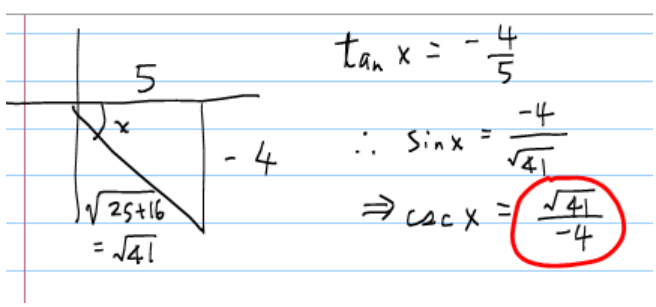
Review for test 4 (Some hints are provided).

1. Remember the graphs for six trigonometric functions.
2. If $f(x) = \cos^{-1}x$ or $\arccos(x)$. Then
   - **a**. the domain and range for f are and respectively.
   - **b**. the graph of f is shown
   - **c**. the domain and range for f are and respectively.
   - **d**. the graph of f is shown below.
3. If $f(x) = \sin^{-1}x$ or $\arcsin(x)$. Then
   - **a**. the domain and range for f are respectively.
   - **b**. the graph of f is shown below.
   - **c**. the domain and range for f are respectively.
   - **d**. the graph of f is shown below.
4. Redo the problem 3 by replacing the function f by $\tan^{-1}x$ and $\cot^{-1}x$.
5. Prove $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ [To prove this is equivalent to prove $\sin^{-1}x = -\cos^{-1}x + \frac{\pi}{2}$. First, you plot $y = \cos^{-1}x$, second you plot $y = -\cos^{-1}x$ (which is a flipping along the $x - axis$ from $y = \cos^{-1}x$) and last you plot $y = -\cos^{-1}x + \frac{\pi}{2}$ and show that this is the same as $y = \sin^{-1}x$.]
6. If $f(x) = \cos^{-1}(x)$, then find the
   - **a**. inverse functions for $f(x+\pi)$ and $f(x) + \ ]\pi$ respectively. [The inverse for $f(x+\pi)$ is $\cos(x) - \pi$; the inverse for $f(x) + \pi$ is $\cos(x - \pi)$.]
   - **b**. inverse functions for $f(x-\pi)$ and $f(x) + \pi$ respectively. [The inverse for $f(x-\pi)$ is $(\cos x) + \pi$,]
   - **c**. inverse functions for $f(x+\pi)$ and $f(x) - \pi$ respectively. [The inverse for $f(x) - \pi$ is $\cos(x+\pi)$
   - **d**. inverse functions for $f(x-\pi)$ and $f(x) - \pi$ respectively.
7. Find the exact vaule of the expression without using a calculator
   - **a**. $\csc(\arctan(-\frac{4}{5})) \ = -\frac{1}{4}\sqrt{41}$, $\cot(\arcsin(\frac{3}{4})) = \frac{1}{3}\sqrt{7}$
     [hint: Set $x = \arctan(-\frac{4}{5})$, this implies that $\tan x = -\frac{4}{5}$ and x has to be in the fourth quadrant because the domain for $y = \tan x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (for us to talk about the $\arctan x$). Next we draw a triangle in the fourth quadrant so that $\tan x = -\frac{4}{5}$, see below



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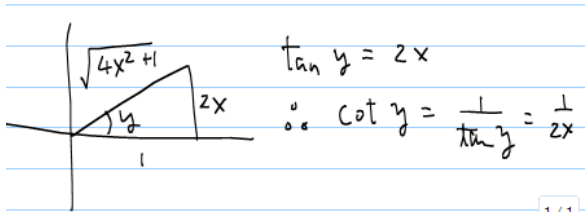
8. Write an algebraic expression that is equivalent to the given expression **if it is meaningful**.

a. $\cot(\arctan 2x), x > 0$. [hint: first set $y = \arctan 2x$, we get $\tan y = 2x$. Since x is positive, $\tan y$ should be also positive, this implies that y is in the first quadrant; draw a triangle with angle y below:



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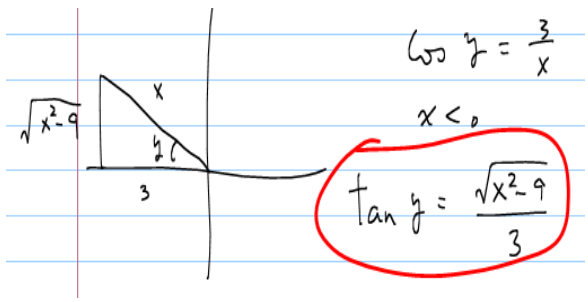
b. $\tan(\arccos \frac{3}{x}), x < 0$. [hint: Set $y = \arccos \frac{3}{x}$, we get $\cos y = \frac{3}{x}$, since $x < 0$, we see $\cos y$ is negative too, so we need y to be in the second quadrant. Draw a triangle (with angle y) in the second quadrant below:



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Notice something wrong with the diagram above, since $x < 0$ but x in this case is the hypotenuse, it can't be negative, so there is no solution in this case.

9. Identify the following identities:

a. $\cot\left(\frac{\pi}{2} - x\right) = \tan x; \sin(-x) = \sin x,$

b. $\cot\left(\frac{\pi}{2} - x\right) = \cot x; \sin(-x) = -\sin x,$

c. $\cot\left(\frac{\pi}{2} - x\right) = \tan x; \cos(-x) = \cos x.$

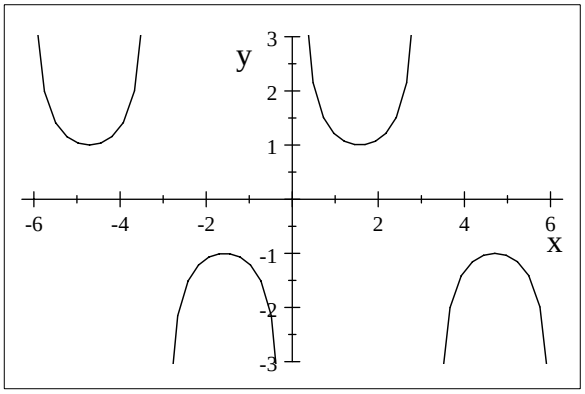
10. If $f(x) = \tan(x)$, then

a. the new function $g(x)$ that is a reflection of $y = f(x)$ along the $y - axis$ is [answer: $g(x) = \tan(-x)$.]

b. the new function $h(x)$ that is a horizontal shifting of $y = g(x)$ to the right $\frac{\pi}{2}$ is [answer: $h(x) = \tan(-(x - \frac{\pi}{2}))$.]

c. the new function $g(x)$ that is a reflection of $y = f(x)$ along the $x - axis$ is [answer: $g(x) = -\tan(x)$].

11. If the graph of $y = \csc(x)$ is given as follow:



a. Then the vertical asymptotes for $y = 3\csc(\frac{x}{4})$ is

b. sketch $y = -3\csc(\frac{x}{4})$ by changing the units from $y = \csc(x)$ without sketching the new graph for $y = -3\csc(\frac{x}{4})$

[Hint: This is explained in number of 11 of 'http://www.radford.edu/~wyang/140/practice4.pdf'.]