New look at polar equations

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Objectives:

1. In the regular Calculus texbooks, for a polar equation, $r = f(\theta)$, we only get a formula of dy/dx at a particular θ. In other words, we get dy/dx at one point.
2. In this note, we will see how new software allows us to visualize the derivative functions for a given polar equation.
3. If we rewrite the polar equation, $r = f(\theta)$, in a paratmetric form (see below), it is easy to prove that the derivative function is indeed what we see from step 2 above.

Finding and graphing the derivative for a polar equation

1. What is the derivative $\frac{dy}{dx}$ for $r = \theta$? How does **ClassPad** do it (open **derivative-polar**)?
2. How do we learn this from the textbook?
3. Parametric equation for $r = f(\theta)$

$$x(\theta) = f(\theta)\cos\theta$$
$$y(\theta) = f(\theta)\sin\theta.$$

So

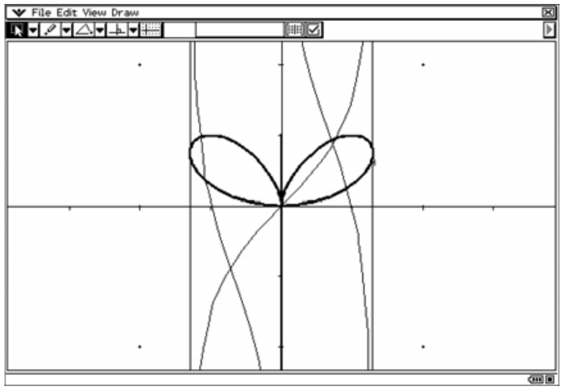
$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + (\cos\theta)f(\theta)}{f'(\theta)\cos\theta - (\sin\theta)f(\theta)},$$

and

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{d\theta}\left(\frac{dy}{d\theta}\right)}{\frac{dx}{d\theta}\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d^2y}{(d\theta)^2}}{\left(\frac{dx}{d\theta}\right)^2}.$$ #

One more example using ClassPad:

1. **Example** *http://www.3dsoftware.com/Math/PlaneCurves/Derivatives/.*
   - $r = a\sin\theta(\cos\theta)^2$; this is equivalent to
   - $(x^2+y^2)^2 = ax^2y$, which is not easy to solve for y. (refer to Maple).
   - In regular text book, we can use implicit differentiation to find the dy/dx at one point, but **can you graph the derivative of such function**?
   - It is easy to use **ClassPad version 3** to show the orginal graph (darker) and its derivative (lighter) below:



More examples

Click ClassPad version 3

1. $r = (\cos\theta)^2$
2. James Stewart-Concepts and contexts, 2nd edition, page A65, $r = \sin(\frac{8\theta}{5})$, where $\theta \in [0, 10\pi]$.
3. $r = 3 + 8\sin\theta$.