Surface Area

The surface area of a polyhedron is sum of the areas of its lateral faces.

Example 1

\[ SA = 2lw + 2hl + 2wh \]
\[ SA = 2(5 \text{ cm})(3 \text{ cm}) + 2(3 \text{ cm})(10 \text{ cm}) + 2(5 \text{ cm})(10 \text{ cm}) \]
\[ SA = 30 \text{ cm}^2 + 60 \text{ cm}^2 + 100 \text{ cm}^2 \]
\[ SA = 180 \text{ cm}^2 \]

Example 2
Area of Top and Bottom = \( \pi r^2 = \pi (1 \text{ ft}) = \pi \text{ ft}^2 \)

Area of Rectangular Side = \( 2\pi rh = 2\pi (1 \text{ ft})(4 \text{ ft}) = 8\pi \text{ ft}^2 \)

\( SA = 2\pi \text{ ft}^2 + 8\pi \text{ ft}^2 = 10\pi \text{ ft}^2 \)

**Example 3**

Find the volume and surface area

Surface Area:

Hypotenuse

\[ c^2 = 10^2 + 15^2 \]
\[ c^2 = 100 + 225 \]
\[ c^2 = 325 \]
\[ c = \sqrt{325} \approx 18 \text{ ft} \]
Triangular Prism:
\[ A = 2 \left( \frac{1}{2}bh \right) + 2lw = (30 \text{ ft})(10 \text{ ft}) + 2(18 \text{ ft})(40 \text{ ft}) = 300 \text{ ft}^2 + 1440 \text{ ft}^2 = 1740 \text{ ft}^2 \]

Rectangular Prism

\[ A_1 = 2h + 2lw + lw \]
\[ A_2 = 2(30 \text{ ft})(14 \text{ ft}) + 2(14 \text{ ft})(40 \text{ ft}) + (30 \text{ ft})(40 \text{ ft}) = 840 \text{ ft}^2 + 1120 \text{ ft}^2 + 1200 \text{ ft}^2 = 3160 \text{ ft}^3 \]
\[ SA = 1740 \text{ ft}^2 + 3160 \text{ ft}^2 = 4900 \text{ ft}^2 \]

Example 4

Find the surface area and volume.

Surface Area

Radius: \[ r = \frac{25 \text{ ft}}{2} = 12.5 \text{ ft} \]

\[ SA = 2 \left( \frac{1}{2} \pi r^2 \right) + \frac{1}{2} (\pi d)l + lw = \pi r^2 + \frac{1}{2} \pi dl = \pi (12.5)^2 + \frac{1}{2} \pi (25)(35) + 25 \cdot 35 \]
\[ = 490.625 + 1373.75 + 875 \]
\[ = 2739.375 \text{ ft}^2 \]
Example 5
Find the surface area of the polyhedron below:

First find the interior angle of the pentagon: 
\[ I = \frac{(5 - 2)180^\circ}{5} = \frac{3(180^\circ)}{5} = \frac{540^\circ}{5} = 108^\circ \]

Now find the interior angle of the triangle: 
\[ \frac{108^\circ}{2} = 54^\circ \]

Next, find the height of the triangle:
\[ \tan 54^\circ = \frac{h}{3} \]
\[ h = 3 \cdot \tan 45^\circ \]
\[ h \approx 4.1 \text{ cm} \]

To find the area of the polygon find the area of the triangle and multiply by 5. (Recall the pentagon is divided into 5 triangles)
\[ A = 5 \left( \frac{1}{2} bh \right) = 5 \left( \frac{1}{2} (3 \text{ cm})(4.1 \text{ cm}) \right) = 30.75 \text{ cm}^2 \]
Now, find the surface area of the polyhedron:

\[ SA = 5(\text{Area of Triangle}) + \text{Area of Pentagon} \]

\[ SA = 5 \left( \frac{1}{2} (6 \text{ cm})(9 \text{ cm}) \right) + 30.75 \text{ cm}^2 \]

\[ SA = 130 \text{ cm}^2 + 30.75 \text{ cm}^2 \]

\[ SA = 160.75 \text{ cm}^2 \]

**Example 6**

How much postal wrap is needed to wrap the rectangular solid package?

\[ SA = 2hw + 2hw + 2lh \]

\[ SA = 2(10)(12) + 2(4)(10) + 2(12)(4) \]

\[ SA = 240 \text{ cm}^2 + 80 \text{ cm}^2 + 96 \text{ cm}^2 \]

\[ SA = 416 \text{ cm}^2 \]