**Section 3.3**
**Rates of Change**

**Position, Velocity, and Time**

**Definitions**

**Position** $s(t)$

**Velocity** $v(t) = s'(t) = \frac{ds}{dt}$

**Acceleration** $a(t) = s''(t) = \frac{d^2 s}{dt^2}$

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**Example 1**

A particle moves according to the law of motion $s = t^3 - 6t^2 + 9t$

a) Find the velocity at time $t$?

$$s = t^3 - 6t^2 + 9t$$
$$v(t) = s'(t) = 3t^2 - 12t + 9$$

b) What is the velocity after 4 seconds?

$$v(3) = 3(4)^2 - 12(4) + 9 = 48 - 48 + 12 = 12\ \frac{m}{s}$$

c) When is the particle at rest?

$$v(t) = 3t^2 - 12t + 9$$
$$3(t^2 - 4t + 3) = 0$$
$$3(t-1)(t-3) = 0$$
$$t - 1 = 0 \ or\ t - 3 = 0$$
$$t = 1 \ or\ t = 3$$

d) When is the particle at moving forward?

| Interval | $(0,1)$ | $(1,3)$ | $(3,\infty)$ |
|---|---|---|---|
| Test Value | $x = .5$ | $x = 2$ | $x = 6$ |
| Sign of $v(t)$ | + | - | + |
| Objects | Moving forward | Moving backwards | Moving forward |

$$v(1) = 3(.5)^2 - 12(.5) + 9 = .75 - 6 + 9 = 3.75$$
$$v(2) = 3(2)^2 - 12(2) + 12 = 12 - 24 + 9 = -3$$
$$v(4) = 3(4)^2 - 12(4) + 9 = 48 - 48 + 9 = 9$$

e) Find the total distance traveled during the first 5 seconds?

Find the distance between 0 seconds and 1 second [0,1]

$$|s(1) - s(0)| = \left|\left(1^3 - 6(1)^2 + 9(1)\right) - \left(0^3 - 6(0)^2 + 9(0)\right)\right| = |4 - 0| = 4$$

Find the distance between 1 second and 3 seconds

$$|s(3) - s(1)| = \left|\left(3^3 - 6(3)^2 + 9(3)\right) - \left(1^3 - 6(1) + 9(1)\right)\right| = |0 - 4| = 4$$

Find the distance between 3 seconds and 5 seconds

$$|s(5) - s(3)| = \left|\left(5^3 - 6(5)^2 + 9(5)\right) - \left(3^3 - 6(3) + 9(3)\right)\right| = |20 - 0| = 20$$

Total Distance = 4 + 4 + 20 = 28

f) Find the acceleration time $t$ and 3 seconds?

$$s = t^3 - 6t^2 + 9t$$
$$v(t) = s'(t) = 3t^2 - 12t + 9$$
$$a(t) = s''(t) = 6t - 12$$

$$a(2) = 6(3) - 12 = 18 - 12 = 6\ \frac{m}{s^2}$$

g) Sketch a graph of the distance, velocity, and acceleration verses time on the same graph.

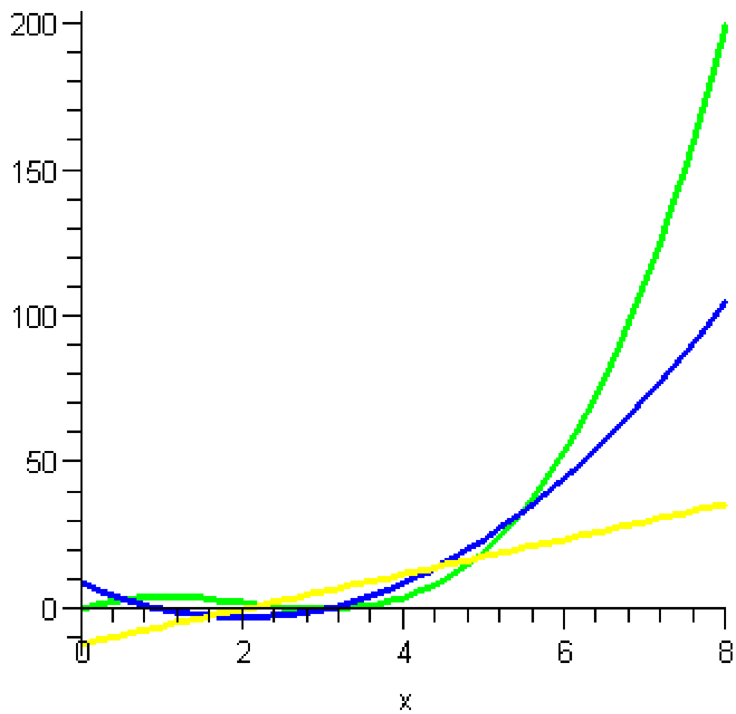
*Using maple to graph the functions, we get the following graphs.*
*Just type in these maple commands and maple will graph the three functions.*

> $f := x \to x^3 - 6\,x^2 + 9\,x$

> $g := x \to 3x^2 - 12\,x + 9$

> $h := x \to 6\,x - 12$

plot( [$f$(x), g(x), h(x) ], x = 0 ..8, color= [green, blue, yellow]
, thickness= [2, 2, 2] )




In the graph the green line is the distance, the blue line is the velocity, and the yellow line is acceleration.

**Example 2**

The position function of a particle is given by $s = t^3 - 3t^2 + 7t$

a) When is the particle reach a velocity of 4 meters per second?

$$s = t^3 - 3t^2 + 7t$$
$$v(t) = 3t^2 - 6t + 7$$

$$3t^2 - 6t + 7 = 4$$
$$3t^2 - 6t + 3 = 0$$
$$3(t^2 - 2t + 1) = 0$$
$$3(t-1)(t-1) = 0$$
$$t - 1 = 0 \Rightarrow t = 0$$

b) When is the acceleration 0? What is the significance of this value of t?

$$s = t^3 - 3t^2 + 7t$$
$$v(t) = 3t^2 - 6t + 7$$
$$a(t) = 6t - 6$$

$$6t - 6 = 0$$
$$6t = 6$$
$$t = 1$$

The object has a constant velocity

**Example 3**

If a stone vertically upward with a velocity of 14 meters per second on the surface of the moon, then its height is given by $s = 14t - .83t^2$

a) What is the velocity of the stone after 4 seconds?

$$s(t0 = 14t - .83t^2$$
$$v(t) = 14 - 2(.83)t$$
$$v(t) = 14 - 1.66t$$
$$v(4) = 14 - 1.66(4) = 14 - 6.64 = 7.36$$

b) What is the velocity of the stone at a height of 30 meters?

$$v(t) = 14 - .1.66t$$
$$s(t) = 14t - .83t^2$$
$$30 = 14t - .83t^2$$
$$.83t^2 - 14t + 30 = 0$$

$$t = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(.83)(30)}}{2(.83)} = \frac{14 \pm \sqrt{192 - 99.6}}{1.66} = \frac{14 \pm \sqrt{92.4}}{1.66} = \frac{14 \pm 9.6}{1.66}$$
$$t = \frac{23.6}{1.66} = 14.2 \ or \ t = \frac{4.4}{1.66} = 2.7$$