

Section 3.2/3.3

Truth Tables

Review of the connectors

Connector	Symbol
Or	\vee
And	\wedge
If – then (Conditional)	\rightarrow
Negation	\sim

Using the connectors in a truth table

Basic Truth tables

Example 1 Construct a truth table for $p \vee q$

p	Q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Think of the statement “Either you like apple or you like oranges”

This statement is true unless “you don’t like oranges” and “you don’t like apples” (See red row in the truth table)

Example 2 Construct a truth table for $p \wedge q$

p	Q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Using the statement “You like apples and oranges”, it turns out that this statement is true only if you both like apples and oranges. (See blue) In the last three cases (rows) the statement is false. (See red)

Example 3

Construct a truth table for $p \rightarrow q$

p	Q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Using the example “If you study for the test, you will pass the test”, it turns out that this is all true except when the hypothesis “If you study for the test” is true, and the conclusion “you will pass the test” is false (See red)

Other examples of Truth Tables

Example 4

Construct a truth table for $(p \wedge q) \rightarrow q$

p	Q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

When a compound statement results with all true statements in the last column it is called a **tautology** (True in all cases)

Example 5

Construct a truth table for $(p \wedge q) \vee p$

p	Q	$p \wedge q$	$(p \wedge q) \vee p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Example 6

Construct a truth table for $(p \wedge q) \vee (\sim p)$

p	Q	$\sim p$	$p \wedge q$	$(p \wedge q) \vee (\sim p)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Example 7

Construct a truth table for $\sim (p \vee q)$

p	Q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Example 8

Construct a truth table for

$(\sim (p \vee q)) \rightarrow q$

p	Q	$p \vee q$	$\sim (p \vee q)$	$(\sim (p \vee q)) \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

Example 9

Construct a truth table for $(p \wedge q) \rightarrow (p \vee q)$

p	Q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

This is a tautology

Example 10

Construct a truth table for $p \vee (\sim r \wedge q)$

p	Q	R	$\sim r$	$(\sim r \wedge q)$	$p \vee (\sim r \wedge q)$
T	T	T	F	F	T
T	T	F	T	F	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Example 11

Construct a truth table for $(p \wedge q) \rightarrow r$

p	Q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Equivalent Statements

Equivalent Statements will have the same result in the last column of their truth tables.

Example 12

Compare the truth tables for $\sim (p \vee q)$ and $\sim p \wedge \sim q$

Truth table for $\sim (p \vee q)$

P	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Truth table for $\sim p \wedge \sim q$

P	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Notice that the last columns of each table are identical. Thus, the arguments are equivalent.

Example 13

Compare the truth tables for $\sim (p \wedge q)$ and $\sim p \vee \sim q$

Truth table for $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Truth table for $\sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Again the truth tables have the same last column. Thus, the statements are equivalent.

De Morgan's Laws

$$\sim (p \vee q) = \sim p \wedge \sim q$$

$$\sim (p \wedge q) = \sim p \vee \sim q$$