You must show your work on all questions to qualify for credit. On multiple-choice questions, be sure to choose the letter corresponding to your answer now; I will not change your grade later if you have worked the problem correctly but chosen the wrong letter.

1.) What is the connective in the statement “Today is Tuesday or this is Belgium”?

or

2.) What is the connective in the statement “Jack likes snakes, but Jill doesn’t”?

but = and

3.) What is the connective in the statement “If all else fails, read the instructions”?

if...then...

4.) Let $p$ be the statement “Love is a thing that can never go wrong,” and let $q$ be “I am Marie of Rumania.” Then the statement “Love is a thing that can never go wrong, and I am Marie of Rumania” is symbolized by $p \land q$.

5.) Let $p$ be the statement “Love is a thing that can never go wrong,” and let $q$ be “I am Marie of Rumania.” Then the statement “Love is a thing that can never go wrong, or I am Marie of Rumania” is symbolized by $p \lor q$.

6.) Let $p$ be the statement “Love is a thing that can never go wrong,” and let $q$ be “I am Marie of Rumania.” Then $\neg p \rightarrow \neg q$ expresses which of the following statements?

If love is not a thing that can never go wrong, then I am not Marie of Rumania.

7.) Let $p$ be the statement “Love is a thing that can never go wrong,” and let $q$ be “I am Marie of Rumania.” Then $p \land q$ expresses which of the following statements?

Love is not a thing that can never go wrong, and I am Marie of Rumania.

8.) Which of the following statements is the contrapositive of the statement “If I don’t go, I won’t get there”?

Recall that the contrapositive of $X \rightarrow Y$ is $\sim Y \rightarrow \sim X$. So it’s:

If I get there, then I went.
9.) Which of the following statements is equivalent to the negation of the statement “If something can go wrong, it will”?

Remember that the negation of \( X \rightarrow Y \) is \( X \land \sim Y \). So it’s:

Something can go wrong, but it won’t.

10.) Make a truth table for the statement \((p \land \sim q) \rightarrow \sim p\).

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11.) State DeMorgan’s Laws.

\[
\sim (p \land q) \equiv p \lor \sim q \\
\sim (p \lor q) \equiv p \land \sim q
\]
12.) Make the truth tables for $\sim (p \rightarrow q)$ and $\sim p \lor q$. What is the logical relation between the two?

$$\begin{array}{c|c|c|c} p & q & p \rightarrow q & \sim (p \rightarrow q) \\ \hline T & T & T & F \\ T & F & F & T \\ F & T & T & F \\ F & F & T & F \\ \end{array}$$

They are negations (each is true when the other is false, false when the other is true).

$$\begin{array}{c|c|c|c|c|c} p & q & p \rightarrow q & \sim (p \rightarrow q) \\ \hline T & T & F & T \\ T & F & F & F \\ F & T & T & T \\ F & F & T & T \\ \end{array}$$

13.) Use a truth table to determine whether the statement $[\sim (p \land (p \lor q)) \rightarrow q]$ is a tautology.

$$\begin{array}{c|c|c|c|c|c|c} p & q & \sim p & p \lor q & \sim p \land (p \lor q) & q & [\sim p \land (p \lor q)] \rightarrow q \\ \hline T & T & F & T & F & T & T \\ T & F & F & T & F & F & T \\ F & T & T & T & T & T & T \\ F & F & T & F & F & F & T \\ \end{array}$$

Yes, it is a tautology (all T’s in last column).

14.) Which of the following statements is the contrapositive of the statement “If you pass this course, then I’m Marie of Rumania?”

If I’m not Marie of Rumania, then you won’t pass this course.
15.) Use a truth table to determine whether the following argument is valid.

\[
\begin{align*}
p \rightarrow q \\
q \lor r \\
p \lor r
\end{align*}
\]

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In the sixth line of the truth table each premise is True, while the conclusion is False, so the argument is invalid.
16.) Determine whether \( p \lor \sim q \) is logically equivalent to \( \sim (p \land \sim q) \).

We make a truth table for each and compare the last columns.

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They are *not* equivalent: the last columns of their truth tables are not identical.

17.) Which of the following is the negation of “It is winter and it is not cold”?

Use DeMorgan’s Law to get:

Either it isn’t winter, or it is cold.

18.) Which of the following is the negation of “Either John likes Mary, or Ellen is wrong”?

Another DeMorgan’s Law question; it’s:

John doesn’t like Mary, and Ellen is right.

19.) Which of the following is equivalent to the negation of “Now is the winter of our discontent made glorious summer by this sun of York; and all the clouds that lour’d upon our house in the deep bosom of the ocean buried”?

Yet another DeMorgan's Law question. Answer:

Now is the winter of our discontent not made glorious summer by this sun of York; or, not all the clouds that lour’d upon our house are buried in the deep bosom of the ocean.
20.) Find the contrapositive of \( \sim (p \lor \sim q) \rightarrow r \).
\[
\sim r \rightarrow (p \lor \sim q)
\]

21.) Symbolize the following argument and use a truth table to determine its validity:

Either Tom is wrong, or Mary is in trouble.
If Mary is not in trouble, then there’s no reason to worry.
There’s reason to worry.

\[
\therefore \text{Tom isn’t wrong.}
\]

We symbolize it thus:

\[
\begin{align*}
& w \lor t \\
& \sim t \rightarrow \sim r \\
& r \\
& \sim w
\end{align*}
\]

In the first line of the truth table each premise is True, while the conclusion is False, so the argument is invalid.
22.) Use an Euler diagram to determine the validity of the following argument:

All math majors are nerds.
Ruth is a math major.

∴ Ruth is a nerd.

\[ \begin{align*}
\text{N = nerds} \\
\text{M = math majors} \\
\text{R = Ruth} \\
\text{VALID}
\end{align*} \]

23.) Use an Euler diagram to determine the validity of the following argument:

Some people who study do well.
Some people who are smart do well.

∴ Some people who study are smart.

\[ \begin{align*}
\text{S = people who study} \\
\text{W = people who do well} \\
\text{T = people who are smart} \\
\text{INVALID}
\end{align*} \]

24.) Suppose that \( p \) is true but that both \( q \) and \( r \) are false. Which of the following must be true?

Make a one-line truth table for each expression. Here's the one for the correct answer:

\[ \begin{array}{cccc}
p & q & r & (q \land r) \rightarrow p \\
\hline
T & F & F & T
\end{array} \]

25.) Which of the following is the negation of “Some cats have no tails”?

The negation of “Some a’s are not b’s” is “All a’s are b’s”, so it’s:

All cats have tails.