Performance Based Learning and Assessment Task

Modeling Quadratics Basketball Video Project

I. **ASSESSMENT TASK OVERVIEW & PURPOSE:**
   Students will use their own video footage of a basketball going into a hoop to write quadratic equations in groups. Students will make inferences about the accuracy of the basketball shot using their equations.

II. **UNIT AUTHOR:**
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III. **COURSE:**
   Algebra 2

IV. **CONTENT STRAND:**
   Algebra

V. **OBJECTIVES:**
   Students will be able to: film a video of a quadratic real-world situation, use the video of half of a quadratic to write the equation of the situation, and use the video of half of a quadratic to predict the rest of the quadratic and solve a real-world problem.

VI. **REFERENCE/RESOURCE MATERIALS:**
   Modeling Quadratics Worksheet (attached), measuring paper hanging on wall, yard sticks or measuring tape, hoops (bendable glow sticks work well), balls (tennis balls work well), ipads/iphones, video editing app such as Splice (free)

VII. **PRIMARY ASSESSMENT STRATEGIES:**
   The task includes an assessment component that performs two functions: (1) for the student it will be a checklist and provide a self-assessment and (2) for the teacher it will be used as a rubric. The attached assessment list will assess the video, quadratic graph and equation, and solution to the problem (will the ball make it in the hoop?).

VIII. **EVALUATION CRITERIA:**
   Assessment List, corresponding rubric.

IX. **INSTRUCTIONAL TIME:**
   Two ninety-minute class periods
Modeling Quadratics Basketball Video Project

**Strand**
Algebra

**Mathematical Objective(s)**
Students will:
- film a video of a quadratic real-world situation
- use the video of half of a quadratic to write the equation of the situation
- use the video of half of a quadratic to predict the rest of the quadratic and solve a real-world problem

**Related SOL**
- AII.4 (The student will solve, algebraically and graphically b) quadratic equations over the set of complex numbers)
- AII.6 (The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions)
- AII.7 (The student will investigate and analyze functions algebraically and graphically. Key concepts include b) zeros; c) x- and y-intercepts)

**NCTM Standards**
- Understand patterns, relations, and functions: understand relations and functions and select, convert flexibly among, and use various representations for them
- Understand patterns, relations, and functions: analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
- Represent and analyze mathematical situations and structures using algebraic symbols: use symbolic algebra to represent and explain mathematical relationships

**Materials/Resources**
Students will use: modeling Quadratics Worksheet, measuring paper hanging on wall, yard sticks or measuring tape, hoops (bendable glow sticks), tennis balls, ipads (or iphones), video editing app (such as Splice)
Assumption of Prior Knowledge

- Students should have a basic knowledge of quadratic equations in their different forms (standard, vertex, etc)
- Students should have a basic knowledge of quadratic function properties (intercepts, vertex equations, symmetrical properties, etc)
- Students should know how to go from a quadratic equation to the graph and vice versa.
- Students should know how to use the video tool on an iPad and the editing app to cut the video short.

Introduction: Setting Up the Mathematical Task

To introduce this project, the teacher should show students the video from Dan Meyer's blog of him shooting a basketball. The video ends with the ball at the top of the quadratic arc without students seeing if the shot made it. The video can be downloaded from here: http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/ (scroll down to Attachments and then download Take 1).

Ask the class to vote whether Dan made the shot or not. The teacher should put a picture of the shot projected on the board for the students to see (slide attached) and ask students how we could solve this problem algebraically. Guide students to the idea that we could assign points to the picture to start creating a graph of the situation. Once the students agree that it would be helpful to see the picture as a graph, show the slide of the coordinate plane overlapped with the picture (slide attached). Tell the students that they are going to create their own videos of this situation. With a group of 3-4 they will video a shot and then edit it so that only half of the shot is visible. They will then trade videos with another group and mathematically write an equation for the quadratic and use their equation to determine if the shot went into the hoop or not. This introduction should take about 20 minutes.

Planned Time Outline: Video Filming

- Before class, create paper measuring tapes with numbers large enough to see in a video and hang them vertically on the wall.
- Divide groups into 3-4 students and give each group an iPad, tennis ball, measuring tape or yard stick, and glow stick hoop.
- Instruct each group to try to center their shot around the vertical measuring tape so that the height of the vertex will be seen in the video. The group should also measure with the yard stick how far apart the hoop and the shooter are so that they have a horizontal measurement as well.
- Teams to video a few shots until they have one they are satisfied with. The shot does not have to go in.
• Once teams are satisfied with their video, collect their materials and have students edit their videos so that they have one cut short so that the viewer only sees half of the shot.
• Have students upload both videos to a class Dropbox.com account for easy access. (Labeling videos is important so that groups can easily find their own half and full videos)

Planned Time Outline: Equation Writing
• Pair groups up and have groups switch half videos. Groups should share information about how high the vertex of their shot went and the horizontal measurements for how far apart the shooter and the hoop were.
• Groups should work by themselves to graph the situation and write the equation of the shot they just watched.
• From their equation, each group should determine if the shot went into the hoop or if it was missed.
• Each student should record their own graph and work on their own paper.
• The teacher should circulate between groups and help facilitate math discourse within groups.
• Students can finish their work for homework if necessary.

Planned Time Outline: Conclusion
• At the beginning of the next class, groups should get back together and share their results with one another. They should watch the full videos on the iPads to see if their predictions based on their equations were correct.
• The teacher should bring the class back together for a whole group discussion. Put Dan Meyer’s shot overlapped with the graph back on the board. Have groups share ways that they were able to write the equation of a quadratic from a half-video. From this discussion, come up with a class equation for Dan's video.
• From the class equation, determine if his shot was successful.
• Reveal the answer by playing the full video (found at the same place on his blog website).

The video filming should take 30 minutes. The equation writing should take 40 minutes. The conclusion should take 30 minutes at the beginning of the next class period.
**Student Exploration**

**Individual Work**
Students are responsible for contributing to the success of their group video. To accomplish this, groups should remain small (3-4 students) so that every student has a role in the filming process. Each student is also responsible for their own worksheet of the graph of their quadratic, equation, and solution if the shot went into the hoop.

**Small Group Work**
Students will be working as a group for the whole activity to accomplish the goal. Students should work collaboratively and cooperatively by listening to one another's ideas and practice mathematical discourse to explain their own.

**Whole Class Sharing/Discussion**
The teacher should facilitate a whole group discussion on different methods for finding the equation of the quadratic from a half-shot video by doing Dan Meyer's video as a class after groups have worked out their own solutions. The teacher should try to get groups to share as many different methods as possible so that groups learn from one another.

**Student/Teacher Actions**
Students should be working with their groups on each part of the activity to work together to build create their video and work through the algebra. In order to facilitate math discourse, the teacher should work with groups by asking questions such as: Explain your process for finding the equation. Why did you choose that particular method? Are there other methods for solving if the shot goes through the hoop? etc. When groups become stuck because of differing ideas, ask group members to explain each other's ideas and ask the group to work through multiple methods if there are differing ideas.

**Monitoring Student Responses**
- Students are to communicate their thinking and their new knowledge but communicating their ideas and mathematical reasoning verbally with math discourse to their group members
- Students are to communicate with each other respectively and supportively
- Teacher should assist groups who have difficulties by asking guiding questions and asking students to compromise on their ideas
- Teacher should summarize the activity by having the class work together to solve Dan Meyer's half shot and see the full video together
Assessment List and Benchmarks

Half-Shot Reference
Step 1: Make your video

Your team should make a video of someone shooting the tennis ball into a hoop. Be sure to take all the measurements necessary to find your quadratic equation.

Height of vertex: 

Horizontal distance from shooter to hoop: 

Step 2: Edit your video to cut it so that only half of the shot can be seen. Upload both the half-version and full-version of your video onto our class Dropbox.com page. Label each video Group#_half and Group#_full.

Step 3: Swap measurements with another group.

Other group's info:

Horizontal distance from shooter to hoop: 

Step 4: Swap half videos. Watch the video and determine the coordinate of the vertex. Also, based on the half video, make a quick prediction if the ball is going to make it.

Height of vertex:  Prediction: 

Step 5: Draw the half shot of the other group. Draw a coordinate plane and plot points based on the other group's information. You can scale the graph to fit your information (for example, every box = 3 inches).
Step 6: Find the equation of the quadratic

Equation of Quadratic: _______________________________

Step 7: Did they make the shot? Justify your answer mathematically as well as with a written explanation.

Step 8: Extension: What is the coordinate where that ball would hit the ground?
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<thead>
<tr>
<th>Num</th>
<th>Element</th>
<th>Point Value</th>
<th>Earned Assessment</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>Video of full shot created and uploaded to dropbox.com</td>
<td>3</td>
<td></td>
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<tr>
<td>2</td>
<td>Video of half shot created and uploaded to dropbox.com</td>
<td>3</td>
<td></td>
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<tr>
<td>3</td>
<td>Graph of other group's half shot drawn</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Equation of other group's quadratic found with work of method clearly shown</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Mathematical justification completed for if the shot was successful or missed</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Found the coordinate for where the ball would hit the ground</td>
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<td></td>
</tr>
<tr>
<td>7</td>
<td>Video quality</td>
<td>2</td>
<td></td>
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<tr>
<td>8</td>
<td>Neatness of graph and math justifications</td>
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<td></td>
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<td></td>
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<td>1</td>
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<td>-------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------</td>
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</tr>
<tr>
<td>1</td>
<td>Video of full shot created and uploaded to dropbox.com</td>
<td>No video of full shot was filmed.</td>
<td>Video of full shot was partially filmed.</td>
</tr>
<tr>
<td>2</td>
<td>Video of half shot created and uploaded to dropbox.com</td>
<td>No video of half shot was created.</td>
<td>Video of full shot was not correctly edited.</td>
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<tr>
<td>3</td>
<td>Graph of other group's half shot drawn</td>
<td>Graph of half shot is not attempted.</td>
<td>Graph of half shot is attempted but with significant errors that make the graph unusable for the video.</td>
</tr>
<tr>
<td>4</td>
<td>Equation of other group's quadratic found with work of method clearly shown</td>
<td>Finding the equation of the quadratic was not attempted.</td>
<td>An incorrect method was used to find the quadratic equation of the half video, and calculated incorrectly.</td>
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<tr>
<td>5</td>
<td>Mathematical justification completed for if the shot was successful or missed</td>
<td>No mathematical justification was provided to support a prediction if the shot was successful or missed.</td>
<td>Incorrect mathematical justification using the quadratic equation was used to prove if the shot was successful or missed.</td>
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<td>Column</td>
<td>Description</td>
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<td>-------</td>
<td>-------------</td>
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<tr>
<td></td>
<td>Found the coordinate where basketball hits the ground</td>
<td>Used an incorrect method to find the coordinate of where the ball hits the ground.</td>
<td>Used a correct method to find the coordinate of where the ball hits the ground, but mistakes in calculations.</td>
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<tr>
<td></td>
<td>Video quality</td>
<td>No videos were created.</td>
<td>Videos are unclear and difficult to see the measurements, hoop, and/or the ball.</td>
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<tr>
<td></td>
<td>Neatness of graph and math justifications</td>
<td>No math work is provided on the worksheet.</td>
<td>Math methods, calculations, graphs, and justifications are disorganized, sloppy, and difficult to read.</td>
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</table>
Benchmark

Screenshot taken from a student video.

Sample videos are available online:

Half-Video

https://vimeo.com/101926364

Full-Video

https://vimeo.com/101925517
Algebra 2
Modeling Quadratics

Step 1: Make your video

Your team should make a video of someone shooting the tennis ball into a hoop. Be sure to take all the measurements necessary to find your quadratic equation.

Height of vertex: 18 in

Horizontal distance from shooter to hoop: 72 in (6 ft)

Step 2: Edit your video to cut it so that only half of the shot can be seen. Upload both the half-version and full-version of your video onto our class Dropbox.com page. Label each video Group#_half and Group#_full.

Step 3: Swap measurements with another group.

Other group's info:

Horizontal distance from shooter to hoop: 48 in (4 ft)

Step 4: Swap half videos. Watch the video and determine the coordinate of the vertex. Also, based on the half video, make a quick prediction if the ball is going to make it.

Height of vertex: 4.5 ft

Prediction: Yes, the shot makes it

Step 5: Draw the half shot of the other group. Draw a coordinate plane and plot points based on the other group's information. You can scale the graph to fit your information (for example, every box = 3 inches).
Step 6: Find the equation of the quadratic

\[ y = a (x-h)^2 + k \]
\[ y = a (x-2.25)^2 + 4.5 \]
\[ y - 10 = (0,2) \]
\[ z = a (0 - 2.25)^2 + 4.5 \]
\[ z = a (5.0625) + 4.5 \]
\[ 2.5 = 5.0625 a \]
\[ -4.94 = a \]

Equation of Quadratic: \[ y = -4.94 (x-2.25)^2 + 4.5 \]

Step 7: Did they make the shot? Justify your answer mathematically as well as with a written explanation.

\[ \text{hoop: (4,3)} \]
\[ 3 = -4.94 (4-2.25)^2 + 4.5 \]
\[ 3 = -4.94 (1.75)^2 + 4.5 \]
\[ 3 = -4.94 (3.0625) + 4.5 \]
\[ 3 = 1.513 + 4.5 \]
\[ 3 = 2.987 \]

Since the hoop is a point very close to the equation of the parabola, the ball should go through the hoop. Although there is a small difference off of the parabola, the hoop is wide enough to still have the ball go through it.

Step 8: Extension: What is the coordinate where that ball would hit the ground?

\[ 0 = -4.94 (x-2.25)^2 + 4.5 \]
\[ -4.5 = -4.94 (x-2.25)^2 \]
\[ \sqrt{4.109} = \sqrt{(x-2.25)^2} \]
\[ \pm 3.018 = x - 2.25 \]
\[ 5.268 = x \]
\[ -2.68 = x \]

Where the ball hits the ground: \((5.268, 0)\)