Study guide for test 1 of math 152.

1. Describe in steps how you will find the shortest distance between \( O = (0, 0) \) and \( f(x) = (x - 2)^2 + 1 \).

2. (a) Derive the Newton’s method \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \). (b) Explain why Newton’s method fails when applied to the equation \( \sqrt{x} = 0 \) with any initial approximation \( x_0 \) other than 0. Illustrate your explanation with a sketch.

3. Let \( f(x) = 3 \ln(5x + 8) \) be a function which is continuous and differentiable on the interval \([2, 4]\). According to the Mean Value Theorem, there exists (at least) a point \( c \in (2, 4) \) such that

\[
f'(c) = \frac{f(4) - f(2)}{4 - 2} = 0.6627491.
\]

Please find effectively such a \( c \) verifying this condition.

4. Find the following limits:
   
   a. \( \lim_{x \to \infty} x^4 e^{-x^2} \).
   
   b. \( \lim_{x \to 0} x^5 \).

5. Find the largest area of a rectangle in the first quadrant under the curve \( y = \sqrt{4 - x^2} \).

6. (Maple) Use Newton’s method to find the inflection point of the curve \( y = e^{\cos x}, 0 \leq x \leq \pi \), correct up to 6 decimal places.

7. Use Newton’s Rule to estimate the intersection of \( \ln(x) \) and \( e^{-3x} \) correct up to 6 decimal places.

8. Prove \( \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^2} \).

9. If the graph of \( f \) is given together with its first and second derivatives below. Label the graphs for \( f, f' \) and \( f'' \).

10. (Maple) Use Newton’s Rule to estimate the zero of \( f(x) = x - \cos x \) correct up to 6 decimal places.

11. Find the limit for the followings. Specify whenever L’ Hospital rule is used.

   a. \( \lim_{x \to 0} (\csc x - \cot x) \)

   b. \( \lim_{x \to 0} (\cos x)^{\frac{1}{x}} \).

   c. \( \lim_{x \to \infty} x^4 e^{-x^2} \).

12. (Maple) If \( f(x) = 2x \sin x \) and \( g(x) = \sec x - 1 \).

   a. Illustrate L’Hospital’s Rule by graphing both \( \frac{f(x)}{g(x)} \) and \( \frac{f'(x)}{g'(x)} \) near \( x = 0 \) to see that these ratios have the same limit as \( x \to 0 \).

   b. Use Maple "limit" command to confirm that \( \lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)} \).

13. Suppose the rate of change for the number \( N \) of bacteria in a culture after \( t \) days is modeled by the
following graph: [Note the following graph represents $y = N'(t)$].

![Graph Image]

a. Does the function $N$ have maximum or minimum during $t = 0$ and $t = 10$.

b. When is the rate of change of $N$ increasing the most?

c. Will the number of bacteria be stabilized eventually?

d. Approximate when $N$ will be increasing at its fastest.

14. Suppose you are given the graph of the marginal cost (derivative for cost) for a company. [The $x$ – axis represents the number of units in thousands, and the $y$ – axis represents the cost in millions]. And we assume the $x$ – intercepts are at $x = 1$, $3$ and $5$.

![Graph Image]

a. When will the company achieve its largest and lowest cost respectively?

b. When will the rate of cost increase the most during $x = 2$ and $x = 5$?

c. When will the rate of cost decrease the most during $x = 0$ and $x = 3$?

d. Explain how you will find where cost function increases the most and decreases the most.