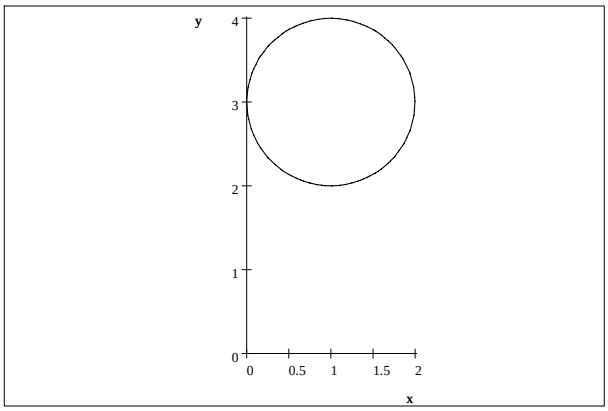
Math 152 Practices for Final Exam

1. (20 points) Set up the integral that will give you the volume of a donut by rotating a circle of radius 1 and 2 units above the $x-axis$ around $x-axis$. Please refer to the graph below.



2. Find the volume of the donut from question above.

3. Find the area bounded by two curves: $y = x^2 - 1$ and $y = -x^2 + 1$.

4. $\int \frac{1}{x^2\sqrt{x^2-4}}dx$

5. $\int \frac{1}{\sqrt{x^2+4}}dx$

6. $\int \frac{\sin x}{1+\cos^2 x}dx$

7. $\int \frac{x}{(1+x^2)^3}dx$

8. $\int \frac{-2x^2+x-4}{x^3+x^2-2x}dx$ [hint: $\frac{-2x^2+x-4}{x^3+x^2-2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$; ans=$2\ln x - \frac{5}{3}\ln(x-1) - \frac{7}{3}\ln(x+2)$]

9. $\int \frac{2x+7}{x^2-x-2}dx$ [hint: $\frac{2x+7}{x^2-x-2} = \frac{A}{x-2} + \frac{B}{x+1}$; ans= $\frac{11}{3}\ln(x-2) - \frac{5}{3}\ln(x+1)$]

10. $\int_{-\infty}^{\infty} \frac{1}{(x+2)^2}dx$

11. $\int_2^{\infty} xe^x dx$

12. $\int \tan^{-1}x\, dx$. (hint: integration by parts)

13. $\int \frac{dx}{(x^2+9)^{3/2}}$.

14. $\int \frac{dx}{x^2\sqrt{x^2-1}}$ (hint: trig substitution $x = \sec\theta$.)

15. $\int \cos^3\theta d\theta$, (hint: write $\cos^3\theta = \cos^2\theta\cos\theta$.)

16. If $f(x) = |x+1| + |x-2|$.

- **a.** Sketch $y = f(x)$.
- **b.** Compute the area of $y = f(x)$ from $x = 0$ to $x = 3$ directly without using integral(s).
- **c.** Use the left end Riemann sum when $n = 100$ to approximate $\int_0^3 f(x)dx$ [hint: the command for an absolute function is **abs(x-1)** and etc.]

Find the area bounded by the following curves.

17. $y = \sqrt{x}, y = x - 6,$ and $x - axis.$

18. $y = \ln(x), y = -x + 4,$ and $x - axis.$

19. $x = 3y - y^2 + 3, x = 0$

20. $y = \dfrac{1}{x^2 + 1},\ y = -x^2 + 3.$

21. If $g(x) = \int_0^x f(t)dt,$ where $f(t) = e^x(x^3 - 3x^2 + x)$ then

a. determine the interval(s) where g is increasing,

b. find the local maximum and minimum of $g(x)$. (Only x value is needed).

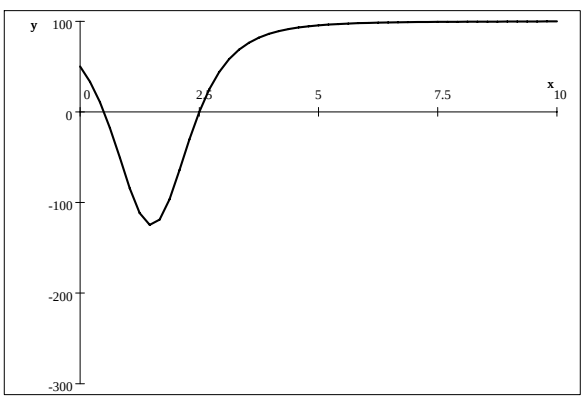
22. $\frac{d}{dx}\int_2^x 3t\sqrt[3]{t^2 + 1}\, dt.$

23. $\frac{d}{dt}\int_3^{\sin t} x^2 \ln x\, dx$

24. $\frac{d}{dt}\int_{2x}^{x^2} e^{-x} \sin x\, dx.$

25. Find $\int_{-1}^{1} \sqrt{1 - x^2}\, dx$. [Interpret the integral as the area under a function.]

26. Suppose the marginal profit (the rate of change for a profit function P) after t months for a company is modeled by the following graph: [Note the following graph represents $y = P'(t)$].



a. Estimate t where the maximum and minimum for the profit function P occur during $t = 0$ and $t = 4$.

b. Estimate t where the profit function P has an inflection point during $t = 0$ and $t = 10$. Explain.

c. Is the company's profit increasing after $t = 6$?