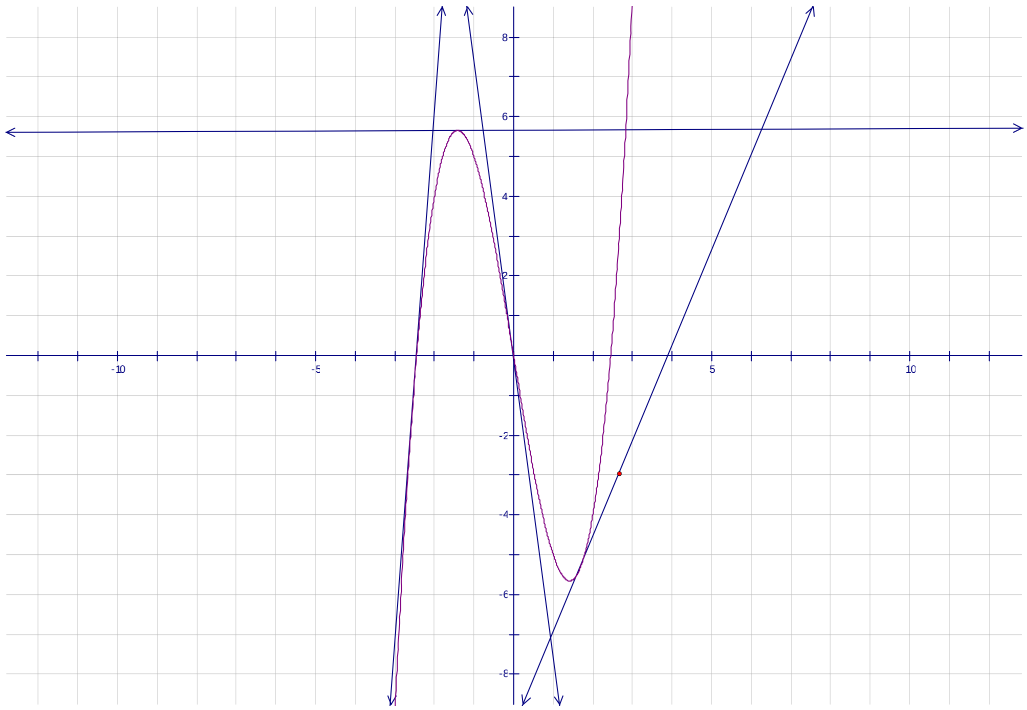
Section 2.8

Limits and Derivatives

The definition of differentiable

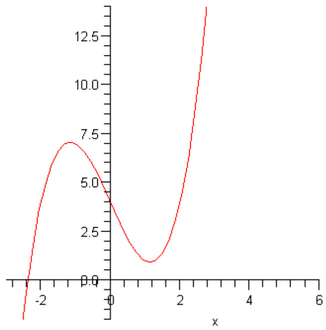
A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a,b) if it is differentiable at every number in the interval.



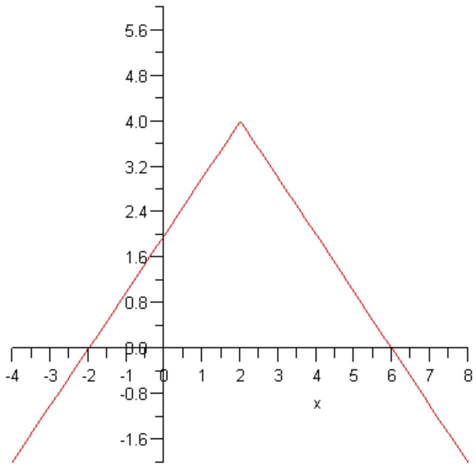
Theorem

If a function is differentiable at a, then it is continuous at a

Example where f is differentiable at $x = 2$



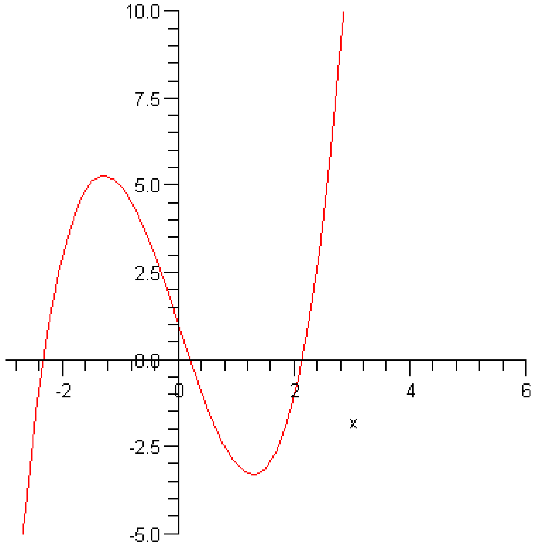
Example where f is not differentiable at $x = 2$



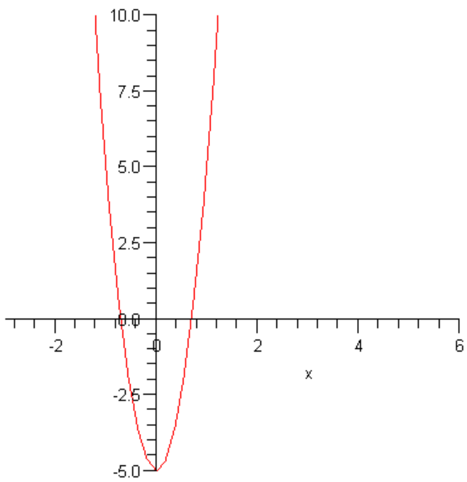
Example 1

Given the graph of the function f , sketch a graph of the derivative of f .

Graph of $f(x)$



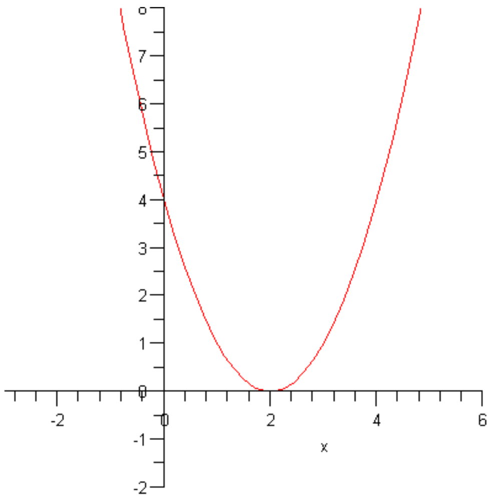
Graph $f'(x)$



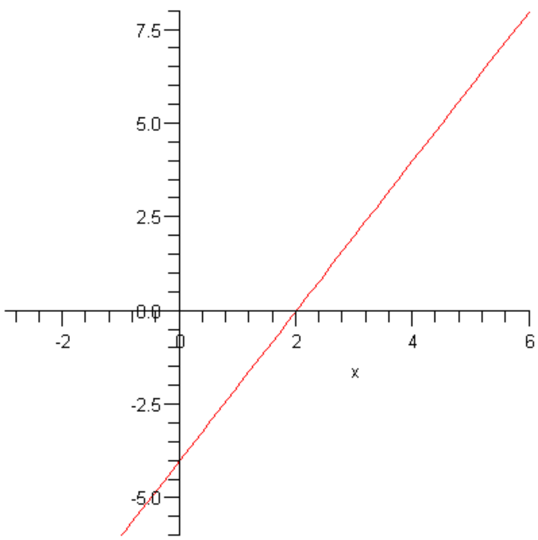
Example 2

Given the graph of the function f , sketch a graph of the derivative of f .

Graph of $f(x)$



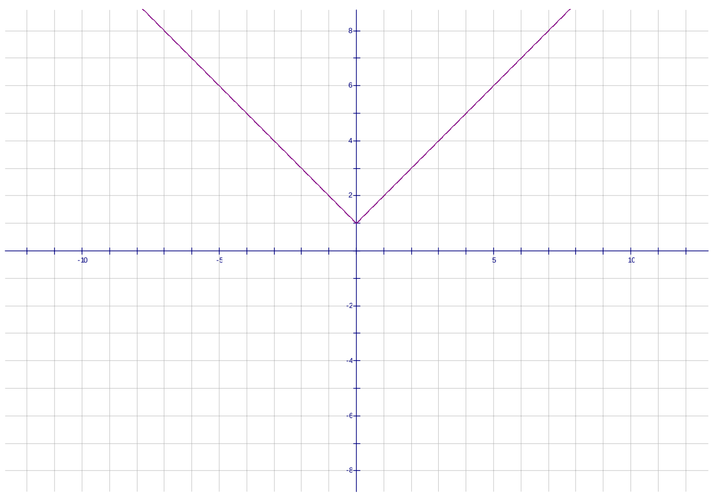
Graph $f'(x)$



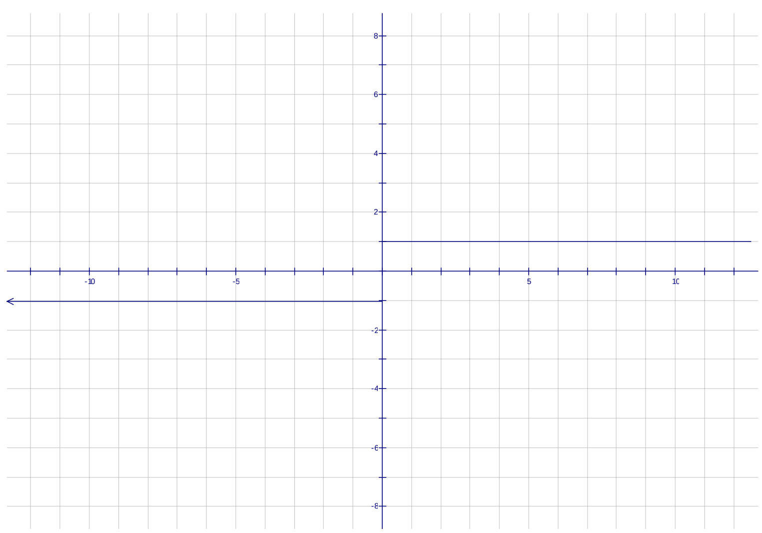
Example 3

Given the graph of the function f , sketch a graph of the derivative of f .

Graph of $f(x)$



Graph $f'(x)$



Example 4

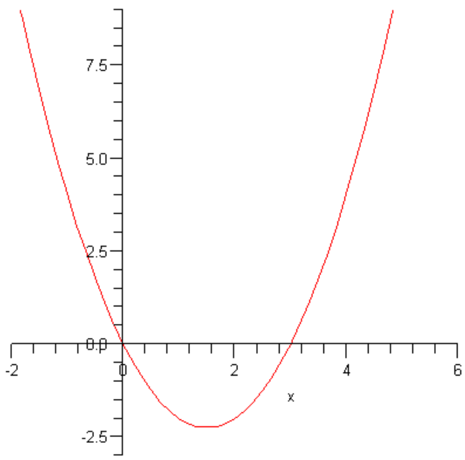
Use the limit definition of a derivative to find the derivative of the function then state the domain of the function and the derivative.

$$f(x) = x^2 - 3x$$

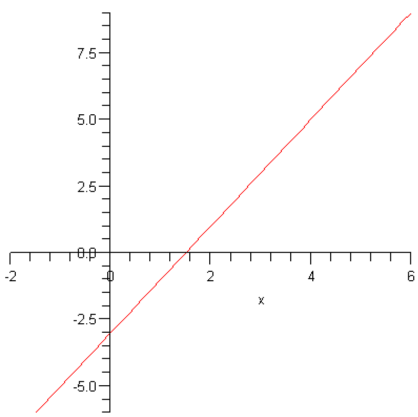
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2xh - 3h}{h} = \lim_{h \to 0} \frac{h(h + 2x - 3)}{h} = \lim_{h \to 0} h + 2x - 3 = 2x - 3$$

Domain of $f(x) = x^2 - 3x$ **:** $(-\infty, \infty)$



Domain of $f(x) = 2x - 3$: $(-\infty, \infty)$



Example 6

Use the limit definition of a derivative to find the derivative of the function then state the domain of the function and the derivative.

$f(x) = 3x - 4$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 4 - (3x-4)}{h} = \lim_{h \to 0} \frac{3x + 3h - 4 - 3x + 4}{h} = \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3$$

Domain of $f(x) = 3x - 4$: $(-\infty, \infty)$

Range of $f(x) = 3$: $(-\infty, \infty)$