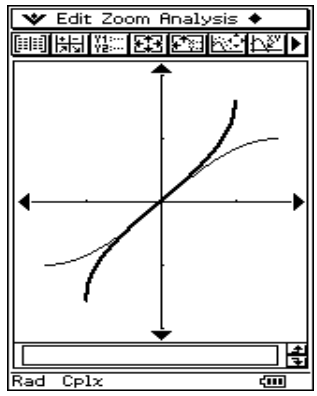
Practice problems.

1. If $\sin t = \frac{4}{5}$, and $0 < t < \frac{\pi}{2}$. then find the followings:
   a. $\sin(\pi - t)$,
   b. $\tan t$,
   c. $\cos(\pi - t)$.
2. If $\cos t = \frac{4}{5}$, and t is in the first quadrant, then find the followings:
   a. $\sin(\frac{\pi}{2} - t)$,
   b. $\tan t$,
   c. $\cos(\pi - t)$.
3. If $t = -\frac{3\pi}{4}$, find $\sin t$ and $\cos t$.
4. If $f(x) = \cos^{-1}x$ or $\arccos(x)$. Then
   a. find the domain and range for f, [answer: domain=$[-1, 1]$ and the range is $[0, \pi]$).
   b. sketch the graph of f and f^{-1}together
   c. explain how you can find $\cos^{-1}1, \cos^{-1}0$, and $\cos^{-1}(\frac{1}{2})$ respectively without using a calculator. [You set $x = \cos^{-1}1$ and apply cosine function both sides, we get $\cos x = 1$, which implies that $x = 0$, we only pick the answer from the domain of $y = \cos x$. Similarly, we can show that $\cos^{-1}0 = \frac{1}{2}\pi$. Finally, we set $x = \cos^{-1}(\frac{1}{2})$, and we get $\cos x = \frac{1}{2}$, this is a special case when $x = \frac{1}{3}\pi$.]
5. If $f(x) = \sin^{-1}x$ or $\arcsin(x)$. Then
   a. find the domain and range for f, [domain is $[-1, 1]$ and range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.]
   b. sketch the graph of f and f^{-1}together.



c. explain how you can find $\sin^{-1}1, \sin^{-1}0,$ and $\sin^{-1}(-\frac{1}{2})$ respectively without using a calculator. [We set $x = \sin^{-1}1,$ and apply sine function on both side. We get $\sin x = 1$ and find $x = \frac{\pi}{2}$. Similarly we get $\sin^{-1}0 = 0$ and $\sin^{-1}(-\frac{1}{2}) = -\frac{1}{6}\pi,$ notice that we need to pick the value in the domain of $\sin x$ to make $\sin x = -\frac{1}{2},$ that is why we pick the one $(-\frac{1}{6}\pi)$ in the fourth quadrant.

6. If $f(x) = (\sin^{-1}x) + 1,$ then

a. find f^{-1}, [answer: $f^{-1}(x) = \sin(x-1)$].

b. find the domain and range for f^{-1}. [Since the domain of $y = \sin x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so the domain for $y = \sin(x-1)$ is $[[-\frac{\pi}{2}+1, \frac{\pi}{2}+1]$ and the range is $[-1, 1]$.

7. If $f(x) = \cos^{-1}(x+1),$ then

a. find f^{-1}, [answer: $f^{-1}(x) = (\cos x) - 1.$]

b. find the domain and range for f^{-1}. [Answer: the domain for $f^{-1}(x) = (\cos x) - 1$ is $[0, \pi]$ and the range is $[-2, 0]$.

8. Relate $\tan(x)$ and $\cot x$ by an equation. [$\tan(x) = \cot(-(x - \frac{\pi}{2})) = \cot(\frac{\pi}{2} - x)$].

9. Which of the following identities (identity) are (is) true? Answer: (c).

a. $\cot\left(\frac{\pi}{2} - x\right) = \tan x; \sin(-x) = \sin x,$

b. $\cot\left(\frac{\pi}{2} - x\right) = \cot x; \sin(-x) = -\sin x,$

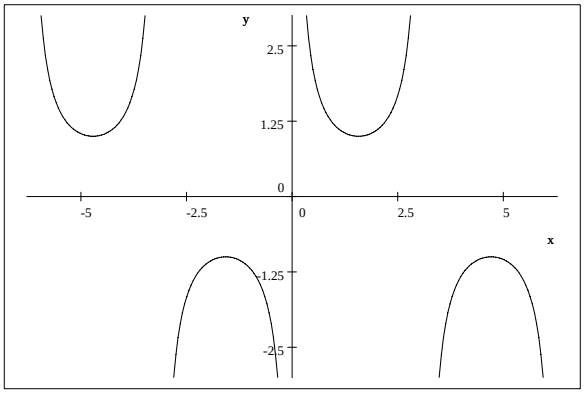
c. $\cot\left(\frac{\pi}{2} - x\right) = \tan x; \cos(-x) = \cos x.$

10. If $f(x) = \tan(x),$ then

a. Write down the new function $g(x)$ that is a reflection of $y = f(x)$ along the $y - axis$.[Answer: $g(x) = \tan(-x)$.]

b. Write down the new function $h(x)$ that is a horizontal shifting of $y = g(x)$ to the right $\frac{\pi}{2}$. [Answer: $h(x) = \tan(-(x - \frac{\pi}{2})) = \tan(\frac{\pi}{2} - x)$.

11. If the graph of $y = \csc(x)$ is given as follow:



a. find period for $y = 3\csc(\frac{x}{4})$, [answer: $\frac{2\pi}{\frac{1}{4}} = 8\pi$]

b. find the vertical asymptotes for $y = 3\csc(\frac{x}{4})$, [We consider $y = \csc(\frac{x}{4}) = \dfrac{1}{\sin(\frac{x}{4})}$.
If we let $\sin x = 0$, we have $x = \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots$ by looking at the zeroes of the graph of $y = \sin x$. Now we need $\sin(\frac{x}{4}) = 0$, therefore we set $\frac{x}{4} = \ldots -2\pi, -\pi, 0, \pi, 2\pi, \ldots$. In other words, $x = -8\pi, -4\pi, 0, -4\pi, 8\pi, \ldots$, these are the vertical asymptotes for $y = \cos(\frac{x}{4})$, which are also the vertical asymptotes for $y = 3\csc(\frac{x}{4})$.

c. sketch $y = -3\csc(\frac{x}{4})$ by changing the horizontal and vertical units from $y = \csc(x)$.

12. If $f(x) = 3\cos(3x + \pi)$, then

a. find the amplitude for f, [answer: the amplitude of f is 3].

b. find the period for f, [we write $y = 3\cos(3x + \pi) = 3\cos\left(3\left(x + \frac{\pi}{3}\right)\right)$, therefore, the period for f is $\frac{2\pi}{3}$.

c.]find the relationship between $y = f(x)$ and $y = 3\cos(3x)$ [Note that $y = f(x) = 3\cos\left(3\left(x + \frac{\pi}{3}\right)\right)$ is being shifted to the left $\frac{\pi}{3}$ units from $y = 3\cos(3x)$.]

13. If $f(x) = \sec(x)$ and $g(x) = \csc(x)$

a. Sketch the graphs of $y = f(x)$ and $y = g(x)$.

b. Write down an equation that will relate these two functions of f and g. [$\sec(x - \frac{\pi}{2}) = \csc(x)$.]

14. Define a function f, whose graph is similar to the sine function, $\sin x$, and satisfies ALL the following conditions:

a. the period of f is 4π,

b. the function f has amplitude of 2,

c. the function f has a maximum value of 6 when $x = \pi$. [Answer: $f(x) = 2\sin(\frac{x}{2}) + 4$.]