1. Physicist's proof that all odd natural numbers are prime: 1 is prime. 3 is prime. 5 is prime. 7 is prime. 9 is experimental error. 11 is prime. 13 is prime. We have now taken sufficiently many readings to verify the hypothesis. Comment!

2. You know that there are 1000 people in a hall. Upon inspection you determine that 999 of these people are men. What can you conclude about the 1000'th person?
Solution: You can't make any conclusion at all about her. Don't even try.

3. The product rule for differentiation says that for every number x and all functions f and g that are differentiable at x, we have
$$(fg)'(x) = f'(x)g(x) + f(x)g'(x).$$
Write down the opening line of a proof of the product rule. Your opening line should start: Suppose that ...
Solution: Suppose that x is a real number and that f and g are functions that are differentiable at the number x.

4. Given that $P(x)$ and $Q(x)$ are statements that contain an unknown x and that S is a set, outline a strategy for the proving the assertion $P(x) \Rightarrow Q(x)$ for every $x \in S$. Write down the opening line of your proof.
Solution: Suppose that $x \in S$ and that the condition $P(x)$ is true. Then write a proof that $Q(x)$ must be true.

5. Given that $P(x)$ is a statement that contains an unknown x and that S is a set, write down an opening line of a proof of the assertion that $P(x)$ is true for every $x \in S$.
Solution: Suppose that $x \in S$.

6. You are given that $P(x)$ and $Q(x)$ are statements that contain an unknown x, that S is a set, that $P(x)$ is true for every $x \in S$ and that $P(x) \Rightarrow Q(x)$. Is it possible to deduce that $Q(x)$ is true for every member x of the set S?
Solution: Yes it is possible. For each x we know that $P(x)$ is true and that $P(x) \Rightarrow Q(x)$ and so we know that $Q(x)$ is true.

7. You are given that $P(x)$ and $Q(x)$ are statements that contain an unknown x, that S is a set, that $P(x)$ is true for every $x \in S$ and that $P(x) \Rightarrow Q(x)$. Is it possible to deduce that $Q(x)$ is true for at least one member x of the set S?
Solution: No it isn't possible to deduce that $Q(x)$ is true for at least one member x of the set S because we have no information to the effect that the set S is nonempty.

8. Write down the contrapositive form of the statement that for every member x of a given set S we have $P(x) \Rightarrow Q(x)$.
Solution: For every member x of the set S we have $(\neg Q(x)) \Rightarrow (\neg P(x))$.

9. Write down the denial of the statement that for every x we have $P(x) \Rightarrow Q(x)$.
Solution: There is at least one x for which $P(x)$ is true and $Q(x)$ is false.

10. Prove that for every number x in the interval $[-2, 2]$, if we define
$$u = \sqrt[3]{\frac{3\sqrt{3}\,x + (2x^2+1)\sqrt{4-x^2}}{6\sqrt{3}}}$$
and
$$v = \sqrt[3]{\frac{3\sqrt{3}\,x - (2x^2+1)\sqrt{4-x^2}}{6\sqrt{3}}}$$
then
$$u^2 + v^2 = 1 + uv.$$
Hint: With an eye on the proof shown earlier, show that $u^3 + v^3 = u + v$.

That earlier proof shows that if x is any number in the interval $[-2, 2]$ then

$$\sqrt[3]{\frac{3\sqrt{3}\,x + (2x^2 + 1)\sqrt{4 - x^2}}{6\sqrt{3}}} + \sqrt[3]{\frac{3\sqrt{3}\,x - (2x^2 + 1)\sqrt{4 - x^2}}{6\sqrt{3}}} = x$$

which we can express as $u + v = 1$. On the other hand,

$$u^3 + v^3 = \frac{3\sqrt{3}\,x + (2x^2 + 1)\sqrt{4 - x^2}}{6\sqrt{3}} + \frac{3\sqrt{3}\,x - (2x^2 + 1)\sqrt{4 - x^2}}{6\sqrt{3}}$$

and we conclude that $u^3 + v^3 = x$. Thus

$$u^3 + v^3 = u + v$$

and, by factorizing the left side, we obtain the desired result

$$u^2 + v^2 = 1 + uv$$

without difficulty.