Section 3.3
Rates of Change

Position, Velocity, and Time

Definitions

Position $s(t)$

Velocity $v(t) = s'(t) = \frac{ds}{dt}$

Acceleration $a(t) = s''(t) = \frac{d^2s}{dt^2}$

Example 1

A particle moves according to the law of motion $s = t^3 - 6t^2 + 9t$

a) Find the velocity at time t?

$$s = t^3 - 6t^2 + 9t$$
$$v(t) = s'(t) = 3t^2 - 12t + 9$$

b) What is the velocity after 4 seconds?

$$v(3) = 3(4)^2 - 12(4) + 9 = 48 - 48 + 12 = 12\ \frac{m}{s}$$

c) When is the particle at rest?

$$s = t^3 - 6t^2 + 9t$$
$$0 = t^3 - 6t^2 + 9t$$
$$0 = t(t^2 - 6t + 9)$$
$$0 = t(t-3)(t-3)$$
$$t = 0 \quad or \quad t - 3 = 0$$
$$t = 0 \qquad\qquad t = 3$$

d) When is the particle at moving forward?

$v(t) = 3t^2 - 12t + 9$
$3(t^2 - 4t + 3) = 0$
$3(t - 1)(t - 3) = 0$
$t - 1 = 0 \text{ or } t - 3 = 0$
$t = 1 \text{ or } t = 3$

Interval	$(0,1)$	$(1,3)$	$(3,\infty)$
Test Value	$x = .5$	$x = 2$	$x = 6$
Sign of $v(t)$	+	-	+
Objects	Moving forward	Moving backwards	Moving forward

$v(1) = 3(.5)^2 - 12(.5) + 9 = .75 - 6 + 9 = 3.75$
$v(2) = 3(2)^2 - 12(2) + 12 = 12 - 24 + 9 = -3$
$v(4) = 3(4)^2 - 12(4) + 9 = 48 - 48 + 9 = 9$

e) Find the total distance traveled during the first 5 seconds?

Find the distance between 0 seconds and 1 second [0,1]

$|s(1) - s(0)| = |(1^3 - 6(1)^2 + 9(1)) - (0^3 - 6(0)^2 + 9(0))| = |4 - 0| = 4$

Find the distance between 1 second and 3 seconds

$|s(3) - s(1)| = |(3^3 - 6(3)^2 + 9(3)) - (1^3 - 6(1) + 9(1))| = |0 - 4| = 4$

Find the distance between 3 seconds and 5 seconds

$|s(5) - s(3)| = |(5^3 - 6(5)^2 + 9(5)) - (3^3 - 6(3) + 9(3))| = |20 - 0| = 20$

Total Distance = 4 + 4 + 20 = 28

f) Find the acceleration time t and 3 seconds?

$$s = t^3 - 6t^2 + 9t$$
$$v(t) = s'(t) = 3t^2 - 12t + 9$$
$$a(t) = s''(t) = 6t - 12$$

$$a(2) = 6(3) - 12 = 18 - 12 = 6\ \frac{m}{s^2}$$

Example 2

The position function of a particle is given by $s = t^3 - 3t^2 + 7t$

a) When is the particle reach a velocity of 4 meters per second?

$$s = t^3 - 3t^2 + 7t$$
$$v(t) = 3t^2 - 6t + 7$$

$$3t^2 - 6t + 7 = 4$$
$$3t^2 - 6t + 3 = 0$$
$$3\left(t^2 - 2t + 1\right) = 0$$
$$3\left(t - 1\right)\left(t - 1\right) = 0$$
$$t - 1 = 0 \Rightarrow t = 0$$

b) When is the acceleration 0? What is the significance of this value of t?

$$s = t^3 - 3t^2 + 7t$$
$$v(t) = 3t^2 - 6t + 7$$
$$a(t) = 6t - 6$$

$$6t - 6 = 0$$
$$6t = 6$$
$$t = 1$$

The object has a constant velocity

Example 3

If a stone vertically upward with a velocity of 14 meters per second on the surface of the moon, then its height is given by $s = 14t - .83t^2$

a) What is the velocity of the stone after 4 seconds?

$$s(t0 = 14t - .83t^2$$
$$v(t) = 14 - 2(.83)t$$
$$v(t) = 14 - 1.66t$$
$$v(4) = 14 - 1.66(4) = 14 - 6.64 = 7.36$$

b) What is the velocity of the stone at a height of 30 meters?

$$v(t) = 14 - .1.66t$$
$$s(t) = 14t - .83t^2$$
$$30 = 14t - .83t^2$$
$$.83t^2 - 14t + 30 = 0$$

$$t = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(.83)(30)}}{2(.83)} = \frac{14 \pm \sqrt{192 - 99.6}}{1.66} = \frac{14 \pm \sqrt{92.4}}{1.66} = \frac{14 \pm 9.6}{1.66}$$
$$t = \frac{23.6}{1.66} = 14.2 \ or \ t = \frac{4.4}{1.66} = 2.7$$