Section 3.8 Valid Arguments

An argument consists of a series of statements called premises followed by a statement called the conclusion. E.g.:

Socrates is a man.

All men are mortal.

Therefore, Socrates is mortal.

An argument is invalid if it is possible for all its premises to be true but its conclusion false. An argument is valid if it is not invalid.

Our method of determining validity is to make a truth table and look for a certain condition which means that the argument is invalid. The truth table must contain a column for each premise and a column for the conclusion. The condition to look for is a row in which all the premises are true but the conclusion is false. If such a row exists, the argument is invalid; if not, the argument is valid—even if there’s no row on which all the premises are true.

E.g., determine whether the following argument is valid:

\[ p \land q \]

\[ \sim p \rightarrow \sim q \]

\[ p \land \sim q \]
E.g., determine whether the following argument is valid:

If I buy a red car, the cops will stop me.

I don’t buy a red car. ______________________

The cops don’t stop me.

E.g., determine whether the following argument is valid:

Either the puppy is cute, or I will not buy it.

The puppy is cute. ______________________

I will buy it.
E.g., determine whether the following argument is valid:
If Tony is wealthy, then he is either intelligent or a good businessman.
Tony is intelligent but not a good businessman.
Tony is not wealthy.

E.g., determine whether the following argument is valid:
If John likes apples, Jane likes oranges.
If Jane doesn’t like cheese, then she doesn’t like oranges.
John likes apples.
Standard valid forms of argument

Modus ponens:
\[ p \rightarrow q \]
\[ p \]
\[ q \]

Modus tollens:
\[ p \rightarrow q \]
\[ \sim q \]
\[ \sim p \]

Hypothetical syllogism:
\[ p \rightarrow q \]
\[ q \rightarrow r \]
\[ p \rightarrow r \]

Disjunctive syllogism:
\[ p \lor q \]
\[ \sim p \]
\[ q \]

Finding a valid conclusion

E.g., find a valid conclusion that uses all of the following premises (Lewis Carroll):

Babies are illogical.

Nobody is despised who can manage a crocodile.

Illogical persons are despised.

To approach this, first translate the statements into symbols. Start with a simple statement if there is one. Use the premises in pairs, crossing them off as you use them. From the first pair of premises, draw a conclusion. From that conclusion and another premise, draw a second conclusion. Continue until all premises have been used.