Solutions to Practice Problems for Test 3

Math 103
Spring 2002

You must show your work on all questions to qualify for credit. On multiple-choice questions, be sure to choose the letter corresponding to your answer now; I will not change your grade later if you have worked the problem correctly but chosen the wrong letter.

1.) How many subsets has the set \( S = \{ \text{Mary, John, Fred} \} \)?
   \[
   2^3 = 8 \text{ subsets}
   \]

2.) If three fair coins are tossed and two fair dice are thrown, how many outcomes are possible?
   \[
   2 \cdot 2 \cdot 2 \cdot 6 \cdot 6 = 288
   \]

3.) Social Security numbers have nine digits. How many social security numbers are possible?
   \[
   10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^9
   \]

4.) An urn contains three red balls and five black balls. The red balls are numbered 1 through 3, and the black balls are numbered 1 through 5. If I reach in and draw out two balls, in how many different ways could I get one of each color?
   \[
   \text{(number of ways to get one red) \cdot (number of ways to get one black)} = 3 \cdot 5 = 15
   \]

5.) How many three-digit numbers can be written using the digits 0, 1, 2, 3, 4, and 5? (Repetitions are allowed, but the first digit cannot be 0.)
   choose a first digit: 5 ways (can’t choose 0)
   choose a second digit: 6 ways
   choose a third digit: 6 ways
   total: \( 5 \cdot 6 \cdot 6 = 180 \) ways
6.) How many of the three-digit numbers that can be written using the digits 0, 1, 2, 3, 4, and 5
(where repetitions are allowed, but the first digit cannot be 0) are multiples of 5? [Hint: what
does a number have to end with to be a multiple of 5?]

A number has to end with a 0 or a 5 to be a multiple of 5, so it’s:

choose a first digit: 5 ways
choose a second digit: 6 ways
choose a third digit: 2 ways

total: $5 \cdot 6 \cdot 2 = 60$ ways

7.) Seven people are waiting to play on a tennis court. The recreation director will call out
their names two at a time. In each pair, the first person called will serve to the second
person called. How many different choices does the director have when calling the first two
players?

The order in which they’re called matters, so it’s $7 \cdot 6 = 42$.

8.) How many distinguishable permutations are there of the letters in the word
PSYCHOPOESIS?

$$\frac{12!}{2! \cdot 3! \cdot 2!}$$

9.) A committee of fourteen people wishes to select a subcommittee of four. In how many
different ways can this be done?

The order in which the subcommittee members is chosen doesn’t matter, so it’s $\binom{14}{4}$.

10.) A committee of six men and eight women wishes to select a subcommittee of four. In how
many different ways can this be done if the subcommittee must have at least two women
on it?

As usual, take care of any special requirements first. Note that after the two women who
must be on the subcommittee are chosen, we can choose the other two members from among
all the men and women remaining.

choose two women: $C(8, 2)$ ways
choose two other members: $C(12, 2)$ ways

total: $C(8, 2)C(12, 2)$ ways

11.) In how many different ways can a five-card hand be dealt (from a standard deck) that
contains two aces, two kings, and one card less than a 5?

Note that there are twelve cards less than 5 (the 2’s, 3’s, and 4’s).

choose two aces: $C(4, 2)$ ways
choose two kings: $C(4, 2)$ ways
choose one card less than a 5: 12 ways

total: $C(4, 2)C(4, 2) \cdot 12$ ways
12.) Is it true that $C(n-1, r) = C(n, r-1)$ for any $n$ and $r$? Why, or why not?

This is nonsense. Try $n = 4, r = 3$.

13.) The nine starting players on the baseball team and the five starting players on the basketball team are to line up for a group photo, with all the members of the baseball team on the left. In how many ways can this be done?

choose an ordering for the basketball players: $5!$ ways
choose an ordering for the baseball players: $9!$ ways
total: $5!9!$ ways

14.) A promoter has thirty tickets to a show. In how many ways can he divide these equally among three valued clients?

We choose ten tickets for each client. Note that once we have chosen ten for Client #1, only twenty are left to choose from, etc.

choose ten tickets for Client #1: $C(30, 10)$ ways
choose ten tickets for Client #2: $C(20, 10)$ ways
choose ten tickets for Client #3: $C(10, 10)$ ways
total: $C(30, 10)C(20, 10)C(10, 10)$ ways

15.) How many different one-card hands (dealt from a standard deck) have either a king or a heart?

Let $K =$ {one-card hands consisting of one king} and $H =$ {one-card hands consisting of one heart}. We want $n(K \cup H)$:

$$n(K \cup H) = n(K) + n(H) - n(K \cap H)$$
$$= 4 + 13 - 1$$
$$= 16$$

Note: $K \cap H$ consists of exactly one hand, the one consisting of the king of hearts.

16.) How many different two-card hands (dealt from a standard deck) consist either of a pair of aces or of two red cards?

Let $A =$ {two-card hands consisting of a pair of aces} and $R =$ {two-card hands consisting of two red cards}. Note that $A \cap R$ contains exactly one two-card hand, the one consisting of the two red aces. Then

$$n(A \cup R) = n(A) + n(R) - n(A \cap R)$$
$$= \binom{4}{2} + \binom{26}{2} - 1$$
$$= 330$$

17.) If $A$ is a set and $n(A) = 25$, how many different 10-element subsets has $A$?

$$\binom{25}{10}$$
18.) Susie has four friends, to each of whom she wishes to give one book. She has ten different books to choose from. How many different selections can she make?

We assume that it matters which friend gets which book; then it matters which order she chooses the books, so the answer is $P(10, 4)$.

19.) Four phones are to be selected from among fifty to be checked for defects. In how many different ways can this be done?

The order in which the phones are selected doesn’t matter, so it’s $C(50, 4)$.

20.) License numbers consist of three letters followed by three digits. How many different license numbers are there in which at least one letter is repeated?

To get at this, subtract the number of license plates with no repeated letters from the total number of possible license plates:

$$26^3 \cdot 10^3 - P(26, 3) \cdot 10^3$$

21.) $C(9, 3) = \frac{9!}{(9 - 3)!3!} = \frac{9!}{6!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 84$

22.) A true-false test has thirty questions. If students need not answer questions, in how many ways can the answer sheet be filled in?

There are three ways to answer each question: T, F, or blank. Thus there are $3^{30}$ ways to fill out the entire answer sheet.