Confidence Intervals in a Nutshell

• If we are dealing with a random variable \( x \) for which the population standard deviation \( \sigma_x \) is known and we wish to estimate the population mean \( \mu \) using sample information from a sample of size \( n \), a confidence interval for \( \mu \) has the form

\[
\bar{x} \pm Z \left( \frac{\sigma_x}{\sqrt{n}} \right)
\]

Here, \( Z \) corresponds to the level of confidence; it’s value is determined using the standard normal \( z \)-table, or the normalcdf command on the TI–83, or easiest of all - the \texttt{zinterval} command.

• If we are dealing with a random variable \( x \) for which the population standard deviation \( \sigma_x \) is not known and we must estimate it using \( S \) and we wish to estimate the population mean \( \mu \) using sample information from a sample of size \( n \), a confidence interval for \( \mu \) has the form

\[
\bar{x} \pm T \left( \frac{S}{\sqrt{n}} \right)
\]

Here, \( T \) corresponds to the level of confidence; it’s value is determined using the Student \( t \)-table (Table 6), or more easily by using the \texttt{tinterval} command.

Examples

Problem 1a

A person would like to estimate the mean number \( \mu \) of cars \( x \) making legal right turns on a red light at a particular intersection. The person observes \( n = 25 \) red light sequences and counts the number of legal right turns. The mean for these 25 sequences is \( \bar{x} = 7.3 \). From past experience the person knows that \( \sigma_x = 1.8 \). Construct a 98% confidence for \( \mu \).

Solution:

We know \( \sigma_x \) so we use the \( z \)-distribution:

\[
z = \frac{\bar{x} - \mu_x}{\frac{\sigma_x}{\sqrt{n}}}.
\]
We have $x = 7.3, \sigma_x = 1.8$, and $n = 25$. The confidence interval is then

$$x \pm Z_{.9900} \left( \frac{\sigma_x}{\sqrt{n}} \right) =$$

$$7.3 \pm 2.33 \left( \frac{1.8}{5} \right) =$$

$$7.3 \pm 0.84 =$$

$$(6.46, 8.14).$$

The $Z = 2.33$ came either from the standard normal table: $p(z \leq 2.33) = .9900$ or from using the TI–83: \texttt{invnorm(.9900,0,1)} = 2.33.

**Conclusion:**

We are 98% confident based on this sample that the population mean $\mu_x$ is between 6.46 and 8.14. ($\mu_x$ will lie in 98% of all confidence intervals constructed in this fashion.)

**TI–83 Solution:**

- \texttt{STATS–TESTS–7 (Z confidence interval)}
- Highlight and click \texttt{STATS}.
- Enter the values of $\sigma_x, \bar{x}, n$, and \texttt{C–Level}.
- Highlight and click \texttt{Calculate}.

**Note:**

Please, please, please do several problems using both the first way to make sure you understand the different parts that go into making up a confidence interval.

**Problem 1b**

Same as problem 1a. However, assume the person doesn’t know $\sigma_x^2$ and has only the sample estimate $S = 1.8$.

**Solution:**

We don’t know $\sigma_x$ so we use the $t$-distribution:

$$t = \frac{\bar{x} - \mu_x}{S} \sqrt{n}.$$
We have \( \bar{x} = 7.3, S = 1.8, n = 25 \). The confidence interval is then

\[
\bar{x} \pm T_{0.9900} \left( \frac{S}{\sqrt{n}} \right) = \\
7.3 \pm 2.49 \left( \frac{1.8}{5} \right) = \\
7.3 \pm 0.9 = \\
(6.40, 8.20).
\]

The \( T = 2.49 \) came from the Student–t table (Table 6), row 24 (since \( df=n-1=24 \)), (red) column .01.

**Conclusion:**
We are 98% confident based on this sample that the population mean \( \mu_x \) is between 6.40 and 8.20. (\( \mu_x \) will lie in 98% of all confidence intervals constructed in this fashion.)

**TI–83 Solution:**

- **STATS–TESTS–8** (T confidence interval)
- Highlight and click **STATS**.
- Enter the values of \( S, \bar{x}, n, \) and C–Level.
- Highlight and click **Calculate**.

Note (again):

Please, please, please do several problems using both the first way to make sure you understand the different parts that go into making up a confidence interval.

**Problem 2a**

In a time and motion study the amount of time required to paint a room is measured for 15 professional painters. The calculated sample mean is \( \bar{x} = 73 \) minutes. Previous studies indicate the population standard deviation is 8 minutes. Construct a 95% confidence interval for the mean time to paint the room for all professional painters. *See if you can do this one as in Problem 1a.*

**Problem 2b**

Same as problem 2a. However, assume that the population standard deviation is not known and that the sample standard deviation for the 15 painters is \( S = 8 \). *See if you can do this one as in Problem 1b.*

**Problem 3**
The mean monthly sales for agents in a large insurance company in the past was $72,000. In an attempt to improve sales, a new training program was developed. 10 agents were selected to participate in the program. At its completion, the sales were monitored for these 10 agents. Their monthly sales (in thousands of dollars) were:

\[63, 87, 95, 75, 83, 78, 69, 79, 103, 98\]

Use this sample to construct a 95% confidence interval for agents completing the new training program.

**TI–83 Solution:**

- Enter the data in list **L1**.
- **STATS–TESTS–8** (T confidence interval)
- Highlight and click **DATA**.
- Enter the value of **C–Level**.
- Highlight and click **Calculate**.

- We obtain the sample statistics \( \bar{x} = 83, S = 12.85, \text{ and } n = 10 \) as well as the desired confidence interval (73.8, 92.2).

**Question:**

Suppose someone asked you to use this data to test their “hypothesis” (theory) that the mean sales for all reps completing this program is equal to $98,000. What’s your answer to them? Same question except that someone thinks that the mean sales for all reps completing this program is equal to $80,000. Next up: Hypothesis Testing.