Math 121 Review for final

1. Find the first derivative for the following functions (You do not need to simplify)
   a. \( f(x) = 3x \sqrt{(2x - 1)^2} \)
   b. \( f(x) = \left( \frac{-2x - 3}{x - 4} \right)^2 \)
   c. \( f(x) = \sqrt{(3x - 1)^2} (-2x - 5) \)
   d. \( f(x) = \frac{3x^2 + 2x - 1}{x} \)
   e. \( f(x) = 3 \sqrt{x^5} + \frac{2}{\sqrt{x}} \)
   f. \( f(x) = \sqrt{x} (\sqrt{x} + 3)^2 \)
   g. \( f(x) = \left( \frac{3x - 1}{x^2 + 3} \right)^2 \)

2. A rectangle is bounded by the \( x - \) and the graph of the semicircle \( y = \sqrt{25 - x^2} \). What length and width should the rectangle have so that its area is a maximum.
   a. Set up the area function \( A \).
   b. Find \( A'(x) \).

3. Suppose you are given the graph of the marginal cost (derivative for cost) for a company. [ The \( x - \) axis represents the number of units in thousands, and the \( y - \) axis represents the cost in millions]. And we assume the \( x - \) intercepts are at \( x = 1, 3 \) and \( 5 \).

   ![Graph of marginal cost function](image)

   a. When will the company achieve its largest and lowest cost respectively?
   b. When will the cost increase the fastest during \( x = 2 \) and \( x = 5 \)?
   c. When will the cost decrease the most during \( x = 0 \) and \( x = 3 \)?
   d. Find the inflection points for the cost function.
   e. Explain in your own words how inflection points are related to the cost function.

4. Suppose the rate of change for the number \( N \) of bacteria in a culture after \( t \) days is modeled by the following graph: [Note the following graph represents \( y = N'(t) \)].
a. Does the function $N$ have a maximum or minimum during $t = 0$ and $t = 10$?

b. When does $N$ increase the slowest?

c. Is the number of bacteria stabilized after 10 days?

5. If the rate of change for the number $N$ (refer to problem 4 above) is given by the following graph:

![Graph]

a. Does the function $N$ have a maximum or minimum during $t = 0$ and $t = 10$?

b. When does $N$ increase the fastest?

c. Is the number of bacteria stabilized after 10 days?

6. If $f(x) = -3\sqrt{(x - 1)^2} + 3$, then

a. find $f'(x)$,

b. use the signs of $f'(x)$ to find the interval(s) where $f$ is increasing or decreasing,

c. find the maximum or minimum for $f$, if any.

7. Suppose that profit function of a company is given by $P(x) = -0.01x^2 + 50x - 10,000$.

a. Find the value of $x$ that maximizes the profit. (Hint: set $P'(x) = 0$ and solve for $x, x = 2500$)

b. Determine the maximum profit. ($P(2500) = 52500$)

c. Use the marginal profit to estimate the profit of the 2501 unit. ($P'(2500) = 0$, i.e. there is no profit for the 2501 unit).

8. Discuss (a) the interval(s) where $f$ is increasing or decreasing (b) the relative maximum or minimum of $f$ if the derivative of the function $f$ is given below.

a. $f'(x) = \frac{x-1}{x+2}$

b. $f'(x) = -(x + 1)^2(x - 3)(x - 5)$
9. If \( f(x) = -3\sqrt{2x - 1} \),
   a. find \( f' \)
   b. find the critical number(s) for \( f \)
   c. use \( f' \) to find the interval where \( f \) is increasing or decreasing
   d. use \( f'' \) to find the relative maximum or minimum for \( f \)
   e. use \( f''' \) to find the interval(s) where \( f \) is concave upward or concave downward
   f. find the inflection point(s).

10. If \( f(x) = \frac{-2x - 3}{\sqrt{x + 5}} \),
    a. find \( f' \)
    b. find the critical number(s) for \( f \)
    c. use \( f' \) to find the interval where \( f \) is increasing or decreasing
    d. use \( f'' \) to find the relative maximum or minimum for \( f \)
    e. use \( f''' \) to find the interval(s) where \( f \) is concave upward or concave downward
    f. find the inflection point(s).

11. If \( R(x) = -\frac{1}{3}x^3 + \frac{9}{2}x^2 - 14x \) represent the revenue function. Explain.
    a. Find the marginal revenue function.
    b. Use the marginal revenue function to estimate the revenue for the 30th unit.
    c. Find the number of units produced which will either maximize or minimize the revenue.

12. If the cost function of producing \( x \) units for some company is given by
    \[
    C(x) = \left( \frac{2x^2}{x + 1} \right)^3.
    \]
    Estimate the cost of producing the second unit.