Math 121 Review for final

1. Find the first derivative for the following functions (You do not need to simplify)

a. $f(x) = 3x\sqrt[5]{(2x-1)^2}$

b. $f(x) = \left(\dfrac{-2x-3}{x-4}\right)^2$

c. $f(x) = \sqrt[3]{(3x-1)^2}\,(-2x-5)$

d. $f(x) = \frac{3x^3+2x-1}{x}$

e. $f(x) = 3\sqrt[3]{x^5} + \frac{2}{\sqrt{x}}$

f. $f(x) = \sqrt[3]{x}\,(\sqrt{x}+3)^2$

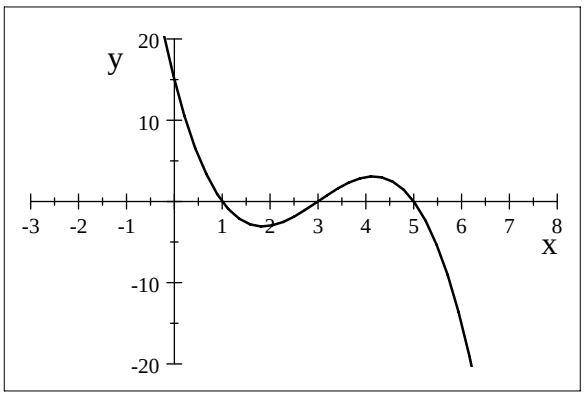
g. $f(x) = \left(\frac{3x-1}{x^2+3}\right)^2$

2. A rectangle is bounded by the $x-$ and the graph of the semicircle $y = \sqrt{25-x^2}$. What length and width should the rectangle have so that its area is a maximum.

a. Set up the area function A.

b. Find $A'(x)$.

3. Suppose you are given the graph of the **marginal cost** (derivative for cost) for a company. [The $x-$ *axis* represents the number of units in thousands, and the $y-$ *axis* represents the cost in millions]. And we assume the $x-$ *intercepts* are at $x = 1,\ 3$ and 5.



a. When will the company achieve its largest and lowest cost respectively?

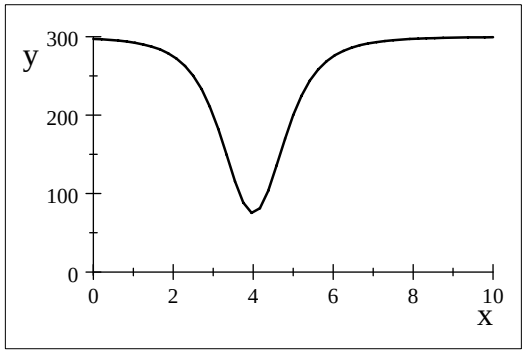
b. When will the cost increase the fastest during $x = 2$ and $x = 5$?

c. When will the cost decrease the most during $x = 0$ and $x = 3$?

d. Find the inflection points for the cost function.

e. Explain in your own words how inflection points are related to the cost function.

4. Suppose the **rate of change** for the number N of bacteria in a culture after t days is modeled by the following graph: [Note the following graph represents $y = N'(t)$].

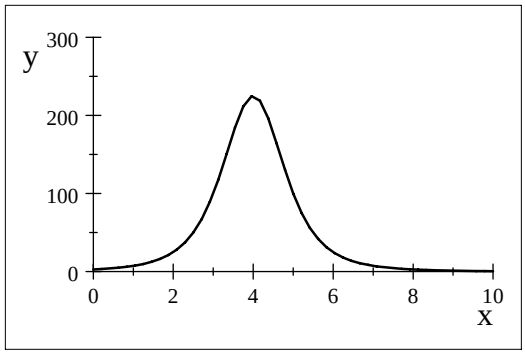


a. Does the function N have a maximum or minimum during $t = 0$ and $t = 10$?

b. When does N increase the slowest?

c. Is the number of bacteria stablized after 10 days?

5. If the **rate of change for the number** N (refer to problem 4 above) is given by the following graph:



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a. Does the function N have a maximum or minimum during $t = 0$ and $t = 10$?

b. When does N increase the fastest?

c. Is the number of bacteria stablized after 10 days?

6. If $f(x) = -3\sqrt[3]{(x-1)^2} + 3$, then

a. find $f'(x)$,

b. use the signs of $f'(x)$ to find the interval(s) where f is increasing or decreasing,

c. find the maximum or minimum for f, if any.

7. Suppose that profit function of a company is given by $P(x) = -0.01x^2 + 50x - 10,000$.

a. Find the value of x that maximzies the profit. (Hint: set $P'(x) = 0$ and solve for $x, x = 2500$)

b. Determine the maximum profit. ($P(2500) = 52500$)

c. Use the marginal profit to estimate the profit of the 2501 unit. ($P'(2500) = 0$, i.e. there is no profit for the 2501 unit).

8. Discuss (**a**) the interval(s) where f is increasing or decreasing (**b**) the relative maximum or minimum of f if the deriative of the function f is given below.

a. $f'(x) = \dfrac{x-1}{x+2}$

b. $f'(x) = -(x+1)^2(x-3)(x-5)$

c. $f'(x) = \dfrac{-2x-3}{\sqrt{x+5}}$

9. If $f(x) = -3\sqrt[3]{(2x-1)}$,

a. find f'

b. find the critical number(s) for f,

c. use f' to find the interval where f is increasing or decreasing

d. use f' to find the relative maximum or minimum for f.

e. use f'' to find the interval(s) where f is concave upward or concave downward

f. find the inflection point(s).

10. If $f(x) = \dfrac{-2x-1}{x-3}$.

a. find f'

b. find the critical number(s) for f,

c. use f' to find the interval where f is increasing or decreasing

d. use f' to find the relative maximum or minimum for f.

e. use f'' to find the interval(s) where f is concave upward or concave downward

f. find the inflection point(s).

11. If $R(x) = \frac{-1}{3}x^3 + \frac{9}{2}x^2 - 14x$ represent the revenue function. Explain.

a. Find the marginal revenue function.

b. Use the **marginal revenue function** to estimate the revenue for the 30*th* unit.

c. Find the number of units produced which will either maximize or minimize the revenue.

12. If the cost function of producing x units for some company is given by

$$C(x) = \left(\frac{2x^2}{x+1}\right)^3.$$

Estimate the cost of producing the second unit.