Section 3.6 DeMorgan’s Laws and Equivalent Statements

The logic versions of DeMorgan’s Laws tell us how to negate conjunctions and disjunctions:

\[ \sim (p \land q) \equiv \sim p \lor \sim q \]

\[ \sim (p \lor q) \equiv \sim p \land \sim q \]

Truth tables prove these correct, but to understand them, consider an example. If \( p = \text{‘John is tall’} \) and \( q = \text{‘John is dark’} \), then \( p \lor q \) is true if and only if John is either tall or dark (or both). Thus it is false if and only if John is not tall and not dark, i.e., \( p \lor q \) is false if and only if \( \sim p \land \sim q \) is true; so the negation of \( p \lor q \) is \( \sim p \land \sim q \).

How the logic versions of DeMorgan’s Laws relate to the set versions: Suppose that \( T \) is the set of tall people and \( D \) is the set of dark people (where these are somehow precisely defined), and let \( p = \text{‘John is in } T \text{’} \) and \( q = \text{‘John is in } D \text{’} \). Then \( p \lor q = \text{‘John is in } T \text{ or in } D \text{ (or both)’} = \text{‘John is in } T \lor D \text{’} \). The negation of \( p \lor q \) then says ‘John is not in \( T \lor D \text{’}, or in other words ‘John is in \( (T \lor D)’ \). Applying the set versions of DeMorgan’s Laws, this last statement can be written ‘John is in \( T’ \cap \sim D’ \), or ‘John is in \( T’ \) and John is in \( D’ \), or finally ‘John is not in \( T \) and John is not in \( D \), which is the statement \( \sim p \land \sim q \).

E.g., negate (a) \( \sim p \land q \)

\( (b) \ p \land \sim q \)

E.g., negate (a) ‘It is not warm and I am freezing’
(b) ‘Either today is Tuesday or tomorrow is my birthday’

(c) ‘Either the door is open or the light is on’

(d) ‘Alice said that either Bob or Carol did it’

(e) Alice did not say that either Bob or Carol did it’. 