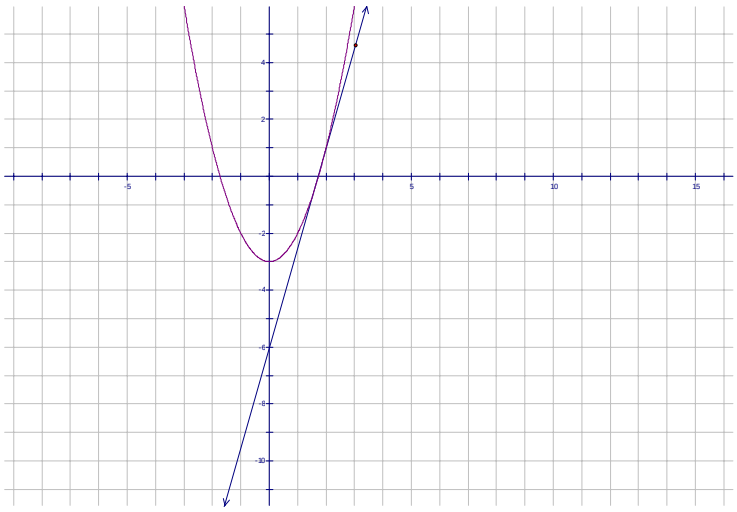
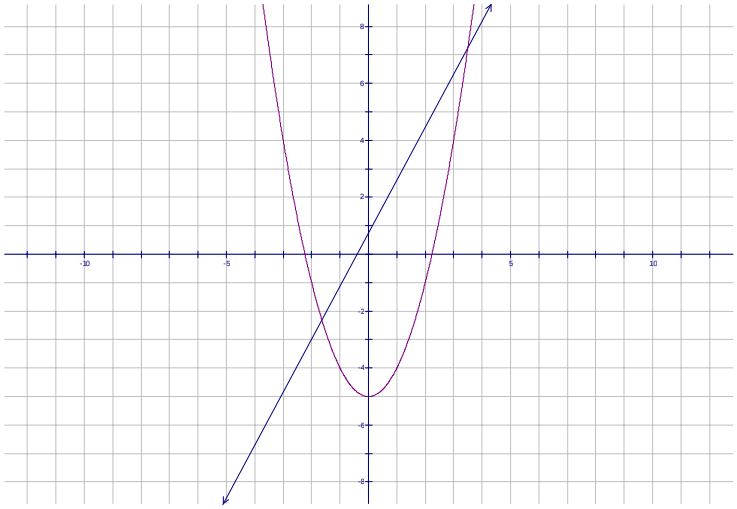
Section 2.1

Secant Lines and Tangent Lines





Example 1 (Page 97 #2)

A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent lines represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
Heartbeats	2530	2661	2806	2948	3080

Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t.

a) $t = 36 \text{ and } t = 42$

$$m = \frac{2948 - 2530}{42 - 36} = \frac{418}{6} = 69.7$$

b) $t = 38 \text{ and } t = 42$

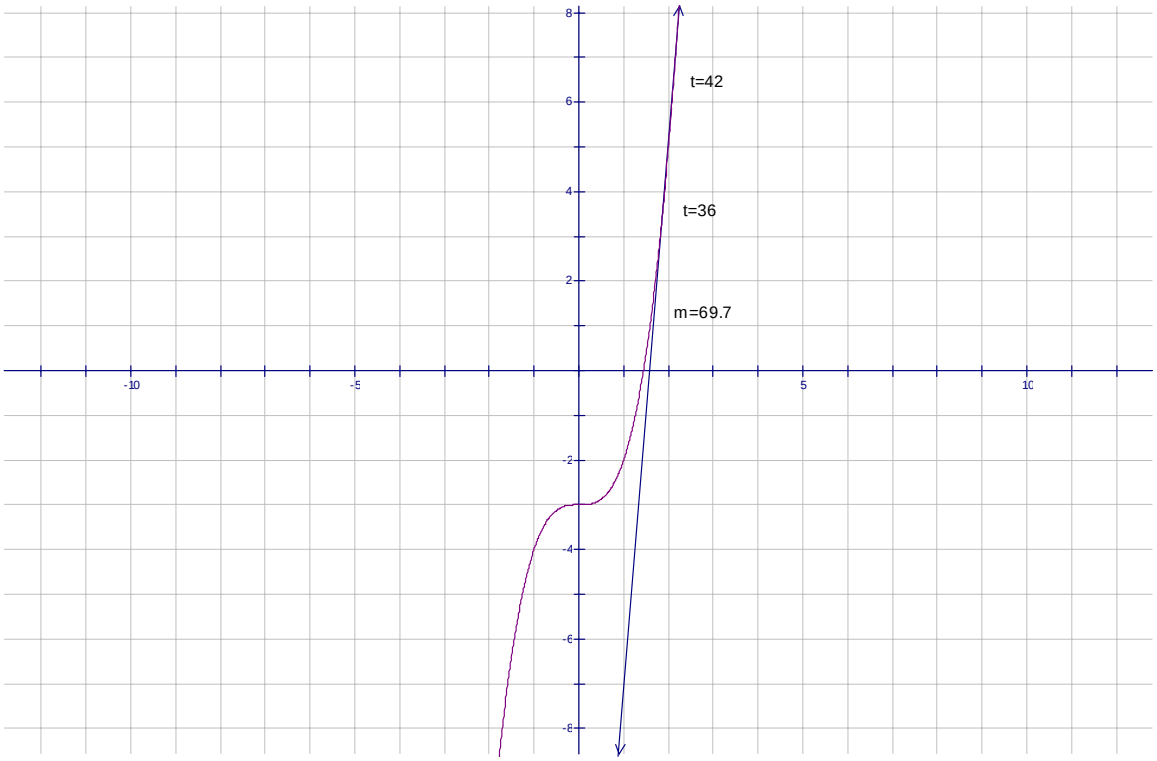
$$m = \frac{2948 - 2661}{42 - 38} = \frac{287}{4} = 71.75$$

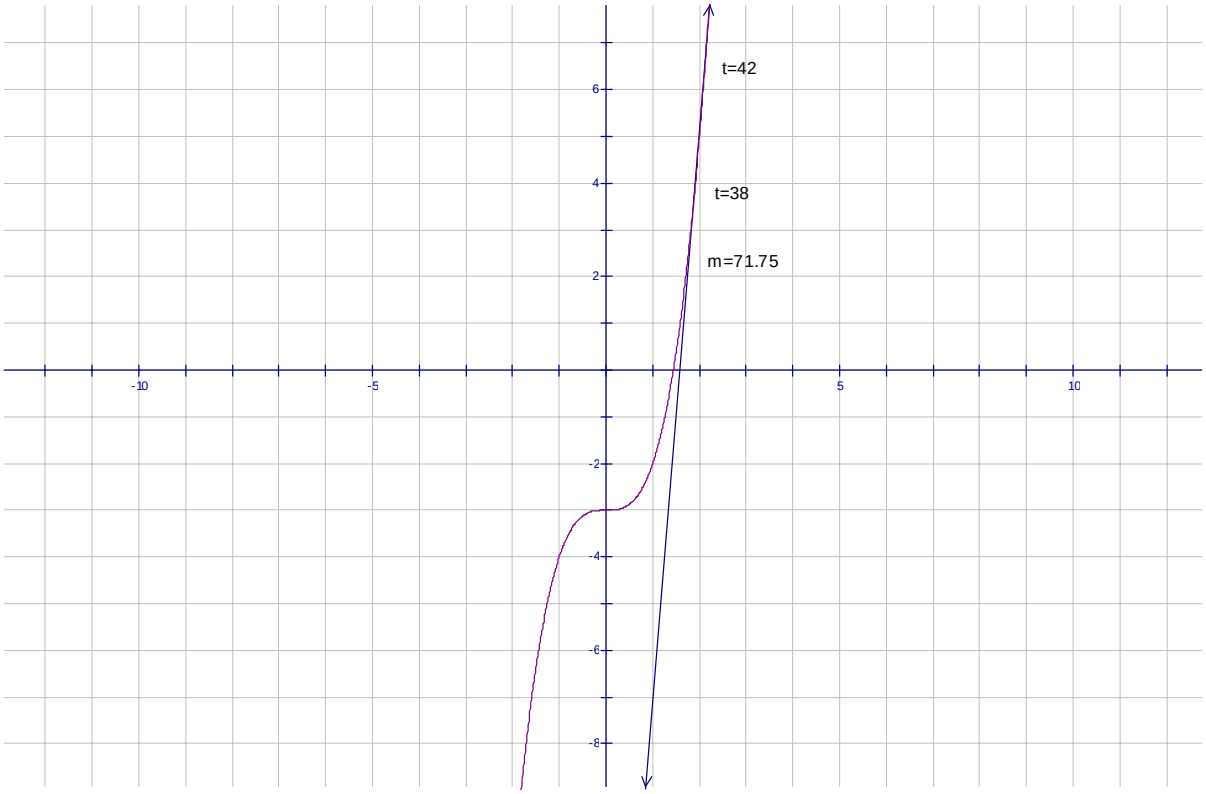
c) $t = 40 \text{ and } t = 42$

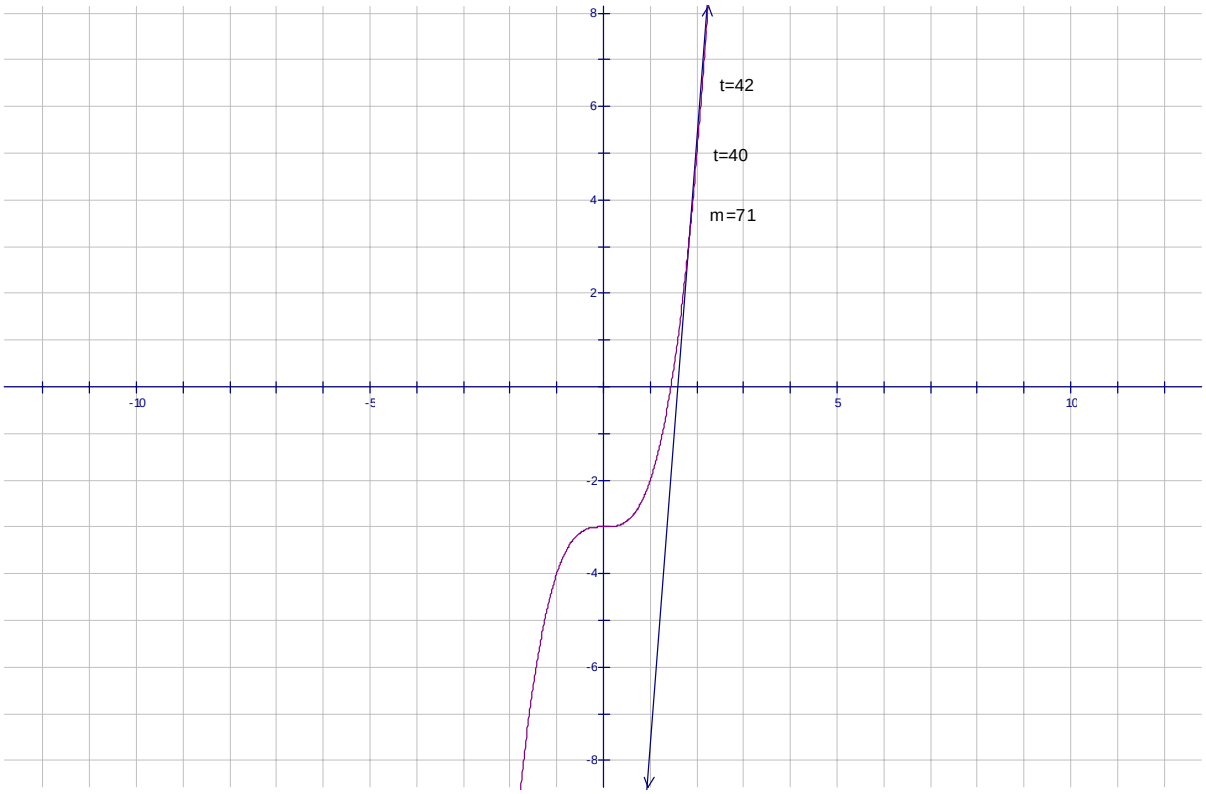
$$m = \frac{2948 - 2806}{42 - 40} = \frac{142}{2} = 71$$

d) $t = 42 \text{ and } t = 44$

$$m = \frac{3080 - 2948}{44 - 42} = \frac{132}{2} = 66$$


t=42
t=36
m=69.7


t=42
t=38
m=71.75



Example 2

The point P(3,1) lies on the curve $y = \sqrt{x-2}$

a) If Q is the point $\left(x, \sqrt{x-2}\right)$, use your calculator the slope of the secant line PQ for the following values of x:

i) 2.5

$y = \sqrt{x-2} = \sqrt{2.5-2} = \sqrt{.5} = .707107 \Rightarrow (2.5,.707107)$

$$m = \frac{1-.707107}{3-2.5} = \frac{.292893}{.5} = 585786$$

ii) 2.9

$y = \sqrt{x-2} = \sqrt{2.9-2} = \sqrt{.9} = .948683 \Rightarrow (2.9,.948683)$

$$m = \frac{1-.948683}{3-2.9} = \frac{.051317}{.1} = .51317$$

iii) 2.99

$$y = \sqrt{x-2} = \sqrt{2.99-2} = \sqrt{.99} = .994987 \Rightarrow (2.9,.994987)$$

$$m = \frac{1-.994987}{3-2.99} = \frac{.5013}{.001} = .5013$$

iv) 2.999

$$y = \sqrt{x-2} = \sqrt{2.999-2} = \sqrt{.999} = .999500 \Rightarrow (2.9,.999500)$$

$$m = \frac{1-.9995}{3-2.999} = .5$$

v) 3.5

$$y = \sqrt{x-2} = \sqrt{3.5-2} = \sqrt{1.5} = 1.22474 \Rightarrow (3.5,1.22474)$$

$$m = \frac{1.22474-1}{3.5-3} = \frac{.22474}{.5} = .44948$$

vi) 3.1

$$y = \sqrt{x-2} = \sqrt{3.1-2} = \sqrt{1.1} = 1.048809 \Rightarrow (3.1,1.048809)$$

$$m = \frac{1.048809}{3.1-3} = \frac{.048809}{.1} = .48809$$

vii) 3.01

$$y = \sqrt{x-2} = \sqrt{3.01-2} = \sqrt{1.01} = 1.004988 \Rightarrow (3.01,1.004988)$$

$$m = \frac{1.004988-1}{3.01-3} = .4988$$

viii) 3.001

$$y = \sqrt{x-2} = \sqrt{3.001-2} = \sqrt{1.001} = 1.000500 \Rightarrow (3.001,1.000500)$$

$$m = \frac{1.000500}{3.001} = .5$$

b) Using the values of part a), guess the value of the slope of the tangent line to the curve of P(3,1)

$$m = \frac{1}{2}$$

c) Using the slope from part b), find the equation of the line passing P(3,1)

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{2}(x - 3)$$
$$y - 1 = \frac{1}{2}x - \frac{3}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2}$$

Example 3

If an arrow is shot upward on the moon with a velocity of $55\ \frac{m}{s}$ its velocity is given by the model $h = 55t - .83t^2$

a) Find the average velocity over the given time intervals

i) $[1,2]$

$t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17$

$t = 2: h = 55(2) - .83(2)^2 = 110 - 3.32 = 106.68$

$$m = \frac{106.68 - 54.17}{2 - 1} = \frac{52.51}{1} = 52.51$$

ii) $[1,1.5]$

$t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17$

$t = 1.5: h = 55(1.5) - .83(1.5)^2 = 82.5 - 1.87 = 80.63$

$$m = \frac{80.63 - 54.17}{1.5 - 1} = \frac{26.46}{.5} = 52.92$$

iii) $[1,1.1]$

$t = 1: h = 55(1) - .83(1)^2 = 55 - .83 = 54.17$

$t = 1.1: h = 55(1.1) - .83(1.1)^2 = 59.50$

$$m = \frac{59.50 - 54.17}{1.1 - 1} = \frac{5.33}{.1} = 53.3$$