

**Math 151 Study List 3  
Solutions**

**Find the derivative of each function.**

1)

$$y = x^4 + 5x^2 + 7x + 6$$

$$y' = 4x^3 + 10x + 7$$

2)

$$y = 5x^4 - 8x^3$$

$$y' = 20x^3 - 24x^2$$

3)

$$f(x) = 8\sin(x)$$

$$f'(x) = 8\cos(x)$$

4)

$$f(x) = \cos(4x^2)$$

$$f'(x) = 8x\cos(4x^2)$$

5)

$$y = (x^2 + 3x)(2x^2 + 6x + 4)$$

$$y' = (2x + 3)(2x^2 + 6x + 4) + (4x^2 + 6)(x^2 + 3x)$$

$$y' = 4x^3 + 12x^2 + 8x + 6x^2 + 18x + 12 + 4x^4 + 12x^3 + 6x^2 + 18x$$

$$y' = 4x^4 + 16x^3 + 24x^2 + 44x + 12$$

6)

$$y = \frac{x^2 + 3}{3x^2 - 4x}$$

$$y' = \frac{(3x^2 - 4x)\frac{d}{dx}(x^2 + 3) - (x^2 + 3)\frac{d}{dx}(3x^2 - 4x)}{(3x^2 - 4x)^2}$$

$$y' = \frac{(3x^2 - 4x)(2x) - (x^2 + 3)(6x - 4)}{(3x^2 - 4x)^2}$$

$$y' = \frac{6x^3 - 8x^2 - (6x^3 - 4x^2 - 18x + 12)}{(3x^2 - 4x)^2}$$

$$y' = \frac{-4x^2 + 18x - 12}{(3x^2 - 4x)^2}$$

7)

$$f(x) = e^{x^2} \sin(3x)$$

$$f'(x) = \frac{d}{dx}(e^{x^2})\sin(3x) + \frac{d}{dx}\sin(3x)(e^{x^2})$$

$$f'(x) = 2xe^{x^2} \sin(3x) + 3e^{x^2} \cos(3x)$$

8)

$$f(x) = 8x^2 e^{x^2-8x}$$

$$f'(x) = \frac{d}{dx}(8x^2)e^{x^2-8x} + \frac{d}{dx}(e^{x^2-8x})(8x^2)$$

$$f'(x) = 16xe^{x^2-8x} + (2x-8)e^{x^2-8x}(8x^2)$$

$$f'(x) = 16xe^{x^2-8x} + 16x^3e^{x^2-8x} - 64x^2e^{x^2-8x}$$

9)

$$f(x) = \frac{e^{3x} - 2}{e^{-2x}}$$

$$f'(x) = \frac{(e^{-2x})(3e^{3x}) - (e^{3x} - 2)(-2e^{-2x})}{(e^{-2x})^2}$$

$$f'(x) = \frac{3e^x + 2e^x + 4e^{-2x}}{e^{-4x}}$$

$$f'(x) = \frac{5e^x + 4e^{-2x}}{e^{-4x}}$$

or

$$f'(x) = e^{4x}(5e^x + 4e^{-2x})$$

10)

$$y = (x^2 + 2x)^4$$

$$y' = 4(2x + 2)(x^2 + 2x)^3$$

$$y' = (8x + 8)(x^2 + 2x)^3$$

11)

$$g(x) = \ln(x^2 - 4x + 4)$$

$$g'(x) = \frac{2x - 4}{x^2 - 4x + 4}$$

12)

$$y = \tan(2x^2 + 3x)$$

$$y' = (4x + 3)\sec^2(2x^2 + 3)$$

13)

$$h(x) = 5x \ln(x^2)$$

$$h'(x) = \frac{d}{dx}(5x) \ln(x^2) + \frac{d}{dx}(\ln(x^2))(5x)$$

$$h'(x) = 5 \ln(x^2) + \frac{2x}{x^2}(5x)$$

$$h'(x) = 5 \ln(x^2) + 10$$

14)  $y = \frac{\ln(4x) - e^x}{\ln(3x^2)}$

$$y = \frac{\ln(4x) - e^x}{\ln(3x^2)}$$

$$y' = \frac{(\ln(3x^2))\left(\frac{4}{4x} - e^x\right) + (\ln(4x) - e^x)\frac{6x}{3x^2}}{[\ln(3x^2)]^2}$$

$$y' = \frac{(\ln(3x^2))\left(\frac{1}{x} - e^x\right) + (\ln(4x) - e^x)\left(\frac{2}{x}\right)}{[\ln(3x^2)]^2}$$

15)

$$f(x) = x^3 \sin(2x^3)$$

$$f'(x) = \frac{d}{dx}(x^3) \sin(2x^3) + \frac{d}{dx}(\sin(2x^3))(x^3)$$

$$f'(x) = 3x^2 \sin(2x^3) + 3x^2 \cos(2x^3)(x^3)$$

$$f'(x) = 3x^2 \sin(2x^3) + 3x^5 \cos(2x^3)$$

16)

$$y = e^{x^2 \sin(2x)}$$

$$y' = \left( \frac{d}{dx}(x^2) \sin(2x) + \frac{d}{dx}(\sin(2x))(x^2) \right) e^{x^2 \sin(2x)}$$

$$y' = (2x \sin(2x) + 2 \cos(2x)(x^2)) e^{x^2 \sin(2x)}$$

$$y' = (2x \sin(2x) + 2x^2 \cos(2x)) e^{x^2 \sin(2x)}$$

17)

$$f(x) = \cos(\ln(x))$$

$$f'(x) = \frac{d}{dx} \ln(x) (-\sin(\ln(x)))$$

$$f'(x) = -\frac{1}{x} \sin(\ln(x))$$

$$f'(x) = -\frac{\sin(\ln(x))}{x}$$

**Find the equation of the tangent line through the given point.**

18)  $y = x^3 - 2$ ;  $(1, -1)$

$$y = x^3 - 2$$

$$y' = 3x^2$$

$$m = 3(1)^2 = 3(1) = 3$$

$$y - (-1) = 3(x - 1)$$

$$y + 1 = 3x - 3$$

$$y = 3x - 4$$

$$19) f(x) = x^2 \sin(x); (\pi, 0)$$

$$f(x) = x^2 \sin(x)$$

$$f'(x) = 2x \sin(x) + \cos x(x^2)$$

$$f'(x) = 2x \sin x + x^2 \cos x$$

$$m = f'(\pi) = 2(\pi) \sin(\pi) + \pi^2 \cos(\pi) = 0 + (-\pi^2) = -\pi^2$$

$$y - 0 = -\pi^2(x - \pi)$$

$$y = -\pi^2(x - \pi)$$

$$20) f(x) = e^{x^2-1}; (1, 1)$$

$$f(x) = e^{x^2-1}$$

$$f'(x) = 2xe^{x^2-1}$$

$$m = f'(1) = 2(1)e^{1^2-1} = 2e^{1-1} = 2e^0 = 2$$

$$y - 1 = 2(x - 1)$$

$$y - 1 = 2x - 2$$

$$y = 2x - 1$$

**Use implicit differentiation to find the derivative of each function**

$$21) x^2 + 3xy + y^2 = 8$$

$$x^2 + 3xy + y^2 = 8$$

$$\frac{d}{dx}(x^2 + 3xy + y^2) = \frac{d}{dx}(8)$$

$$2x + 3y + 3xy' + 2yy' = 0$$

$$3xy' + 2yy' = -2x - 3y$$

$$(3x + 2y)y' = -2x - 3y$$

$$y' = \frac{-2x - 3y}{3x + 2y}$$

22)

$$5 \sin(x) \cos(y) = 5$$

$$\frac{d}{dx} 5 \sin x \cos y = \frac{d}{dx} 5$$

$$-5 \cos x \cos y + 5 \sin x \sin y y' = 0$$

$$5 \sin x \sin y y' = 5 \cos x \cos y$$

$$y' = \frac{5 \cos x \cos y}{5 \sin x \sin y}$$

$$y' = \cot x \cot y$$

23)

$$x^3 + 4x^2 y^2 = 1 + ye^{x^2}$$

$$\frac{d}{dx} (x^3 + 4x^2 y^2) = \frac{d}{dx} (1 + ye^{x^2})$$

$$3x^2 + 8xy^2 + 4x^2 2yy' = 0 + y'e^{x^2} + 2xe^{x^2} y$$

$$4x^2 2yy' - y'e^{x^2} = -3x^2 + 2xe^{x^2} y - 8xy^2$$

$$(8x^2 y - e^{x^2}) y' = -3x^2 + 2xe^{x^2} y - 8xy^2$$

$$y' = \frac{-3x^2 + 2xe^{x^2} y - 8xy^2}{8x^2 y - e^{x^2}}$$

**Use implicit differentiation to find the equation of a tangent line to the curve at the given point.**

$$24) x^2 + 2xy + y^2 = 4; (1,1)$$

$$x^2 + 2xy + y^2 = 4$$

$$\frac{d}{dx}(x^2 + 2xy + y^2) = \frac{d}{dx}4$$

$$2x + 2y + 2xy' + 2yy' = 0$$

$$2xy' + 2yy' = -2x - 2y$$

$$(2x + 2y)y' = -2x - 2y$$

$$y' = \frac{-2x - 2y}{2x + 2y}$$

$$y' = \frac{-(2x + 2y)}{2x + 2y}$$

$$y' = -1$$

$$m = -1 \text{ when } x = 1 \text{ and } y = 1$$

$$y - 1 = -1(x - 1)$$

$$y - 1 = -1x + 1$$

$$y = -x + 2$$