

## Math 151 Study List 1

### I) Functions

1) Find the domain of  $f(x) = \frac{1}{x-3}$

**The function is undefined at  $x = 3$**

**Domain:**  $(-\infty, 3) \cup (3, \infty)$

2) Find the inverse of  $f(x) = x^5 - 2$

$$f(x) = x^5 - 2$$

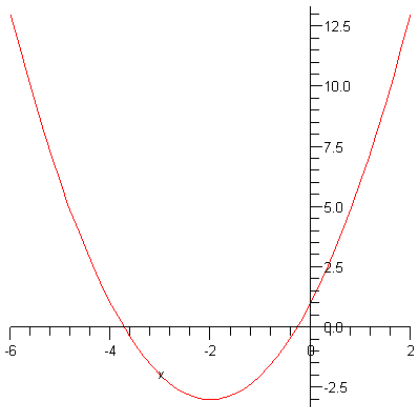
$$y = x^5 - 2$$

$$x = y^5 - 2$$

$$x + 2 = y^5$$

$$\sqrt[5]{x+2} = \sqrt[5]{y^5} \Rightarrow f^{-1}(x) = \sqrt[5]{x+2}$$

3) Graph the function and state its range and domain.  $f(x) = (x+2)^2 - 3$



**Domain:**  $(-\infty, \infty)$

**Range:**  $[-3, \infty)$

4) Find the inverse of  $f(x) = e^{x+2} - 3$

$$f(x) = e^{x+2} - 3$$

$$y = e^{x+2} - 3$$

$$x = e^{y+2} - 3$$

$$x + 3 = e^{y+2}$$

$$\ln(x + 3) = \ln(e^{y+2})$$

$$\ln(x + 3) = y + 2$$

$$y = \ln(x + 3) - 2$$

$$f^{-1}(x) = \ln(x + 3) - 2$$

5) Given the functions  $f(x) = x^2 - 3x + 3$  and  $g(x) = 4x - 8$ , find the following:

a) Find  $f(3)$

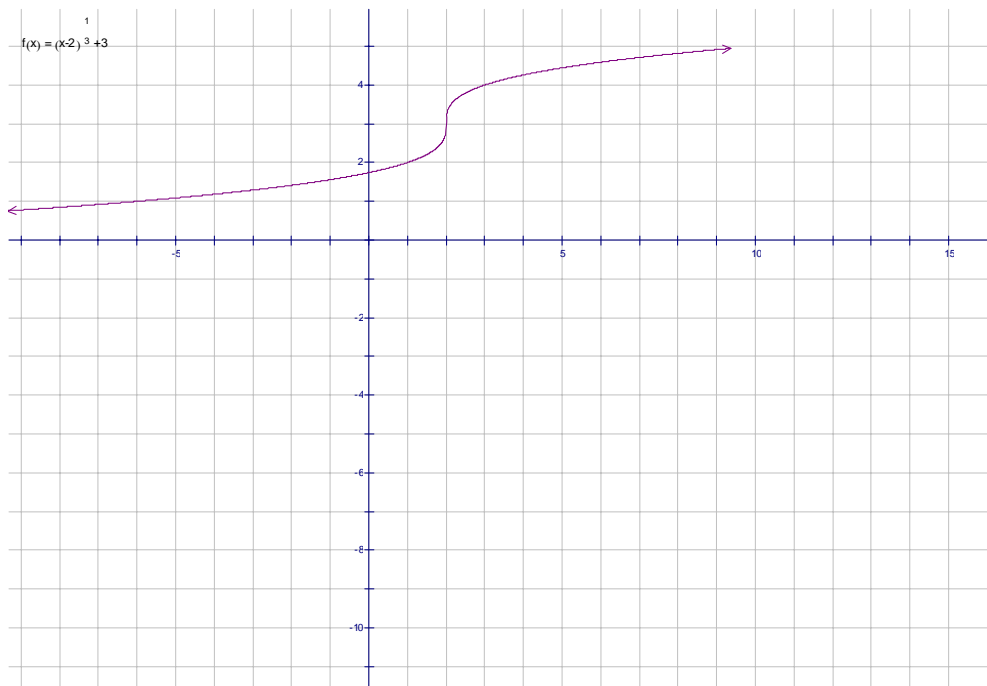
$$f(3) = 3^2 - 3(3) + 3 = 3$$

b) Find  $f(g(x))$  and  $g(f(x))$

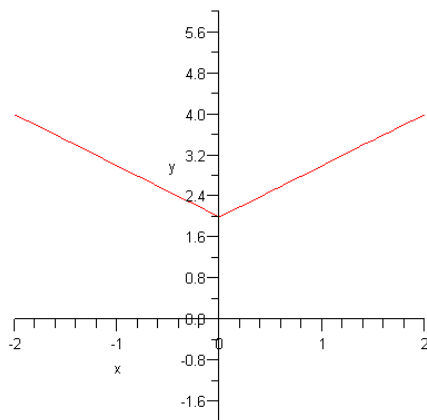
$$\begin{aligned} f(g(x)) &= f(4x - 8) = (4x - 8)^2 - 3(4x - 8) + 3 = 16x^2 - 64x + 64 - 12x + 24 + 3 \\ &= 16x^2 - 76x + 91 \end{aligned}$$

$$\begin{aligned} g(f(x)) &= 4(x - 3x + 3) - 8 = 4x^2 - 12x + 12 - 8 \\ &= 4x^2 - 12x + 4 \end{aligned}$$

6) Graph  $g(x) = \sqrt[3]{x-2} + 3$



7) Is the function below one-to-one? Does it have an inverse?



The following function fails the horizontal line test. Therefore, it is not one-to-one. Since the function is not one-to-one, the function has no inverse.

## II) Logarithms, Natural Logarithms, and the exponential function.

1) Solve  $e^{3x-4} = 4$

$$e^{3x-4} = 4$$

$$\ln(e^{3x-4}) = \ln(4)$$

$$3x - 4 = \ln(4)$$

$$3x = \ln(4) + 4$$

$$x = \frac{\ln(4) + 4}{3}$$

2) Solve  $\ln(5x - 4) = 7$

$$\ln(5x - 4) = 7$$

$$e^{\ln(5x-4)} = e^7$$

$$5x - 4 = e^7$$

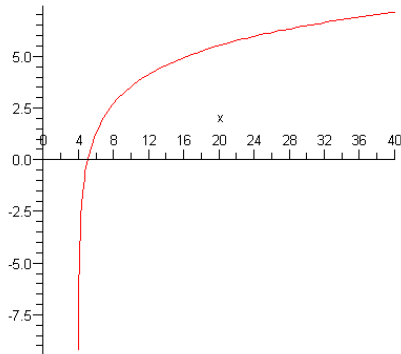
$$5x = e^7 + 4$$

$$x = \frac{e^7 + 4}{5}$$

3) Graph  $f(x) = 2\ln(x - 4)$ . State the domain and range of the function.

**Omit**

Here is the graph



4) Find  $\log_5 \frac{1}{625}$

$$5^x = \frac{1}{625}$$

$$5^x = \frac{1}{5^4}$$

$$5^x = 5^{-4}$$

$$x = -4$$

5) Simplify the following logarithmic expression to an expression with only one natural log term.  $\ln(x+4) + 3\ln(x-4) - 2\ln(x-2)$

$$\ln(x+4) + 3\ln(x-4) - 2\ln(x-2)$$

$$\ln(x+4) + \ln(x-4)^3 - \ln(x-2)^2$$

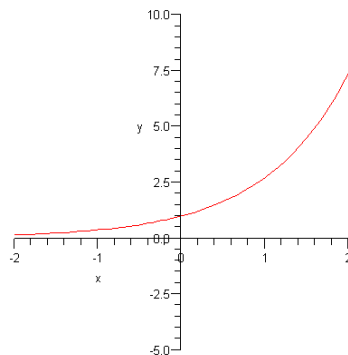
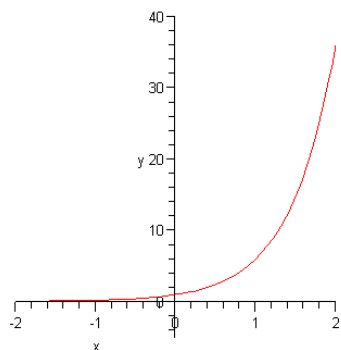
$$\ln\left[(x+4)(x-4)^3\right] - \ln(x-2)^2$$

$$\ln\left[\frac{(x+4)(x-4)^3}{(x-2)^2}\right]$$

6) Simplify  $e^{2\ln x + 3\ln y}$

$$e^{2\ln x + 3\ln y} = e^{\ln x^2 + \ln y^3} = e^{\ln x^2 y^3} = x^2 y^3$$

7) Graph both  $g(x) = e^x$  and  $f(x) = 6^x$  and then compare the graphs of these functions.



8) Find  $\log_3 243$

$$\log_3 243$$

$$3^x = 243$$

$$3^x = 3^5$$

$$x = 5$$

9) Given that the population of a bacteria is represented by the function

$f(t) = 400e^{-.3t}$  where  $t$  represent the amount of time in months. Find out how long it will take for the population of bacteria to reach 10,000.

$$f(t) = 400e^{-.3t}$$

$$10,000 = 400e^{-.3t}$$

$$\frac{10,000}{400} = \frac{400e^{-.3t}}{400}$$

$$25 = e^{-.3t}$$

$$\ln(25) = \ln(e^{-.3t})$$

$$\ln(25) = -.3t$$

$$t = \frac{\ln(25)}{-.3} = 10.7 \text{ years}$$

### III) Trigonometry

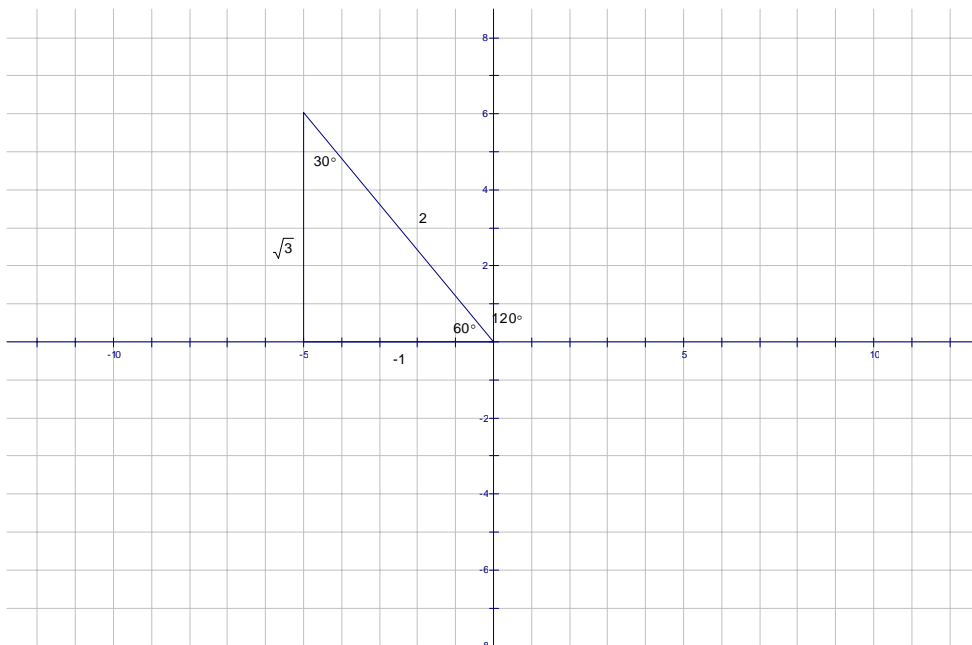
1) Convert  $\frac{5\pi}{4}$  to degrees

$$\frac{5\pi}{4} \cdot \frac{180}{\pi} = 5(45) = 225^\circ$$

2) Convert  $120^\circ$  to radians

$$120^\circ \cdot \frac{\pi}{180} = \frac{2\pi}{3}$$

3) Given the radian measure of  $\frac{2\pi}{3}$  in standard position, find the values of the six trigonometric values



$$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}; \cos \frac{2\pi}{3} = \frac{x}{r} = -\frac{1}{2}; \tan \frac{2\pi}{3} = \frac{y}{x} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$$

$$\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}; \sec \frac{2\pi}{3} = \frac{r}{x} = -\frac{2}{1}; \cot \frac{2\pi}{3} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

- 4) Given that  $\sin \theta = \frac{3}{5}$  and the angle  $\theta$  lies in Quadrant II, find the value of the other five trigonometric functions.

$$y = 3 \text{ and } r = 5$$

$$x^2 + y^2 = r^2$$

$$x^2 + 3^2 = 5^2$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = -4$$

$$\cos \theta = \frac{x}{r} = -\frac{4}{5}; \tan \theta = \frac{y}{x} = -\frac{3}{4}$$

$$\csc \theta = \frac{r}{y} = \frac{5}{3}; \sec \theta = \frac{r}{x} = -\frac{5}{4}; \cot \theta = \frac{x}{y} = -\frac{4}{3}$$

5) Find  $\sin\left(\frac{\pi}{6}\right)$

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

6) Find  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}; \frac{2\pi}{3}$$

7) Solve  $2\sin x - \sqrt{3} = 0$

$$2\sin x - \sqrt{3} = 0$$

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}; \frac{2\pi}{3}$$