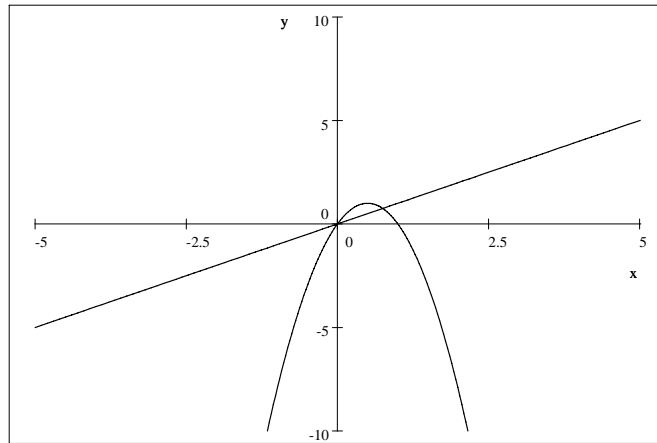


Example 1. If $f(x) = kx(1 - x)$ and $x_{n+1} = f(x_n)$. Discuss the convergence and divergence of the sequence.



Remark: for $f(x) = kx(1 - x)$, if $x_{n+1} = f(x_n)$ converges, it will converge to either 0 or $x = \frac{k-1}{k}$ by solving $x = f(x)$.

Case 1: When $k > 3$, for example $k = 4$, the sequence $\{x_n\}$ converges to 0.75 only when $x_0 = 0.25$; otherwise the sequence diverges.

0.25	0.24
0.75	0.7296
0.75	0.78914
0.75	0.6656
0.75	0.8903
0.75	0.39066
0.75	0.95218
0.75	0.18215
0.75	0.59589
0.75	0.96322
0.75	0.1417

Case 2. When $1 < k < 3$, for example $k = 2.8$, the sequence converges to $k = \frac{1.8}{2.8} = 0.64286$.

Case 3. When $k = 3$, the sequence converges to $2/3$.

Example 2. If $f(x) = kx^2$ and $x_{n+1} = f(x_n)$. Discuss the convergence and divergence

of the sequence.

	1	2
	0.5	2.0
	0.125	2.0
	7.8125×10^{-3}	2.0
	3.0518×10^{-5}	2.0
Case 1. $f(x) = 0.5x^2$ Iterates:	4.6566×10^{-10}	Iterates: 2.0 Iterates:
	1.0842×10^{-19}	2.0
	5.8775×10^{-39}	2.0
	1.7272×10^{-77}	2.0
	1.4917×10^{-154}	2.0
	1.1125×10^{-308}	2.0
2. 1	1. 5	
2. 205	1. 125	
2. 431	0.63281	
2. 9549	0.20023	
4. 3657	2.0045×10^{-2}	
9. 5299	Iterates: 2.009×10^{-4}	.
45. 409	2.0181×10^{-8}	
1031.0	2.0364×10^{-16}	
5.3148×10^5	2.0735×10^{-32}	
1.4124×10^{11}	2.1498×10^{-64}	
9.9741×10^{21}	2.3108×10^{-128}	

	1	4
	0.25	4.0
	1. 5625 × 10 ⁻²	4.0
	6. 1035 × 10 ⁻⁵	4.0
	9. 3132 × 10 ⁻¹⁰	4.0
Case 2. $f(x) = 0.25x^2$ Iterates:	2. 1684 × 10 ⁻¹⁹	Iterates: 4.0 Iterates:
	1. 1755 × 10 ⁻³⁸	4.0
	3. 4545 × 10 ⁻⁷⁷	4.0
	2. 9833 × 10 ⁻¹⁵⁴	4.0
	2. 225 1 × 10 ⁻³⁰⁸	4.0
	1. 2377 × 10 ⁻⁶¹⁶	4.0

7
12. 25
37. 516
351. 86
30951.
2. 3948 × 10⁸
1. 4338 × 10¹⁶
5. 1396 × 10³¹
6. 6039 × 10⁶²
1. 0903 × 10¹²⁵
2. 9718 × 10²⁴⁹

3

2.25

1.2656

0.40045

0.04009

Iterates: 4.0181×10^{-4}

4.0363×10^{-8}

4.0729×10^{-16}

4.1471×10^{-32}

4.2996×10^{-64}

4.6216×10^{-128}

3

5

1.8

5.0

0.648

5.0

8.3981×10^{-2}

5.0

1.4106×10^{-3}

5.0

Case 3. $f(x) = 0.2x^2$ Iterates: 3.9793×10^{-7} Iterates: 5.0 Iterates:

3.167×10^{-14}

5.0

2.0060×10^{-28}

5.0

8.048×10^{-57}

5.0

1.2954×10^{-113}

5.0

3.3562×10^{-227}

5.0

7
9.8
19.208
73.789
1089.0
 2.3717×10^5
 1.125×10^{10}
 2.5314×10^{19}
 1.2816×10^{38}
 3.2849×10^{75}
 2.1581×10^{150}

4
3.2
2.048
0.83886
0.14074
Iterates: 3.9614×10^{-3}
 3.1386×10^{-6}
 1.9701×10^{-12}
 7.7626×10^{-25}
 1.2052×10^{-49}
 2.9048×10^{-99}