Section 2.2 Notation and Description

Definition of set

A set is a collection of distinct objects. Note that this means that sets do not have duplicate elements.

—Think of a basket that can hold anything, including other baskets.

E.g., the set of letters in the word zoo is \{z, o\}.

A set must be well-defined: that is, it must be possible to determine, for any object, whether or not it is in the set.

Well-defined: \{x \mid x > 1,000,000\}

Not well-defined: \{x \mid x \text{ is a large number}\}

Describing sets

Two ways to specify a set: listing and description

Listing: list all elements of the set, or list enough

E.g., \{1, 2, 3\}, \{1, 2, 3, \ldots\}

Description: specify a property; then all objects with that property are in the set. Use set-builder notation.

E.g., \{x \mid x \text{ is a brown dog with white spots}\}

E.g., \{x \mid 0 < x < 1\} (this one is too big to list)

An object in a set is called an element or member of that set. If the object \(x\) is in the set \(A\), we write \(x \in A\). If \(x\) is not in \(A\), we write \(x \notin A\).

E.g., 1 \(\in\) \{1, 2, \(\alpha\)\} but 3 \(\notin\) \{1, 2, \(\alpha\)\}.

Special sets

The set with no elements is called the empty set and is denoted \(\emptyset\) or sometimes just \{\}.

The set of all objects currently under discussion is called the universal set or universe and is denoted \(U\). Often this set is understood, but sometimes we must specify it.
Equality of sets

Two sets are said to be equal if they contain exactly the same elements. 

E.g., \( \{1, 2, 3\} = \{2, 3, 1\} \). (Note that the order of elements doesn’t matter.)

Finite and infinite sets

A set with a whole number of elements is called a finite set. If a set is not finite, it is infinite.

E.g., \( \{x \mid x \text{ is a labrador retriever}\} \) is a finite set; \( \{1, 2, 3, \ldots\} \) and \( \{x \mid x > 0\} \) are infinite.

E.g., Is the set of all water molecules that have ever been part of Earth’s oceans finite or infinite?

Number of elements in a set

If a set \( A \) is finite, then the number of elements in \( A \) is written \( n(A) \).

E.g., if \( A = \{1, 2, 7\} \), then \( n(A) = 3 \).

Two finite sets \( A \) and \( B \) are sometimes said to be equivalent if \( n(A) = n(B) \).