

Show your work.

1. If $f'(x) = -\frac{2}{\sqrt{1-x^2}}$, $f(0) = \pi$. Find f .

Solution: (Method 1) If you use $f(x) = \int -\frac{2}{\sqrt{1-x^2}} dx = -2 \sin^{-1}x + C$, then since

$\arcsin 0 = 0$, we have $C = \pi$, which implies that $f(x) = -2 \sin^{-1}x + \pi$.

Method II. If you use $f(x) = \int -\frac{2}{\sqrt{1-x^2}} dx = 2 \cos^{-1}x + C$, then since $\arccos 0 = \frac{1}{2}\pi$,

this implies that $C = 0$, which means that $f(x) = 2 \cos^{-1}x$. It follows from Method I and Method II above that $-2 \sin^{-1}x + \pi = 2 \cos^{-1}x$. (Try to prove this by using graphs!!)

2. Find $\int \left(3x^{-1} + 8 \sin x + \frac{2}{1+x^2} \right) dx$ [Ans. $3 \ln x - \pi - 8 \cos x + 2 \arctan x + C$].
3. Use Newton's Rule to estimate the zero of $\tan x - (3x + 1) = 0$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$, correct up to 6 decimal places. [Notice there are three zeros in the range, $x = -1.205883768447, -0.527538342223$, and $x = 1.378477437013$.]
4. Prove $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$.