

Math 151
Section 3.5
Chain Rule

The Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example 1

$$y = (3x^2 + 5)^5$$

$$\text{Let } y = u^5 \text{ where } u = 3x^2 + 5 \Rightarrow \frac{du}{dx} = 6x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = 5u^4 \frac{du}{dx}$$

$$y' = 5(3x^2 + 5)^4 (6x)$$

$$y' = (30x)(3x^2 + 5)^4$$

Example 2

$$y = (x^2 + 3x)^7$$

$$\text{Let } y = u^7 \text{ where } u = x^2 + 3x \Rightarrow \frac{du}{dx} = 2x + 3$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$y' = 7u^{7-1} \frac{du}{dx}$$

$$y' = 7u^6 \frac{du}{dx}$$

$$y' = 7(x^2 + 3x)^6 (2x + 3)$$

$$y' = (14x + 21)(x^2 + 3x)^6$$

Example 3

Find the derivative of $y = \sqrt{x^2 - 6x}$

$$y = \sqrt{x^2 - 6x}$$

$$y = (x^2 - 6x)^{\frac{1}{2}}$$

$$\text{Let } y = u^{\frac{1}{2}} \text{ where } u = x^2 - 6x \Rightarrow \frac{du}{dx} = 2x - 6$$

$$y' = \frac{1}{2} u^{\frac{1}{2}-1} \frac{du}{dx}$$

$$y' = \frac{1}{2} u^{-\frac{1}{2}} \frac{du}{dx}$$

$$y' = \frac{1}{2} (x^2 - 6x)^{-\frac{1}{2}} (2x - 6)$$

$$y' = \frac{2x - 6}{2(x^2 - 6x)^{\frac{1}{2}}}$$

$$y' = \frac{2x - 6}{\sqrt{x^2 - 6x}}$$

Example 4

Find the derivative of $f(x) = \sqrt{x^3 - 3x}$

$$y = \sqrt{x^3 - 3x}$$

$$y = (x^3 - 3x)^{\frac{1}{2}}$$

$$\text{Let } y = u^{\frac{1}{2}} \text{ where } u = x^3 - 3x \Rightarrow \frac{du}{dx} = 3x^2 - 3$$

$$y' = \frac{1}{2} u^{\frac{1}{2}-1} \frac{du}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \frac{du}{dx} = \frac{1}{2} (x^3 - 3x)^{-\frac{1}{2}} (3x^2 - 3) = \frac{3x^2 - 3}{2(x^3 - 3x)^{\frac{1}{2}}} = \frac{3x^2 - 3}{2\sqrt{x^3 - 3x}}$$

Example 5

$$y = \sqrt[3]{x^2 + 4}$$

$$y = (x^2 + 4)^{\frac{1}{3}}$$

$$\text{Let } y = u^{\frac{1}{3}} \text{ where } u = x^2 + 4 \Rightarrow \frac{du}{dx} = 2x$$

$$y' = \frac{1}{3} u^{\frac{1}{3}-1} \frac{du}{dx}$$

$$y' = \frac{1}{3} u^{-\frac{2}{3}} \frac{du}{dx}$$

$$y' = \frac{\frac{du}{dx}}{3u^{\frac{2}{3}}}$$

$$y' = \frac{2x}{3(x^2 + 4)^{\frac{2}{3}}}$$

$$y' = \frac{2x}{3\sqrt[3]{x^2 + 4}}$$

Example 6

Find the derivative of $y = \cos(5x)$

$$y = \cos(5x)$$

$$u = 5x$$

$$\frac{du}{dx} = 5$$

$$y = \cos u$$

$$y' = \cos u \cdot \frac{du}{dx}$$

$$y' = 5 \cos(5x)$$

Example 7

Find the $y = e^{3x^2}$

$$y = e^{3x^2}$$

$$u = 3x^2$$

$$\frac{du}{dx} = 6x$$

$$y' = e^u \frac{du}{dx}$$

$$y' = e^{3x^2} (6x)$$

$$y' = 6xe^{3x^2}$$

Example 8

Find the $y = e^{x^2-6x}$

$$y = e^{x^2-6x}$$

$$u = x^2 - 6x$$

$$\frac{du}{dx} = 2x - 6$$

$$y' = e^u \frac{du}{dx}$$

$$y' = e^{x^2-6x} (2x - 6)$$

$$y' = (2x - 6)e^{x^2-6x}$$

Example 9

Find the derivative of $y = e^{3x} \cos(6x)$

$$y = e^{3x} \cos(6x)$$

$$y' = \frac{d}{dx} e^{3x} \cos(6x) + \frac{d}{dx} (\cos(6x)) e^{3x}$$

$$y' = 3e^{3x} \cos(6x) - 6e^{3x} \sin(6x)$$

$$y' = 3e^{3x} (\cos(6x) - 2 \sin(6x))$$

Example 10

Find the derivative of $y = 4xe^{x^2}$

$$y = 4xe^{x^2}$$

$$y' = \frac{d}{dx} (4x)e^{x^2} + \frac{d}{dx} e^{x^2} (4x)$$

$$y' = 4e^{x^2} + (4x)(2x)e^{x^2}$$

$$y' = 4e^{x^2} + 8x^2 e^{x^2}$$

$$y' = (4 + 8x^2)e^{x^2}$$

Example 11

Find the slope of tangent to the function $y = (x^2 + 2x)^3$ at the point (1,27)

$$y = (x^2 + 2x)^3$$

$$\text{Let } y = u^3 \text{ where } u = x^2 + 2x \Rightarrow \frac{du}{dx} = 2x + 2$$

$$y' = 3u^{3-1} \frac{du}{dx}$$

$$y' = 3u^2 \frac{du}{dx}$$

$$y' = 3(x^2 + 2x)^2 (2x + 2)$$

$$y' = (6x + 6)(x^2 + 2x)^2$$

Slope

$$y' = (6(1) + 6)(1^2 + 2(1))^2 = (6 + 6)(1 + 2)^2 = 12(3)^2 = 12(9) = 108$$

$$m = 108$$