Math 152 Practices for Final Exam

1. (20 points) Set up the integral that will give you the volume of a donut by rotating a circle of radius 1 and 2 units above the $x$-axis around $x$-axis. Please refer to the graph below.

2. Find the volume of the donut from question above.

3. Find the area bounded by two curves: $y = x^2 - 1$ and $y = -x^2 + 1$.

4. $\int \frac{1}{x^2 \sqrt{x^2 - 4}} \, dx$

5. $\int \frac{1}{x^2 + 4} \, dx$

6. $\int \frac{\sin x}{1 + \cos^2 x} \, dx$

7. $\int \frac{x}{(1 + x^2)^3} \, dx$

8. $\int -\frac{2x^2 + x - 4}{x^3 + x^2 - 2x} \, dx$ [hint: $\frac{2x^2 + x - 4}{x^3 + x^2 - 2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$; ans = $2 \ln x - \frac{5}{3} \ln(x-1) - \frac{2}{3} \ln(x+2)]$

9. $\int \frac{2x + 7}{x^2 - x - 2} \, dx$ [hint: $\frac{2x + 7}{x^2 - x - 2} = \frac{A}{x-2} + \frac{B}{x+1}$; ans = $\frac{11}{3} \ln(x-2) - \frac{5}{3} \ln(x+1)$]

10. $\int_{-\infty}^{\infty} \frac{1}{(x+2)^2} \, dx$

11. $\int_{-\infty}^{\infty} xe^x \, dx$

12. $\int \tan^{-1} x \, dx$. (hint: integration by parts)

13. $\int \frac{dx}{(x^2 + 9)^{3/2}}$

14. $\int \frac{dx}{x^2 \sqrt{x^2 - 1}}$ (hint: trig substitution $x = \sec \theta$.)

15. $\int \cos^3 \theta \, d\theta$, (hint: write $\cos^3 \theta = \cos^2 \theta \cos \theta$.)

16. If $f(x) = |x + 1| + |x - 2|$.
   a. Sketch $y = f(x)$.
   b. Compute the area of $y = f(x)$ from $x = 0$ to $x = 3$ directly without using integral(s).
   c. Use the left end Riemann sum when $n = 100$ to approximate $\int_{0}^{3} f(x) \, dx$ [hint: the command for an absolute function is $\text{abs}(x-1)$ and etc.]
Find the area bounded by the following curves.
17. \( y = \sqrt{x}, y = x - 6, \text{and} \ x - axis. \)
18. \( y = \ln(x), y = -x + 4, \text{and} \ x - axis. \)
19. \( x = 3y - y^2 + 3, x = 0 \)
20. \( y = \frac{1}{x^2 + 1}, y = -x^2 + 3. \)
21. If \( g(x) = \int_{0}^{x} f(t) \, dt, \text{where} \ f(t) = e^t(x^3 - 3x^2 + x) \) then
   \begin{enumerate}
   \item determine the interval(s) where \( g \) is increasing,
   \item find the local maximum and minimum of \( g(x) \). (Only \( x \) value is needed).
   \end{enumerate}
22. \( \frac{d}{dx} \int_{2}^{x} 3t \sqrt{t^2 + 1} \, dt. \)
23. \( \frac{d}{dt} \int_{3}^{\sin t} x^2 \ln x \, dx \)
24. \( \frac{d}{dt} \int_{0}^{x^2} e^{-x} \sin x \, dx. \)
25. Find \( \int_{-1}^{1} \sqrt{1 - x^2} \, dx. \) [Interpret the integral as the area under a function.]
26. Suppose the marginal profit (the rate of change for a profit function \( P \)) after \( t \) months for a company is modeled by the following graph: [Note the following graph represents \( y = P'(t) \).]

![Graph of y = P'(t)](image)

\begin{enumerate}
\item Estimate \( t \) where the maximum and minimum for the profit function \( P \) occur during \( t = 0 \) and \( t = 4 \).
\item Estimate \( t \) where the profit function \( P \) has an inflection point during \( t = 0 \) and \( t = 10 \). Explain.
\item Is the company’s profit increasing after \( t = 6 \)?
\end{enumerate}