Making Mathematics Fun, Accessible and yet Challenging

Wei-Chi Yang
wyang@radford.edu
Department of Mathematics and Statistics
Radford University
URL: http://www.radford.edu/wyang
USA

Abstract

The advent of evolving technological tools would inevitably influence the teaching and learning of mathematics. By motivating students with graphical and geometric animations and by capturing numerical measurements after animations makes learning mathematics fun and accessible, and yet challenging when solving problems analytically.

One indisputable fact is that the impact of technology varies when technological tools advance. Many students often got frustrated in complicated algebraic manipulations early to lose confidence in learning mathematics. We often found college students can do better in mathematics if their algebraic manipulation skills are better. On the other hand, we found students could understand general concepts of Calculus if we leave complicated algebraic computations behind and motive them with graphical and geometrical representations. In this short note, we will see how we can inspire students learn mathematics through graphical and geometrical representations first. This can be achieved by incorporating technological tools that are equipped with dynamic geometry and computer algebra system. By motivating students with graphical and geometric animations and capturing numerical data after the animations makes learning mathematics fun and accessible. In the meantime, thanks to advanced technological tools, we will see how a traditional static uninteresting problem can be made into many more challenging problems, which shall prompt students to solve the problems analytically. Examples on Geometry, Trigonometry, Precalculus, Calculus and etc. will be discussed.

1 Mathematics contents can be made accessible

We demonstrate how technological tools have allowed us to explore problems in Trigonometry, Pre-Calculus and Calculus.

1.1 Trigonometry and Pre-Calculus

There is a close relationship between unit circle and trigonometric functions such as $\sin x$, $\cos x$ and others. The Example 1 and the corresponding video clip is to motivate the understandings of the graphs of $y = \sin t, y = \cos t, y = \cos(2t)$,and $y = \sin(2t)$ by using a unit circle. The Example 2 is to motivate students the relationship between a circle and a parametric equation of the form $[a + r\cos\theta, a + r\sin\theta]$.

Example 1 *Unit Circle and the Trigonometric Functions.*
(http://mathandtech.org/CASIO_Video/Unitcircle/unit-circle.html)

Example 2 *A circle is drawn (arbitrarily) using a Dynamic Geometry System, then pick a point P=(x(t),y(t)) on the circle, where t is ranging from 0 to* 2π. *What are the graphs of* $(t, x(t))and(t, y(t))$ *respectively? A video clip can be found at*
http://mathandtech.org/CASIO_Video/Trig_Shifting2/Trig_Shifting2.html.

The following Example 3 shows how we explore the inverse functions for trigonometric and exponential functions, and how shiftings will affect the inverse functions. For example, If the inverse of a function f exists and $a > 0$ then to find the inverse of $f(x + a)$ is equivalent to shift $y = f^{-1}(x)$ down $a - unit$.

Example 3 *Discussions on Inverse Trig and Exponential Functions*
(http://mathandtech.org/CASIO_Video/Inv_Functions/Inv_Function.html).

1.2 Calculus

The Example 4 shows how we can enhance the understandings of the derivatives for $\sin x$ and $\tan x$ respectively. In particular, we make use of the equations, $\sin^2 t + \cos^2 t = 1$ and $1 + \tan^2 t = \sec^2 t$, to motivate the derivatives of $\sin x$ and $\tan x$ respectively.

Example 4 *Derivative of sine and tangent.*
(http://mathandtech.org/CASIO_Video/Derivative_sine_tan/sin-tan.html).
We explore the the exponential functions and their corresponding derivatives

We use the similar idea of incorporating DGS to understand the concept of implicit differentiation.

Remark 5 *Use DGS and CAS to explore the derivative* $\frac{dy}{dx}$ *for an ellipse. In general, Use DGS and CAS to explore the Implicit Differentiation. A video clip can be found at*
http://mathandtech.org/CASIO_Video/Imp_Diff/Imp_Diff.html.

We next give some motivation of why $\frac{d}{dx}(a^x) = \ln(a)a^x$.

Example 6 *Exploring the exponential derivatives.*
(http://mathandtech.org/CASIO_Video/Exp_Diff/Derivative-of-the-Exponential-Function.html).

We will see what regular Calculus text book did not teach us in the following example regarding the Polar Differentiation.

Remark 7 *Use DGS and CAS to explore the Polar Differentiation.*
http://mathandtech.org/CASIO_Video/Polar_Diff/Polar_Diff.html.

2 Expand our knowledge with DGS and CAS

We use the following examples to demonstrate how we make use of animations to make several classical Calculus problems (that can be found in many textbooks) more accessible to students before we solve these problems analytically.

Example 8 *Finding the Maximum Area of a Triangle by Folding a Paper. We fold a piece of rectangular paper from upper left-hand corner to the base of the paper. Slide the corner along the base. Find the largest triangle DEF that can be formed.*
(http://mathandtech.org/CASIO_Video/Max_Triangle/Max_Trig.html).

Example 9 *The ladder problem.*
(http://mathandtech.org/CASIO_Video/Ladder/ladder.html).

Example 10 *Rope problem.*
(http://mathandtech.org/CASIO_Video/Rrope/rope.html)

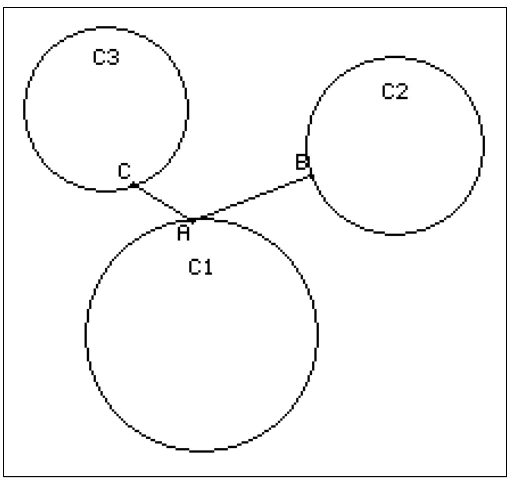
Example 11 *Shrinking Circle-A limit problem.*
(http://mathandtech.org/CASIO_Video/Shrinking_Circle/shrinking_circle.html).

Example 12 *Finding* $\lim_{x\to 0}\left(\dfrac{a^x-1}{x}\right)$ *using geometric approach*
http://mathandtech.org/CASIO_Video/Limit2/Limit2.html.

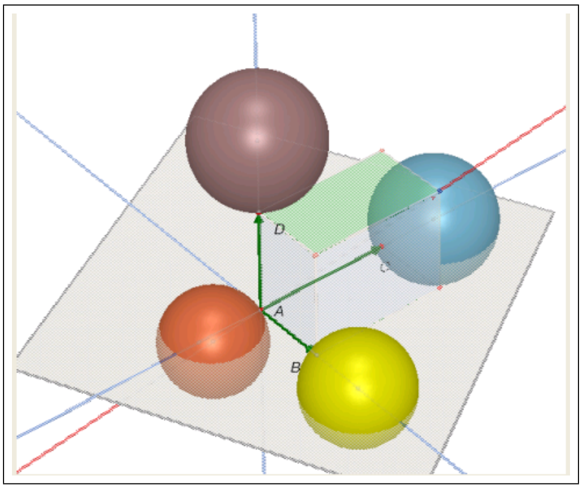
3 Mathematics contents can be challenging

We use the following examples to demonstrate the importance of integrating Calculus with Linear Algebra through use of technological tools, we can tackle some difficult problems geometrically and graphically.

Example 13 *We are given three curves in the plane, see* $C1$*,* $C2$ *and* $C3$ *below. We need to find points* A*,* B*, and* C *on* $C1, C2$ *and* $C3$ *respectively so that the distance* $AB + AC$ *achieves its minimum.*



Example 14 *Shortest sum distance from one surface to three other surfaces. We are given four surfaces in the space, represented by the orange surface, called S_1; yellow surface, called S_2 ; blue surface called S_3 and the purple surface, called S_4 respectively. We want to find points A, B, C and D on S_1, S_2, S_3 and S_4 respectively so that the distance $AB + AC + AD$ achieves its minimum.*



This is the generalization from our 2D example to 3D, which gives good application to linear independency and Lagrange Multipliers in 3D from geometric point of view.

The last example demonstrates when we extend results from two dimensions, many accessible to Calculus students, to three dimensional ones, we often encounter concepts that to be taught in differential geometry or even topology. This is one reason why we, as teachers, need to learn more mathematics.

Example 15 *Shrinking sphere problem.*
(https://php.radford.edu/~ejmt/Content/Papers/eJMT_v1n1p4.pdf).

Software Packages

[1] [ClassPad] ClassPad Manager, a product of CASIO Computer Ltd., http://classpad.net or http://classpad.org/.

[2] [Cabri3D] Cabri 3D, a product of CABRILOG,

http://www.cabri.com/v2/pages/en/index.php.

[3] [Maple] Maple 11, a product of Maplesoft, http://www.maplesoft.com/.