**Math 151 Study List 2**
**Solutions**

**Evaluate each limit.**

1) $\lim_{x\to 2} x^3 - 2x^2$

$\lim_{x\to 2} x^3 - 2x^2 = 2^3 - 2(3)^2 = 8 - 18 = -10$

2) $\lim_{x\to 3} \frac{1}{x-3}$

$\lim_{x\to 3} \frac{1}{x-3} = \frac{1}{3-3} = \frac{1}{0}$ Limit does not exist.

3) $\lim_{x\to 6} \frac{x-6}{x^2-36}$

$\lim_{x\to 6} \frac{x-6}{x^2-36} = \lim_{x\to 6} \frac{x-6}{(x-6)(x+6)} = \lim_{x\to 6} \frac{1}{x+6} = \frac{1}{6+6} = \frac{1}{12}$

4) $\lim_{x\to 1} \frac{x^3-1}{x-1}$

$\lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 0} \frac{x^3-1}{x-1} = \lim_{x\to 0} \frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 0}(x^2+x+1) = 1^2+1+1 = 1+1+1 = 3$

5) Omit

6) $\lim_{x\to 0} \frac{\sqrt{5-x}-\sqrt{5}}{x}$

$\lim_{x\to 0} \frac{\sqrt{5-x}-\sqrt{5}}{x} = \lim_{x\to 0} \cdot \frac{(\sqrt{5-x}-\sqrt{5})}{x} \cdot \frac{(\sqrt{5-x}+\sqrt{5})}{\sqrt{5-x}+\sqrt{5}} = \lim_{x\to 0} \frac{(\sqrt{5-x})^2 + \sqrt{5}\sqrt{5-x} - \sqrt{5}\sqrt{5-5} - 5}{x(\sqrt{5-x}-\sqrt{5})}$

$\lim_{x\to 0} \frac{5-x-5}{x(\sqrt{5-x}-\sqrt{5})} = \lim_{x\to 0} \frac{-x}{x(\sqrt{5-x}-\sqrt{5})} = \lim_{x\to 0} \frac{-1}{\sqrt{5-x}-\sqrt{5}} = \frac{-1}{\sqrt{5}+\sqrt{5}} = \frac{-1}{2\sqrt{5}}$

**In Exercises 5-10 find the derivative of the function using the limit definition of a derivative.**

7) $f(x) = 4x - 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4(x+h) - 2 - (4x-2)}{h} = \lim_{h \to 0} \frac{4x + 4h - 2 - 4x + 2}{h} = \lim_{h \to 0} \frac{4h}{h} = 4$$

8) $f(x) = x^2 - 4x + 7$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) + 7 - (x^2 - 4x + 7)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h} = \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h} = \lim_{h \to 0} \frac{h(2x + h - 4)}{h}$$

$$= \lim_{h \to 0} (2x + h - 4) = 2x - 4$$

9) $f(x) = x^3 - 4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - 4 - (x^3 - 4)}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2 h + 3xh^2 + h^3 - 4 - x^3 + 4}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 h + 3xh^2 + h^3}{h} = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h \to 0} 3x^2 = 3xh + h^2 = 3x^2$$

10) $f(x) = \dfrac{1}{x+6}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h+6} - \frac{1}{x+6}}{h} = \lim_{h \to 0} \frac{\frac{x+6}{(x+h+6)(x+6)} - \frac{x+h+6}{(x+h+6)(x+6)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+6-x-h-6}{(x+h+6)(x+6)}}{h} = \lim_{h \to 0} \frac{\frac{-h}{(x+h+6)(x+6)}}{h} = \lim_{h \to 0} \frac{-1}{(x+h+6)(x+6)} = -\frac{1}{(x+6)^2}$$

11) $f(x) = \sqrt{3x - 1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{3(x+h) - 1} - \sqrt{3x-1}}{h} \cdot \frac{\sqrt{3(x+h) - 2} + \sqrt{3x-1}}{\sqrt{3(x+h) - 1} + \sqrt{3x-1}}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{3(x+h) - 1}\right)^2 + \sqrt{3(x+h)}\sqrt{3x-1} - \sqrt{3(x+h) - 1}\sqrt{3x-1} - (3x-1)}{h\left(\sqrt{3(x+h) - 1} + \sqrt{3x-1}\right)} = \lim_{h \to 0} \frac{3x + 3h - 1 - 3x + 1}{h\left(\sqrt{3x + 3h - 1} + \sqrt{3x-1}\right)}$$

$$= \lim_{h \to 0} \frac{3h}{h\left(\sqrt{3x + 3h - 1} + \sqrt{3x-1}\right)} = \lim_{h \to 0} \frac{3}{\sqrt{3x + 3h - 1} + \sqrt{3x-1}} = \frac{3}{2\sqrt{3x-1}}$$

**In Exercises 12-13, find the equation of the line that is tangent to the given function at the provided point.**

12) $f(x)=\frac{1}{2}x+1;\ (0,1)$

$$f(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{2}(x+h)+1-\frac{1}{2}-\left(\frac{1}{2}x+1\right)}{h}=\lim_{h\to 0}\frac{\frac{1}{2}x+\frac{1}{2}h+1-\frac{1}{2}x-1}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2}h}{h}=\frac{1}{2}$$

$$m=f'(0)=\frac{1}{2}$$

$$y-1=\frac{1}{2}(x-1)$$

$$y-1=\frac{1}{2}x$$

$$y=\frac{1}{2}x+1$$

13) $f(x)=x^2-2x+3;\ (1,2)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-2(x+h)+3-\left(x^2-2x+3\right)}{h}$$

$$=\lim_{h\to 0}\frac{x^2+2xh+h^2-2x-2h+3-x^2+2x-3}{h}=\lim_{h\to 0}\frac{2xh+h^2-2h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h-2)}{h}=\lim_{h\to 0}2x+h-2=2x-2$$

$$m=f'(1)=2(1)-2=0$$

$$y-2=0(x-1)$$

$$y-2=0$$

$$y=2$$

**Limits of Infinity**

14) Evaluate $\lim_{x\to\infty}\frac{2x^2+8x+6}{5x^2-4x+9}$

$$\lim_{x\to\infty}\frac{2x^2+8x+6}{5x^2-4x+9}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^2}+\frac{8x}{x^2}+\frac{6}{x^2}}{\frac{5x^2}{x^2}-\frac{4x}{x^2}+\frac{9}{x^2}}=\lim_{h\to\infty}\frac{2+\frac{8}{x}+\frac{6}{x^2}}{5-\frac{4}{x}+\frac{9}{x^2}}=\frac{2+0+0}{5-0+0}=\frac{2}{5}$$

15) Evaluate $\lim_{x\to\infty}\frac{x^2-3x+7}{2x^3+4x^2+2x}$

$$\lim_{x\to\infty}\frac{x^2-3x+7}{2x^3+4x^2+2x}=\lim_{x\to\infty}\frac{\frac{x^2}{x^3}-\frac{3x}{x^3}+\frac{7}{x^3}}{\frac{2x^3}{x^3}+\frac{4x^2}{x^3}+\frac{2x}{x^3}}=\lim_{h\to\infty}\frac{\frac{1}{x}-\frac{3}{x^2}+\frac{7}{x^3}}{2+\frac{4}{x}+\frac{2}{x^2}}=\frac{0-0+0}{2+0+0}=\frac{0}{2}=0$$

16) Evaluate $\lim_{x\to\infty}\frac{x+4}{x-5}$

$$\lim_{x\to\infty}\frac{x+4}{x-5}=\lim_{x\to\infty}\frac{\frac{x}{x}+\frac{4}{x}}{\frac{x}{x}-\frac{5}{x}}=\lim_{h\to\infty}\frac{1+\frac{4}{x}}{1-\frac{5}{x}}=\frac{1+0}{1-0}=1$$

17) Evaluate $\lim_{x\to\infty}\left(\sqrt{16x^2-x}-4x\right)$

$$\lim_{x\to\infty}\left(\sqrt{16x^2-x}-4x\right)\frac{\left(\sqrt{16x^2-x}+4x\right)}{\left(\sqrt{16x^2-x}+4x\right)}=\lim_{h\to 0}\frac{\left(\sqrt{16x^2-x}\right)^2+\sqrt{16x^2-x}\sqrt{4x}-\sqrt{4x}\sqrt{16x^2-x}-16x^2}{\sqrt{16x^2-x}+4x}$$

$$=\lim_{x\to\infty}\frac{16x^2-x-16x^2}{\sqrt{16x^2-x}+4x}=\lim_{x\to\infty}\frac{-x}{\sqrt{16x^2-x}+4x}=\lim_{x\to\infty}\frac{\frac{-x}{x}}{\sqrt{\frac{16x^2}{x^2}-\frac{x}{x^2}}+\frac{4x}{x}}=\lim_{x\to 0}\frac{-1}{\sqrt{16-\frac{1}{x}}+4}$$

$$=\frac{-1}{\sqrt{16}+4}=\frac{-1}{4+4}=-\frac{1}{8}$$

**Give the intervals where the function is continuous**

18) $f(x)=3x^3+2x+4$

$(-\infty,\infty)$

19) $g(x)=\dfrac{3}{x-2}$

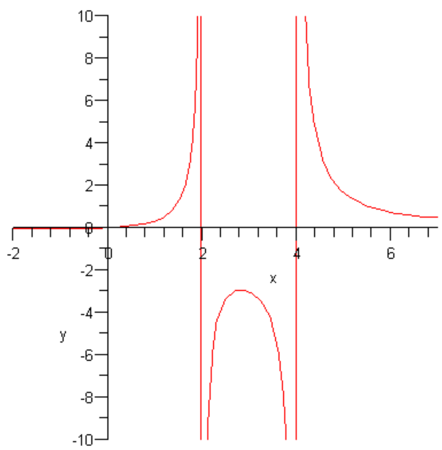
$(-\infty,2)\cup(2,\infty)$

20) $h(x)=\dfrac{4}{x^2-4}$

$h(x)=\dfrac{4}{x^2-4}=\dfrac{2}{(x-2)(x+2)}$

$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

21)

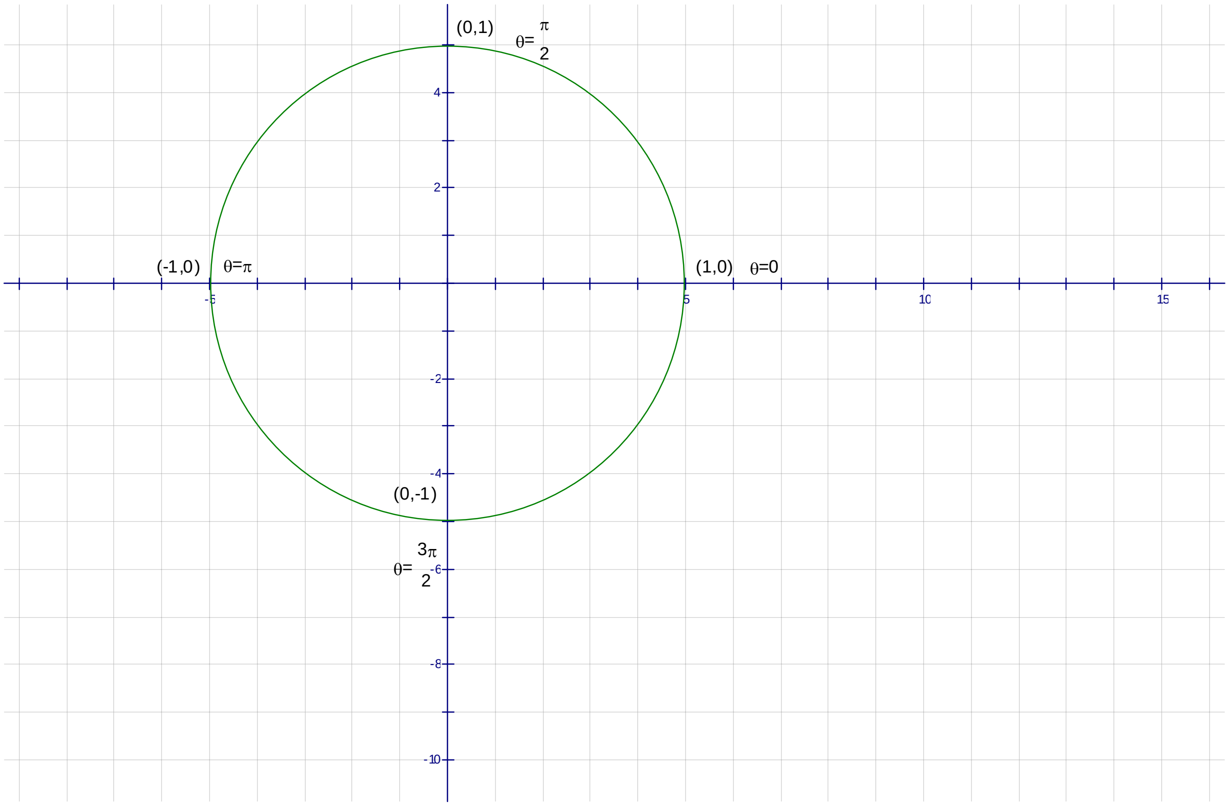



$(-\infty,2)\cup(2,4)\cup(4,\infty)$

## Parametric Equations

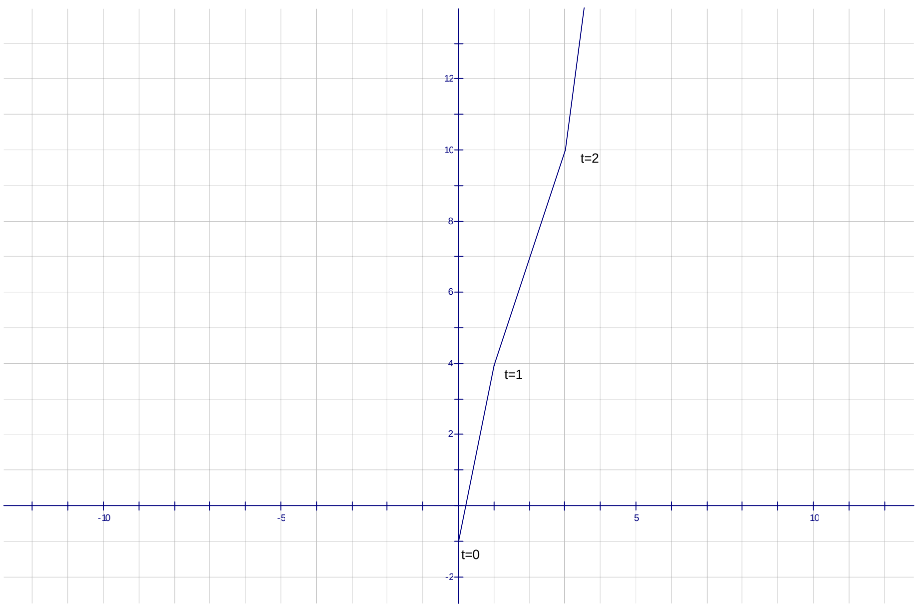
22) Graph the following parametric equations.

$x = \cos\theta,\ y = \sin\theta,\ 0 \le \theta \le 2\pi$



23) Graph the following parametric equations
$x = 2t - 1, y = t^2 + 3t,\ 0 \le t \le 5$

| $t$ | $x = 2t - 1$ | $y = t^2 + 3t$ |
|---|---|---|
| 0 | $x = 2(0) - 1 = -1$ | $y = 0^2 + 3(0) = 0$ |
| 1 | $x = 2(1) - 1 = 1$ | $y = 1^2 + 3(1) = 4$ |
| 2 | $x = 2(2) - 1 = 3$ | $y = 2^2 + 3(2) = 10$ |
| 3 | $x = 2(3) - 1 = 5$ | $y = 3^2 + 3(3) = 18$ |




**Book problems (Solutions should be in back of the textbook)**