Performance Based Learning and Assessment Task

Exploring Types of Variation Activity

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
The students are instructed to explore data to model different types of variation. The students will also create word problems followed by questions that can be answered using their data.

II. UNIT AUTHOR:
Amy Corns, Patrick County High School, Patrick County Public Schools.

III. COURSE:
Algebra II

IV. CONTENT STRAND:
Statistics

V. OBJECTIVES:
The student will be able to:
- Organize and collect data about the research topic
- Analyze the data to successfully create applicable word problems and questions using the data.

VI. REFERENCE/RESOURCE MATERIALS:
Laptop & Calculator

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will be graded on the accuracy of successfully modeling each type of variation. Students will also be assessed on the quality and neatness of their work. There will also be a self-assessment that will provide the student with a checklist and a rubric for the teacher.

VIII. EVALUATION CRITERIA:
The self-assessment and teacher assessment will count 21 points each for a total of 42% of the overall score. The following rubric gives a detailed outline of the scoring points. A benchmark will also be completed by each group which makes up the remaining 58% of the students final grade.

IX. INSTRUCTIONAL TIME:
This activity is estimated to use 2 class blocks. (1 block to plan and organize the project. 1 additional block to finalize and complete their project)
Exploring Types of Variation Activity

Strand
Algebra II: Statistics

Mathematical Objective(s)
The goal of this activity is be able to use the internet to research different example that will model the various types of Variation.(Direct, Inverse, Joint, and Combined) They will use their findings to create applicable word problems and related questions to answer based on the models.

Related SOL

All.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

NCTM Standards

• Formulate questions that can be addressed with data and collect, organize, and display relevant data needed to answer them.
• Select and use appropriate statistical methods to analyze data
• Use mathematical models to represent and understand quantitative relationships

Materials/Resources

• See attached results benchmark
• Graphing Calculator

Assumption of Prior Knowledge

• Students have basic knowledge of the different types of variation.
• Students should be able to conduct research using a search engine on the internet.
• Students may have difficulty searching for real world examples on the internet. The teacher will need to guide the groups by giving helpful hints, appropriate search words and/or suggestions.
• The relevant contexts the student will encounter with this activity are: how different types of variation can be modeled using real world contexts.

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the relationship between real world data and different types of variation. Each group will collect/explore data to model each type of variation. Students will be divided into groups of 2 or 3 persons in each group. Below, you will find a detailed outline of what is specifically required.
Exploring Types of Variation Activity:

You have 1 class day to discuss and plan your data collection. After, you plan and collect your data, you will have 1 additional day to analyze the data and formulate your word problems and related questions. Use your prior knowledge and the laptops to explore a real world example of each type of variation that we have discussed in class. (Direct, Inverse, Joint, and Combined)

Each group must:
1) Find a real world example to model the each specific type of variation.
2) Create a word problem using the specific type of variation.
3) Create 3 questions that can be answered using the word problem model for each type of variation.

**Student Exploration**

Students will be working together in groups of 2-3 students in each group through-out this project. Groups will be chosen by the teacher based on student strengths and weaknesses. The teacher will be circulating and offering guidance when necessary. The teacher should listen to make sure the student’s data numbers are making sense. This will also give the teacher a chance to have in depth discussions to address possible questions that may arise. After the projects have been turned in for grading, the class will have a discussion about their findings and the conclusions that were drawn. The class will discuss the positive and negatives aspects from this assignment.

**Student/Teacher Actions:**

- On day 1, groups should be discussing which Real World examples they will choose. Teachers will listen carefully and make appropriate and encouraging suggestions and comments. After planning, the groups may begin creating the word problems to model each type of variation.
- On day 2, groups will finish creating the word problems. Finally, they will complete the project by formulating sample questions that can be answered using their data.

**Monitoring Student Responses**

- Students are to communicate their thinking by asking questions to group members, making suggestions, and being active listeners to others in the group.
- Students are to communicate with each other in a supportive manner;
- Teachers are to carefully clarify questions and provide possible problem-solving strategies to overcome difficulties without giving the direct solutions to the students.
Assessment List and Benchmarks

- Students will complete each of the following:

  1. Benchmark

  2. Self-Assessment
Exploring Types of Variation

Benchmark (58 points)

Group Member Names: __________________________________________________________

Sally is studying the different types of real world variation. She wants to create different practice questions for each type of variation. Can you help her??

Directions: You have 1 class day to discuss and plan your data collection. After, you plan and collect your data, you will have 1 additional day to analyze the data and formulate your word problems and questions. Use your prior knowledge and the laptops to explore a real world example of each type of variation that we have discussed in class. (Direct, Inverse, Joint, and Combined)

Each group must: 1) Find a real world example to model the each specific type of variation.

2) Create a word problem using the specific type of variation.

3) Create 3 questions that can be answered using the word problem model for each type of variation.

4) Provide an answer key to each of the formulated questions.

1) Direct Variation Word Problem (6 points)

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a) Question 1: (2 points)

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________________________________________________________________________

1) Direct Variation Word Problem (6 points)

________________________________________________________________________
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a) Question 1: (2 points)

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________________________________________________________________________
b) Question 2: (2 points)

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____________________________________________________________________

c) Question 3: (2 points)

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____________________________________________________________________

2) Inverse Variation Word Problem (6 points)

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a) Question 1: (2 points)

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____________________________________________________________________

b) Question 2: (2 points)

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____________________________________________________________________

b) Question 3: (2 points)

____________________________________________________________________
____________________________________________________________________

3) Joint Variation Word Problem (6 points)

____________________________________________________________________
____________________________________________________________________
a) Question 1: (2 points)

b) Question 2: (2 points)

c) Question 3: (2 points)

4) Combined Variation Word Problem (6 points)

a) Question 1: (2 points)
c) Question 3: (2 points)

_____________________________________________________________________
_____________________________________________________________________

5) Define the following in your own words (10 points)

Direct Variation __________________________________________________________

Inverse Variation _________________________________________________________

Joint Variation __________________________________________________________

Combined Variation _______________________________________________________

### Exploring Types of Variation Self/Teacher Assessment

<table>
<thead>
<tr>
<th>NUM</th>
<th>Element</th>
<th>Point Value</th>
<th>Self</th>
<th>Teacher</th>
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<td>1</td>
<td>Is the each type of variation correctly modeled?</td>
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<td>2</td>
<td>Are the models organized and clear to understand?</td>
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<td>3</td>
<td>Is there a Word Problem that models each type of variation?</td>
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<td>4</td>
<td>Are there 3 questions formulated for each type of variation?</td>
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<td>5</td>
<td>Are the definitions given for each type of variation clear to understand and accurate?</td>
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<td>6</td>
<td>Is the answer key to the questions calculated accurately?</td>
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<td>7</td>
<td>Were all elements of the benchmark complete?</td>
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<td>Almost all of the 4 types of variation are correctly modeled.</td>
<td>Few of the types of variation are correctly modeled.</td>
<td>No types of variation are correctly modeled.</td>
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<td>Few of the types of variation are modeled by a word problem.</td>
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<td>Almost all types of variation have 3 questions formulated.</td>
<td>Few of the types of variation have 3 questions formulated.</td>
<td>None of the types of variation have 3 questions formulated.</td>
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<td>A clear and accurate definition is given for almost all types of variation.</td>
<td>A clear and accurate definition is given for a few of the types of variation.</td>
<td>No clear and accurate definitions are given for the types of variation.</td>
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<td>Few of the elements of the benchmark were complete.</td>
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<td>TOTAL</td>
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Exploring Types of Variation

Benchmark (SAMPLE)

Group Member Names: __________________________________________________________

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4) Provide an answer key to each of the formulated questions.

1) Direct Variation Word Problem

Jack gets a $5.00 allowance per week. Let “s” represents the total money saved and “w” represent the number of weeks Jack saves his money.

a) Question 1: Write an equation of Direct Variation to represent this model.

b) Question 2: Explain why this word problem models a Direct Variation.

c) Question 3: If Jack wants to buy a bicycle that costs $45.00, how many weeks would he have to save his allowance money?
2) Inverse Variation Word Problem

Sally travels to the beach at an average speed of 60 mph. Her total driving time was 4 hours. On the way home, she drives an average speed of 40 mph.

a) Question 1: Write an equation to model an Inverse Variation equation.

b) Question 2: Based on the model, how long does it take Sally to drive home from the beach?

c) Question 3: Explain why this is an inverse variation model.

3) Joint Variation Word Problem

Area of a Triangle is \( \frac{1}{2} \) the base times the height of the triangle.

a) Question 1: Write an equation to represent a Joint Variation model.

b) Question 2: Explain why this is a joint variation model.

c) Question 3: Solve the equation for the height of the triangle.

4) Combined Variation Word Problem

The Combined Gas Law is known as \( PV = nRT \) where \( P \) represents pressure, \( V \) represents volume, \( n \) represents a mole, \( T \) represents temperature and \( R \) is a constant.

d) Question 1: Solve this equation for \( P \).

e) Question 2: Explain why this is a Combined Variation model.

f) Question 3: Solve the equation for the constant.

5) Define the following in your own words

**Direct Variation:** A relationship between two variables where \( x \) is being multiplied by a constant \( (k) \) to equal the output \( (y) \). A direct variation equation uses the form \( y = kx \).

**Inverse Variation:** A relationship between two variables that relate in an opposite manner. An inverse variation can be modeled using \( xy = k \) or \( y = \frac{k}{x} \).
Joint Variation: A relationship similar to direct variation except using multiple variables.

A joint variation equation can be modeled by $y = kxz$ where $k$ is a constant.

Combined Variation: A type of variation that combines direct and inverse variation. A combined variation equation can be modeled by $y = \frac{kx}{z}$. 
1) a) $s = 5w$

b) This example models a direct variation equation $y = kx$. In this example, $y$ is the output modeled by total savings. $5$ is the constant because he is saving $5.00 each week. $x$ is modeled by the number of weeks the allowance is saved.

c) Using the model $s = 5w$ and substituting $45.00$ in for $s$. $w$ would equal 9 weeks.

2) a) Let “$s$” represent speed and “$t$” represent total driving time. The equation would be $(s)(t) = k$

b) If $(60)(4) = k$, then $k = 240$. Then, $(40)t = 240$. Thus, $t = 6$ hours to drive home.

c) This example models an inverse variation equation $xy = k$. You know this is inverse because as Speed decreases, the total driving time increases.

3) a) $A = \frac{1}{2} bh$

b) This example models a joint variation because multiple variables are being multiplied by a constant to equal an output.

c) $h = \frac{2A}{b}$

4) a) $P = \frac{nRT}{V}$

b) This example models a combined variation because it combines direct variation with inverse variation. $P$ varies directly with $nRT$ and inversely with $V$.

c) $\frac{PV}{nT} = R$
5) Answers may Vary!