Confidence Intervals in a Nutshell

- If we are dealing with a random variable $x$ for which the population standard deviation $\sigma_x$ is known and we wish to estimate the population mean $\mu$ using sample information from a sample of size $n$, a confidence interval for $\mu$ has the form

  \[ \bar{x} \pm Z \left( \frac{\sigma_x}{\sqrt{n}} \right) \]

  Here, $Z$ corresponds to the level of confidence; it’s value is determined using the standard normal $z$-table or the normalcdf command on the TI-83.

- If we are dealing with a random variable $x$ for which the population standard deviation $\sigma_x$ is not known and we must estimate it using $S$ and we wish to estimate the population mean $\mu$ using sample information from a sample of size $n$, a confidence interval for $\mu$ has the form

  \[ \bar{x} \pm T \left( \frac{S}{\sqrt{n}} \right) \]

  Here, $T$ corresponds to the level of confidence; it’s value is determined using the Student $t$-table (Table 6).
Examples

Problem 1a
A person would like to estimate the mean number \( \mu \) of cars \( x \) making legal right turns on a red light at a particular intersection. The person observes \( n = 25 \) red light sequences and counts the number of legal right turns. The mean for these 25 sequences is \( \bar{x} = 7.3 \). From past experience the person knows that \( \sigma_x = 1.8 \). Construct a 98% confidence for \( \mu \).

Problem 1b
Same as problem 1a. However, assume the person doesn’t know \( \sigma_x \) and has only the sample estimate \( S = 1.8 \).

Problem 2a
In a time and motion study the amount of time required to paint a room is measured for 15 professional painters. The calculated sample mean is \( \bar{x} = 73 \) minutes. Previous studies indicate the population standard deviation is 8 minutes. Construct a 95% confidence interval for the mean time to paint the room for all professional painters.

Problem 2b
Same as problem 2a. However, assume that the population standard deviation is not known and that the sample standard deviation for the 15 painters is \( S = 8 \).

Problem 3
The mean monthly sales for agents in a large insurance company in the past was $72,000. In an attempt to improve sales, a new training program was developed. 10 agents were selected to participate in the program. At its completion, the sales were monitored for these 10 agents. Their monthly sales (in thousands of dollars) were:

\[
63, 87, 95, 75, 83, 78, 69, 79, 103, 98
\]

Use this sample to construct a 95% confidence interval for agents completing the new training program.