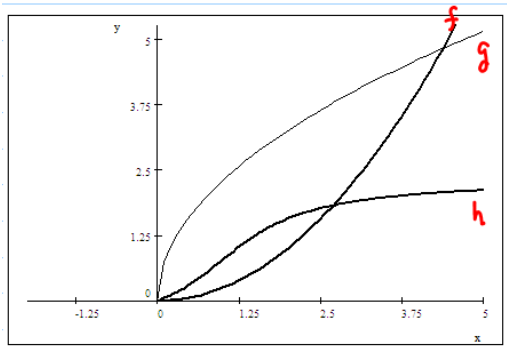
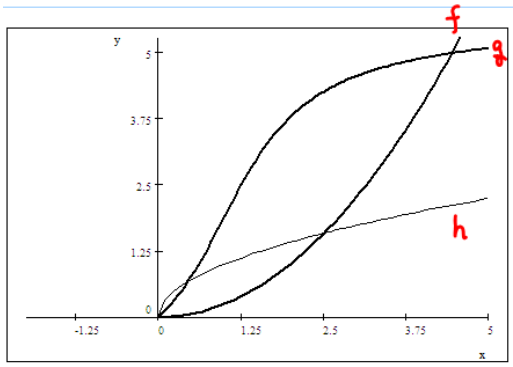
Review for test 4.

1. For $f(x) = x^2 + 3x$.
   a. Use $h = 0.001$ and $h = -0.001$ to find $\dfrac{f(1+h) - f(1)}{h}$ respectively.
   b. Use the definition of $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to prove that $f'(x) = 2x + 3$.
2. Find the derivative for the following functions (You do not need to **simplify** your answers)
   a. $f(x) = 2x\sqrt{3x - 2}$
   b. $f(x) = \sqrt{x(2x - 3)}$
   c. $f(x) = 3x\sqrt[3]{(2x - 1)^2}$
   d. $f(x) = \dfrac{2x - 3}{x - 4}$
   e. $f(x) = (3x - 1)^2(2x - 5)$
   f. More on product/quotient rules from the text, page 129, #23-37 odd numbers
   g. More on chain rule from the text, page 139, #23,25,27,55,57,59.
3. Find the following limits:
   a. $\lim_{x \to 1} \dfrac{x^2 - 4}{x - 2}$
   b. $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$
   c. $\lim_{x \to (-1)^+} \sqrt{x + 1}$
4. Suppose $P(x)$ is a profit function (where x denotes the number of units produced). Explain the followings:
   a. If $P'(100) > 0 > P'(101)$ then sketch $y = P(x)$ for this scenario. Should one increase or decrease the number of units produced? Explain.
   b. If $P'(100) < P'(101) < 0$ then sketch $y = P(x)$ for this scenario. Should one increase or decrease the number of units produced?
5. If $C(x) = \frac{1}{3}x^3 - \frac{9}{2}x^2 + 14x$ represent the cost function. Explain.
   a. Find the exact cost for the 34th unit. [hint: $C(34) - C(33)$]
   b. Find the marginal cost function.
   c. Use the **marginal cost** to estimate the cost for the 34*th* unit. [hint: Use $C'(33)$].
   d. Find the number of units produced which will either maximize or minimize the cost.
6. If $P(x) = -0.25x^2 + 2000x - 250$ represent the profit function.
   a. Find the marginal profit.
   b. What is profit (or loss) initially (when $x = 0$)?
   c. Use the **marginal profit** to estimate the profit for the 32*nd* unit.
   d. Find the number of units produced which will maximize the profit.
   e. What is the maximum profit?
7. Given three distance functions for three runners, f, g and h are shown as follows, where x represents time and y represent distance. Then answer the following questions:



a. Does any one of the runners lead the race from beginning to the end?
b. Who starts out faster?
c. Which runner lose the race near the end?
d. If the race finishes at $y = 5$ units, who will win?

8. Given three distance functions for three runners, f, g and h are shown as follows, where x represents time and y represent distance. Then answer the following questions:



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