \frac{59.50 - 54.17}{1.1-1} = \frac{5.33}{.1} = 53.3
\]