

Section 2.4

Continuity

Definition: A function f is continuous at a number a if

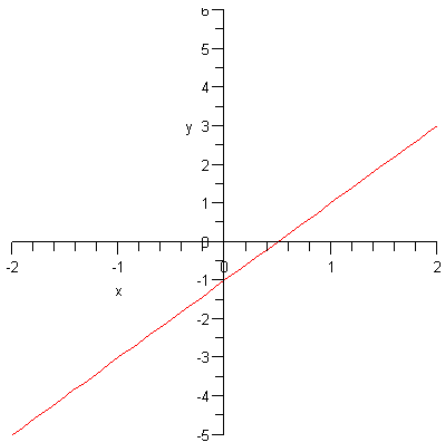
$$\lim_{x \rightarrow a} f(x) = f(a)$$

Notice that the definition implicitly requires three things if f is continuous at a number a :

- 1) $f(a)$ is defined
 - 2) $\lim_{x \rightarrow a} f(x)$ exist
 - 3) $\lim_{x \rightarrow a} f(x) = f(a)$
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Example 1

Discuss the continuity of $f(x)$ at $x = 1$

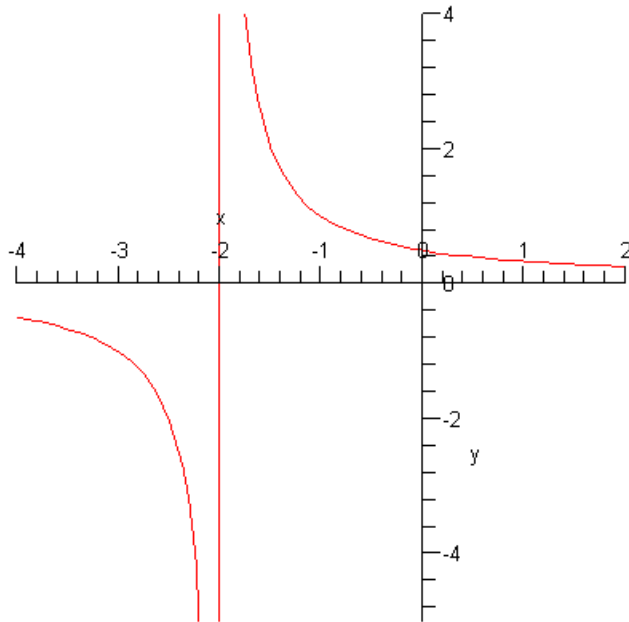


- 1) $f(1) = 1$, so is defined at $x = 1$
- 2) $\lim_{x \rightarrow 1} f(x) = 1$, so the limit exist at $x = 1$
- 3) $\lim_{x \rightarrow 1} f(x) = f(1) = 1$

Therefore, the function is continuous at $x = 1$.

Example 2

Discuss the continuity of $f(x)$ at $x = -2$



1) $f(-2)$ is undefined

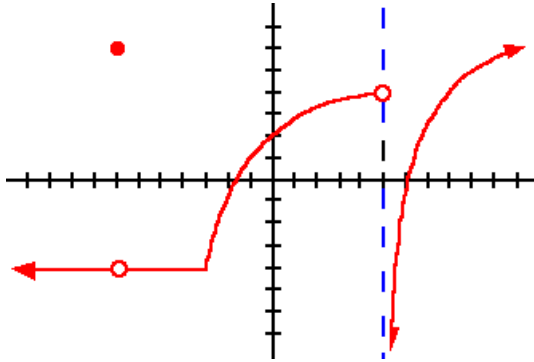
2) $\lim_{x \rightarrow -2} f(x)$ does not exist

3) $\lim_{x \rightarrow -2} f(x) \neq f(1)$

Therefore, the function is not continuous at $x = -2$.

Example 3

Discuss the continuity of $f(x)$ at $x = -6$ and $x = 5$



Look $x = -6$

1) $f(-6) = 6$ is defined

2) $\lim_{x \rightarrow -6} f(x)$ exist

3) $\lim_{x \rightarrow -6} f(x) \neq f(-6)$

Therefore, the function is not continuous at $x = -6$.

Look $x = 5$

1) $f(5)$ is undefined

2) $\lim_{x \rightarrow 5} f(x)$ exist

3) $\lim_{x \rightarrow 5} f(x) \neq f(5)$

Therefore, the function is not continuous at $x = 5$.

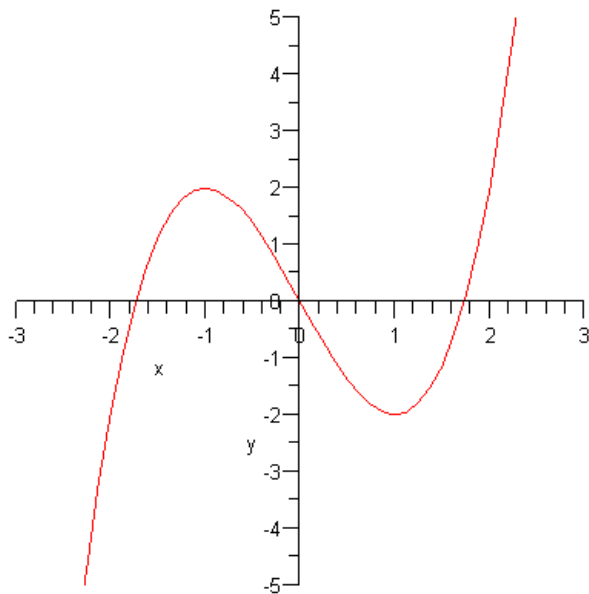
The graph of a continuous function is piecewise smooth.

Continuous Intervals

A function is continuous on an interval if the function is continuous at every point of the interval.

Example 4

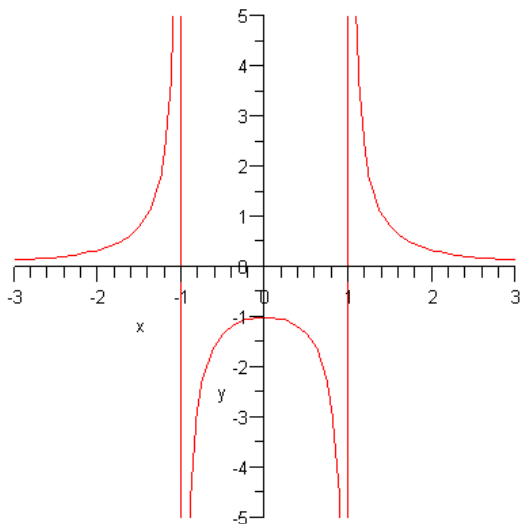
Discuss the continuity of the given function.



The graph is piecewise smooth, since it has breaks, holes, or asymptotes. Therefore, the function is continuous on $(-\infty, \infty)$

Example 5

Discuss the continuity of the function.



The function is discontinuous at $x = -1$ and $x = 1$
The function is continuous on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Theorem: Polynomial functions are continuous everywhere.

Theorem: Rational functions are continuous on their domain

Example 6

Give the intervals where the function is continuous.

$$f(x) = x^3 + 2x^2$$

Continuous on $(-\infty, \infty)$

Example 7

Give the intervals where the function is continuous.

$$f(x) = \frac{1}{x^2 - 4}$$

$$f(x) = \frac{1}{x^2 - 4} = \frac{1}{(x-2)(x+2)} \Rightarrow f \text{ is undefined at } x = -2 \text{ and } x = 2$$

Continuous on $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Example 8

Give the intervals where the function is continuous.

$$f(x) = \frac{x-3}{x^2 + x - 12}$$

$$f(x) = \frac{x-3}{x^2 + x - 12} = \frac{x-3}{(x-3)(x+4)} \Rightarrow f \text{ is undefined at } x = 3 \text{ and } x = -4$$

Continuous on $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

Example 9

Give the intervals where the function is continuous.

$$f(x) = x - \cos x$$

Continuous on $(-\infty, \infty)$

Example 10: Give the intervals where the function is continuous.

$$f(x) = x + e^{x+3}$$

Solution: Continuous on $(-\infty, \infty)$

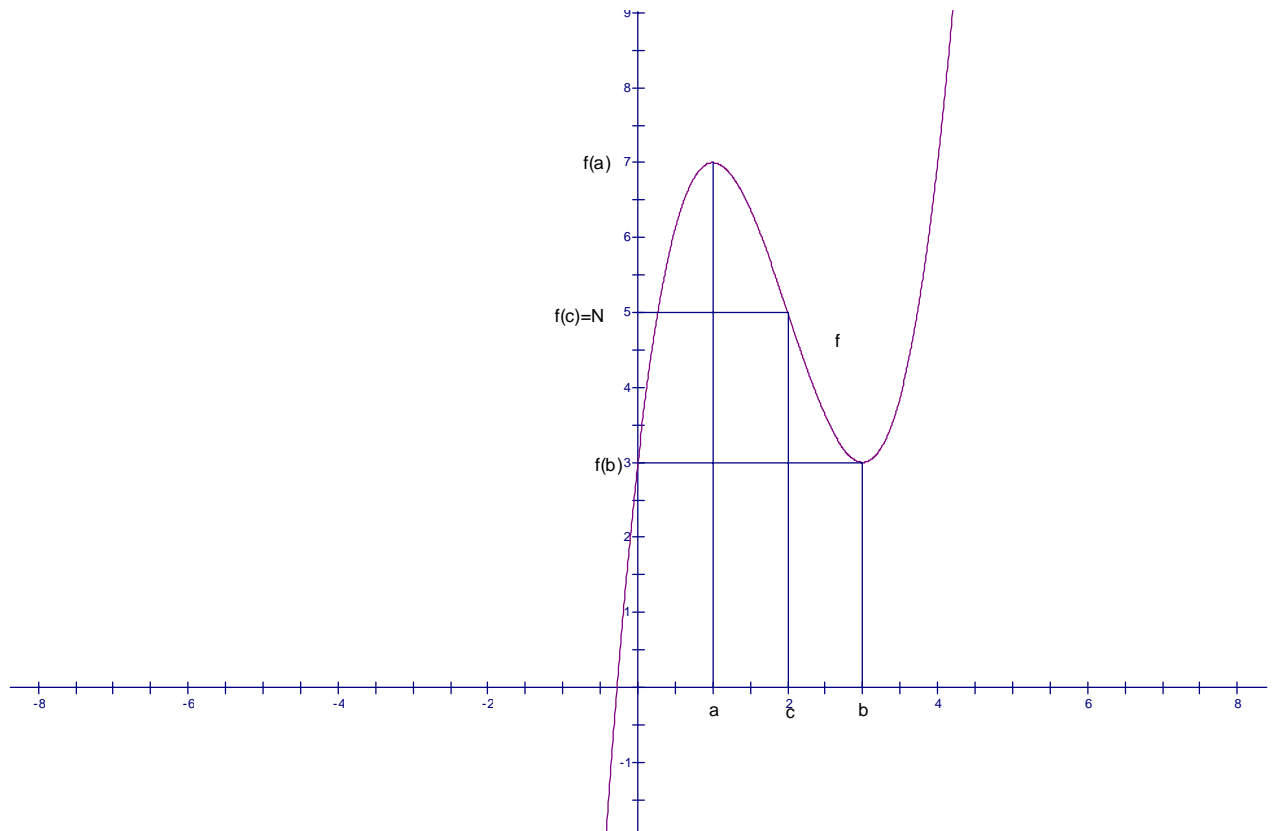
Theorem

If b is a real number and f and g are continuous at $x = c$, then the following are also continuous at c .

- 1) Scalar multiple: bf
 - 2) Sum: $f + g$
 - 3) Difference: $f - g$
 - 4) Product: fg
 - 5) Quotient: $\frac{f}{g}$, if $g(c) \neq 0$
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Intermediate Value Theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let N be any real number between $f(a)$ and $f(b)$. Then there exist a number c in (a, b) such that $f(c) = N$



Exercises

Give the intervals where the function is continuous

1) $f(x) = x^2 - x + 1$

2) $f(x) = \frac{1}{x^2 + 1}$

3) $f(x) = x + \sin x$

4) $f(x) = \cos\left(\frac{\pi x}{2}\right)$

5) $f(x) = \frac{1}{x+1}$

6) $f(x) = \frac{x-3}{x^2-9}$

7) $f(x) = \frac{x+2}{x^2-3x-10}$

8) $f(x) = e^{x-1}$

9) $f(x) = e^x + 1$

10) $f(x) = \ln(x-4)$